

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/1.2.4.2-d-x-
 $\int \frac{dx}{x^m(a-x^q+b-x^n+c-x^{-2-n-q}-x^p)}$

Nasser M. Abbasi

September 20, 2021

Compiled on September 20, 2021 at 4:40pm

Contents

1	Introduction	3
2	detailed summary tables of results	19
3	Listing of integrals	63
4	Appendix	717

Chapter 1

Introduction

Local contents

1.1	Listing of CAS systems tested	4
1.2	Results	5
1.3	Performance	9
1.4	list of integrals that has no closed form antiderivative	11
1.5	list of integrals solved by CAS but has no known antiderivative	12
1.6	list of integrals solved by CAS but failed verification	13
1.7	Timing	13
1.8	Verification	14
1.9	Important notes about some of the results	14
1.10	Design of the test system	16

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [130]. This is test number [36].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (130)	0.00 (0)
Mathematica	99.23 (129)	0.77 (1)
Fricas	99.23 (129)	0.77 (1)
Maple	98.46 (128)	1.54 (2)
Giac	81.54 (106)	18.46 (24)
Mupad	55.38 (72)	44.62 (58)
IntegrateAlgebraic	53.08 (69)	46.92 (61)
Sympy	40.77 (53)	% 59.23 (77)
Maxima	18.46 (24)	81.54 (106)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

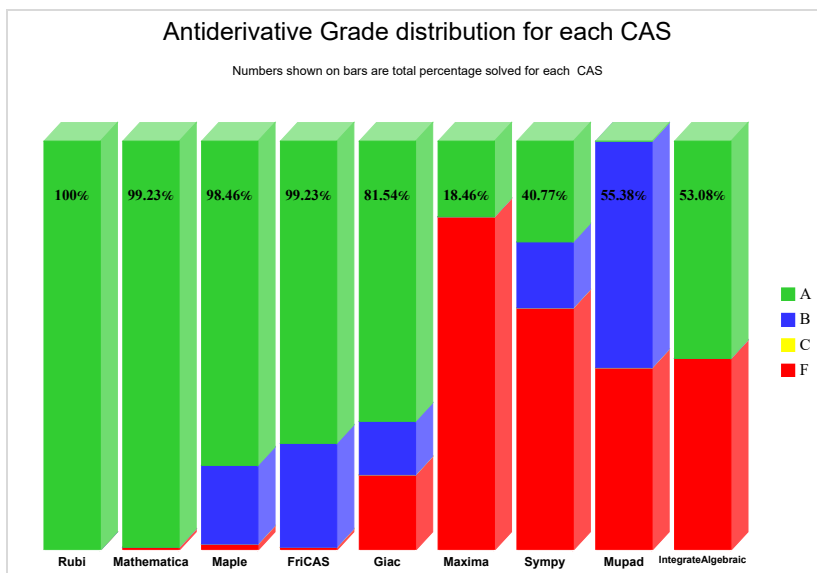
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

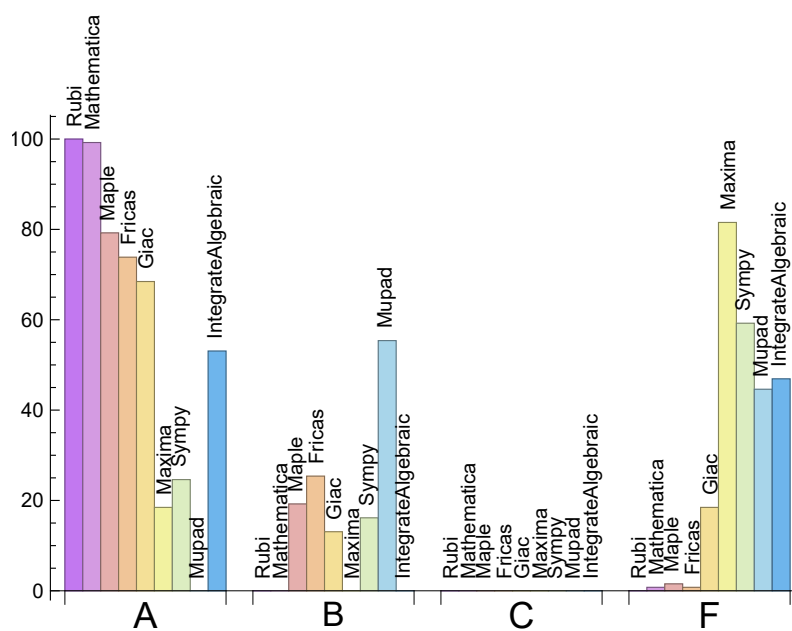
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	99.23	0.00	0.00	0.77
Maple	79.23	19.23	0.00	1.54
Fricas	73.85	25.38	0.00	0.77
Giac	68.46	13.08	0.00	18.46
IntegrateAlgebraic	53.08	0.00	0.00	46.92
Sympy	24.62	16.15	0.00	59.23
Maxima	18.46	0.00	0.00	81.54
Mupad	N/A	55.38	0.00	44.62

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	1	0.00 %	0.00 %	100.00 %
IntegrateAlgebraic	61	100.00 %	0.00 %	0.00 %
Giac	24	29.17 %	29.17 %	41.67 %
Maxima	106	78.30 %	0.00 %	21.70 %
Sympy	77	68.83 %	31.17 %	0.00 %
Mupad	58	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

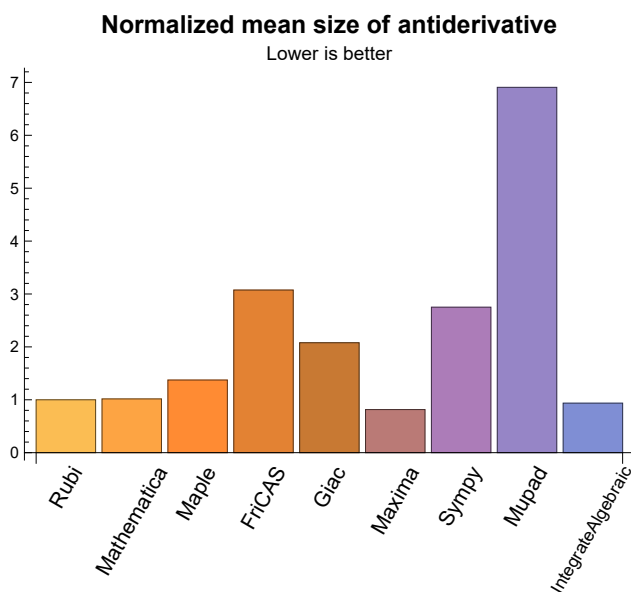
1.3 Performance

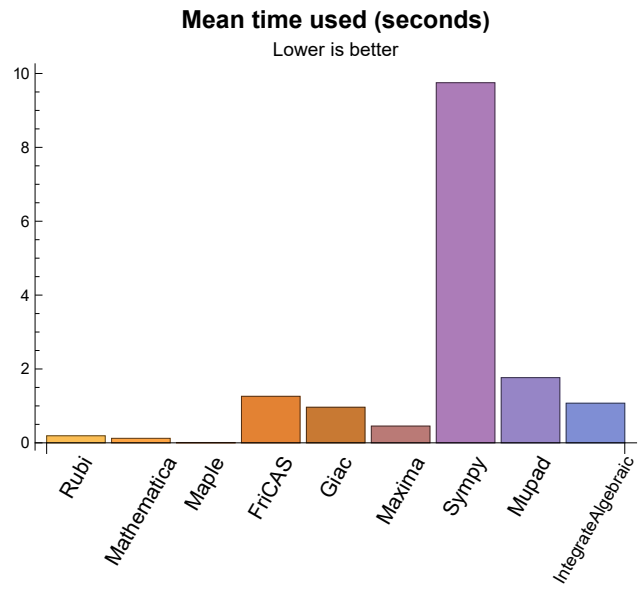
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	124.34	1.00	89.00	1.00
Mathematica	0.12	115.71	1.02	93.00	0.98
Maple	0.01	195.08	1.37	122.50	1.31
Maxima	0.45	32.71	0.81	28.50	0.81
Fricas	1.26	470.89	3.07	272.00	2.74
Sympy	9.75	233.81	2.75	148.00	1.25
Giac	0.96	362.54	2.08	76.00	1.09
Mupad	1.77	1266.07	6.91	172.00	2.35
IntegrateAlgebraic	1.07	114.42	0.94	100.00	0.89

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by

failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

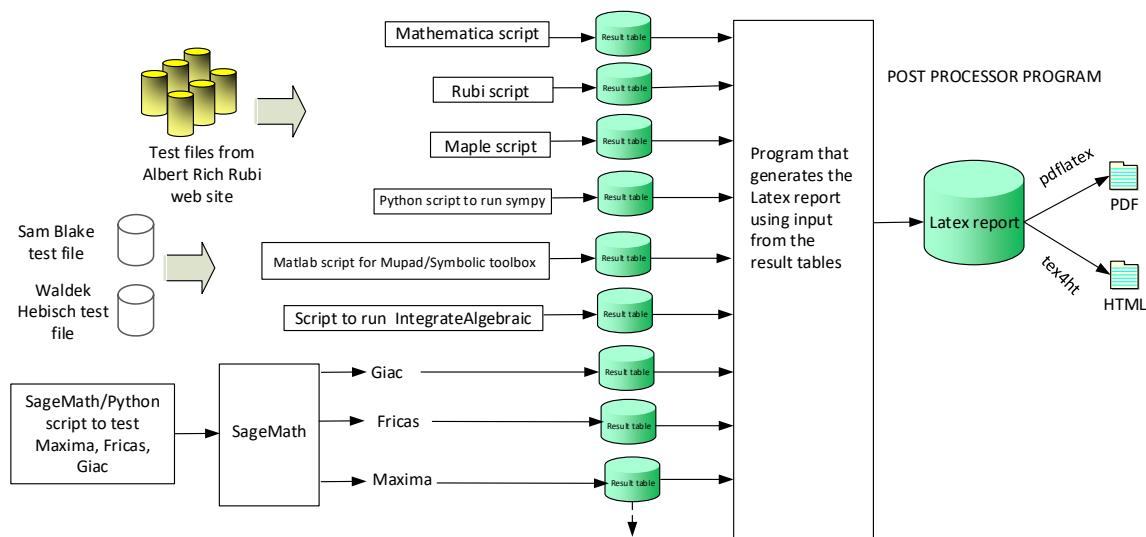
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. integer. 1 if result was verified or 0 if not verified.
The following field present only in Rubi and Mathematica Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

Local contents

2.1	List of integrals sorted by grade for each CAS	20
2.2	Detailed conclusion table per each integral for all CAS systems	24
2.3	Detailed conclusion table specific for Rubi results	57

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1	Rubi	21
2.1.2	Mathematica	21
2.1.3	Maple	21
2.1.4	Maxima	22
2.1.5	FriCAS	22
2.1.6	Sympy	22
2.1.7	Giac	23
2.1.8	Mupad	23
2.1.9	IntegrateAlgebraic	23

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129 }

B grade: { }

C grade: { }

F grade: { 130 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 83, 84, 85, 86, 87, 88, 91, 93, 95, 96, 97, 104, 105, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129 }

B grade: { 19, 24, 25, 26, 27, 28, 35, 37, 47, 48, 65, 72, 79, 81, 89, 90, 92, 94, 98, 99, 100, 101, 102, 103, 110 }

C grade: { }

F grade: { 112, 130 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 127 }

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129 }

B grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 110 }

C grade: { }

F grade: { 130 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 92, 94, 96, 98 }

B grade: { 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97 }

C grade: { }

F grade: { 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 90, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 50, 51, 52, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 104, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129 }

B grade: { 65, 72, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102, 106, 107, 110 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 60, 61, 62, 63, 64, 105, 111, 112, 130 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 119, 123, 127 }

C grade: { }

F grade: { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 124, 125, 126, 128, 129, 130 }

2.1.9 IntegrateAlgebraic

A grade: { 4, 5, 9, 10, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 76, 77, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 130 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.007	0.002	0.000	0.440	0.886	0.062	0.542	0.032	0.000
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.007	0.002	0.000	0.436	0.977	0.062	0.454	0.032	0.000
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.004	0.000	0.002	0.429	0.837	0.061	0.536	0.032	0.001

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	21
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.84
time (sec)	N/A	0.006	0.001	0.001	0.445	0.531	0.064	0.361	0.031	0.025

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	15	16	16	20
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80	1.00
time (sec)	N/A	0.006	0.001	0.001	0.426	0.728	0.061	0.490	0.025	0.016

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	46	48	46	45	0
N.S.	1	1.00	1.00	0.83	0.81	0.85	0.89	0.85	0.83	0.00
time (sec)	N/A	0.054	0.008	0.000	0.427	0.902	0.075	0.576	0.034	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	46	49	46	45	0
N.S.	1	1.00	1.00	0.83	0.81	0.85	0.91	0.85	0.83	0.00
time (sec)	N/A	0.029	0.007	0.001	0.436	0.774	0.075	0.484	0.024	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	48	46	48	46	45	0
N.S.	1	1.00	1.00	0.83	0.89	0.85	0.89	0.85	0.83	0.00
time (sec)	N/A	0.027	0.006	0.000	0.447	0.877	0.072	0.396	0.022	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	44	49	46	45	58
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83	1.07
time (sec)	N/A	0.032	0.007	0.002	0.428	0.757	0.076	0.493	0.025	0.040

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	44	48	46	45	58
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83	1.07
time (sec)	N/A	0.032	0.009	0.002	0.425	0.957	0.078	0.482	0.024	0.036

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	84	132	0	297	381	86	112	0
N.S.	1	1.00	0.94	1.48	0.00	3.34	4.28	0.97	1.26	0.00
time (sec)	N/A	0.094	0.109	0.005	0.000	0.862	0.841	0.453	0.141	0.001

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	73	101	0	235	306	67	172	0
N.S.	1	1.00	1.04	1.44	0.00	3.36	4.37	0.96	2.46	0.00
time (sec)	N/A	0.059	0.063	0.003	0.000	0.805	0.623	0.548	2.033	0.001

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	57	56	0	185	216	55	112	0
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00	0.00
time (sec)	N/A	0.040	0.030	0.003	0.000	0.763	0.326	0.421	0.133	0.001
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	38	35	0	120	124	34	46	0
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35	0.00
time (sec)	N/A	0.026	0.006	0.001	0.000	0.956	0.222	0.497	0.035	0.001
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	62	0	211	564	62	213	0
N.S.	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44	0.00
time (sec)	N/A	0.047	0.065	0.007	0.000	0.915	4.358	0.505	2.298	0.001
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	112	0	269	0	79	339	0
N.S.	1	1.00	0.95	1.38	0.00	3.32	0.00	0.98	4.19	0.00
time (sec)	N/A	0.098	0.081	0.007	0.000	1.038	0.000	0.325	2.504	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	102	150	0	358	0	105	447	0
N.S.	1	1.00	0.98	1.44	0.00	3.44	0.00	1.01	4.30	0.00
time (sec)	N/A	0.146	0.136	0.007	0.000	0.908	0.000	0.508	0.587	0.001

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	131	214	0	445	0	136	524	0
N.S.	1	1.00	0.96	1.56	0.00	3.25	0.00	0.99	3.82	0.00
time (sec)	N/A	0.203	0.104	0.010	0.000	1.120	0.000	0.457	2.596	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	132	352	0	837	842	161	261	0
N.S.	1	1.00	0.88	2.35	0.00	5.58	5.61	1.07	1.74	0.00
time (sec)	N/A	0.159	0.196	0.010	0.000	0.894	1.792	0.567	2.456	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	109	209	0	635	729	125	279	0
N.S.	1	1.00	0.96	1.83	0.00	5.57	6.39	1.10	2.45	0.00
time (sec)	N/A	0.103	0.146	0.009	0.000	0.865	1.366	0.595	2.491	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	81	97	0	387	280	88	135	0
N.S.	1	1.00	1.21	1.45	0.00	5.78	4.18	1.31	2.01	0.00
time (sec)	N/A	0.039	0.091	0.007	0.000	1.043	0.600	0.493	2.129	0.001
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	69	70	0	338	253	76	110	0
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67	0.00
time (sec)	N/A	0.038	0.067	0.003	0.000	0.856	0.564	0.383	2.183	0.001
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	70	68	0	341	265	76	119	0
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80	0.00
time (sec)	N/A	0.035	0.071	0.004	0.000	0.920	0.588	0.459	0.084	0.001
Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	107	237	0	781	0	126	620	0
N.S.	1	1.00	0.99	2.19	0.00	7.23	0.00	1.17	5.74	0.00
time (sec)	N/A	0.148	0.184	0.014	0.000	1.404	0.000	0.505	2.871	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	131	328	0	975	0	171	775	0
N.S.	1	1.00	0.89	2.22	0.00	6.59	0.00	1.16	5.24	0.00
time (sec)	N/A	0.198	0.264	0.015	0.000	1.358	0.000	0.454	2.833	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	175	418	0	1226	0	229	914	0
N.S.	1	1.00	0.87	2.07	0.00	6.07	0.00	1.13	4.52	0.00
time (sec)	N/A	0.250	0.343	0.016	0.000	1.686	0.000	0.441	2.956	0.001

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	218	515	0	1407	0	282	1120	0
N.S.	1	1.00	0.87	2.04	0.00	5.58	0.00	1.12	4.44	0.00
time (sec)	N/A	0.323	0.318	0.017	0.000	2.465	0.000	0.550	3.064	0.001

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	272	619	0	1640	0	347	1260	0
N.S.	1	1.00	0.86	1.95	0.00	5.16	0.00	1.09	3.96	0.00
time (sec)	N/A	0.392	0.378	0.020	0.000	3.175	0.000	0.439	3.143	0.001

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	180	310	0	390	0	283	-1	211
N.S.	1	1.00	0.70	1.21	0.00	1.52	0.00	1.10	-0.00	0.82
time (sec)	N/A	0.588	0.237	0.009	0.000	1.232	0.000	0.873	0.000	0.841
Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	150	265	0	326	0	230	-1	173
N.S.	1	1.00	0.73	1.29	0.00	1.59	0.00	1.12	-0.00	0.84
time (sec)	N/A	0.370	0.161	0.009	0.000	0.676	0.000	0.746	0.000	0.585
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	119	167	0	260	0	166	-1	137
N.S.	1	1.00	0.73	1.02	0.00	1.60	0.00	1.02	-0.01	0.84
time (sec)	N/A	0.058	0.213	0.008	0.000	1.246	0.000	0.881	0.000	0.432
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	100	146	0	220	0	125	-1	116
N.S.	1	1.00	0.84	1.23	0.00	1.85	0.00	1.05	-0.01	0.97
time (sec)	N/A	0.078	0.133	0.005	0.000	1.106	0.000	0.946	0.000	0.116

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	134	126	0	638	0	0	-1	134
N.S.	1	1.00	0.77	0.73	0.00	3.69	0.00	0.00	-0.01	0.77
time (sec)	N/A	0.126	0.095	0.006	0.000	1.409	0.000	0.000	0.000	0.421

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	131	173	0	653	0	0	-1	128
N.S.	1	1.00	0.76	1.00	0.00	3.77	0.00	0.00	-0.01	0.74
time (sec)	N/A	0.124	0.117	0.006	0.000	1.431	0.000	0.000	0.000	0.412

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	112	207	0	226	0	0	-1	100
N.S.	1	1.00	0.98	1.82	0.00	1.98	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.148	0.096	0.006	0.000	1.269	0.000	0.000	0.000	0.535

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	131	234	0	272	0	0	-1	117
N.S.	1	1.00	0.85	1.51	0.00	1.75	0.00	0.00	-0.01	0.75
time (sec)	N/A	0.256	0.123	0.008	0.000	1.333	0.000	0.000	0.000	0.787

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	160	387	0	336	0	0	-1	150
N.S.	1	1.00	0.78	1.89	0.00	1.64	0.00	0.00	-0.00	0.73
time (sec)	N/A	0.386	0.168	0.011	0.000	1.349	0.000	0.000	0.000	1.034
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	422	422	236	649	0	664	0	521	-1	323
N.S.	1	1.00	0.56	1.54	0.00	1.57	0.00	1.23	-0.00	0.77
time (sec)	N/A	1.203	0.384	0.010	0.000	1.383	0.000	1.461	0.000	5.161
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	364	364	197	479	0	558	0	429	-1	266
N.S.	1	1.00	0.54	1.32	0.00	1.53	0.00	1.18	-0.00	0.73
time (sec)	N/A	1.039	0.246	0.010	0.000	1.331	0.000	1.351	0.000	4.082
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	180	431	0	474	0	365	-1	220
N.S.	1	1.00	0.62	1.50	0.00	1.65	0.00	1.27	-0.00	0.76
time (sec)	N/A	0.519	0.219	0.009	0.000	1.174	0.000	0.994	0.000	3.416

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	163	289	0	384	0	284	-1	175
N.S.	1	1.00	0.82	1.46	0.00	1.94	0.00	1.43	-0.01	0.88
time (sec)	N/A	0.178	0.173	0.007	0.000	1.205	0.000	0.925	0.000	2.639

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	132	265	0	320	0	232	-1	143
N.S.	1	1.00	0.80	1.61	0.00	1.94	0.00	1.41	-0.01	0.87
time (sec)	N/A	0.128	0.060	0.005	0.000	1.312	0.000	1.013	0.000	1.583

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	166	222	0	791	0	0	-1	168
N.S.	1	1.00	0.73	0.98	0.00	3.48	0.00	0.00	-0.00	0.74
time (sec)	N/A	0.255	0.224	0.006	0.000	1.421	0.000	0.000	0.000	2.188

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	158	254	0	757	0	0	-1	160
N.S.	1	1.00	0.72	1.16	0.00	3.46	0.00	0.00	-0.00	0.73
time (sec)	N/A	0.242	0.180	0.006	0.000	1.319	0.000	0.000	0.000	2.069

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	162	338	0	757	0	0	-1	167
N.S.	1	1.00	0.74	1.54	0.00	3.46	0.00	0.00	-0.00	0.76
time (sec)	N/A	0.238	0.190	0.007	0.000	1.587	0.000	0.000	0.000	2.047
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	175	435	0	815	0	0	-1	187
N.S.	1	1.00	0.68	1.69	0.00	3.17	0.00	0.00	-0.00	0.73
time (sec)	N/A	0.351	0.289	0.009	0.000	1.546	0.000	0.000	0.000	2.200
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	141	501	0	332	0	0	-1	175
N.S.	1	1.00	0.72	2.54	0.00	1.69	0.00	0.00	-0.01	0.89
time (sec)	N/A	0.363	0.115	0.007	0.000	1.633	0.000	0.000	0.000	2.114
Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	249	177	534	0	394	0	0	-1	211
N.S.	1	1.00	0.71	2.14	0.00	1.58	0.00	0.00	-0.00	0.85
time (sec)	N/A	0.504	0.166	0.010	0.000	1.766	0.000	0.000	0.000	2.669

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	105	144	0	226	0	0	-1	122
N.S.	1	1.00	0.73	1.01	0.00	1.58	0.00	0.00	-0.01	0.85
time (sec)	N/A	0.174	0.093	0.009	0.000	1.015	0.000	0.000	0.000	0.370

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	89	88	0	188	0	108	-1	91
N.S.	1	1.00	0.86	0.85	0.00	1.83	0.00	1.05	-0.01	0.88
time (sec)	N/A	0.078	0.052	0.006	0.000	1.133	0.000	0.915	0.000	0.241

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	65	0	129	0	37	-1	54
N.S.	1	1.00	0.93	0.92	0.00	1.82	0.00	0.52	-0.01	0.76
time (sec)	N/A	0.037	0.035	0.006	0.000	1.058	0.000	0.912	0.000	0.058

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	70	66	0	130	0	59	-1	49
N.S.	1	1.00	1.56	1.47	0.00	2.89	0.00	1.31	-0.02	1.09
time (sec)	N/A	0.016	0.019	0.005	0.000	1.275	0.000	0.919	0.000	0.194

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	89	88	0	194	0	0	-1	79
N.S.	1	1.00	1.16	1.14	0.00	2.52	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.054	0.051	0.007	0.000	1.275	0.000	0.000	0.000	0.295
Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	112	152	0	232	0	0	-1	100
N.S.	1	1.00	0.94	1.28	0.00	1.95	0.00	0.00	-0.01	0.84
time (sec)	N/A	0.149	0.084	0.007	0.000	1.293	0.000	0.000	0.000	0.442
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	183	283	0	616	0	0	-1	192
N.S.	1	1.00	0.70	1.08	0.00	2.35	0.00	0.00	-0.00	0.73
time (sec)	N/A	0.506	0.230	0.009	0.000	1.551	0.000	0.000	0.000	2.805
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	141	199	0	486	0	195	-1	145
N.S.	1	1.00	0.70	0.99	0.00	2.42	0.00	0.97	-0.00	0.72
time (sec)	N/A	0.305	0.152	0.009	0.000	1.362	0.000	0.888	0.000	2.164

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	112	166	0	414	0	110	-1	116
N.S.	1	1.00	0.73	1.08	0.00	2.71	0.00	0.72	-0.01	0.76
time (sec)	N/A	0.175	0.119	0.007	0.000	1.179	0.000	0.983	0.000	1.831

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	37	53	0	73	0	45	75	56
N.S.	1	1.00	0.92	1.32	0.00	1.82	0.00	1.12	1.88	1.40
time (sec)	N/A	0.040	0.075	0.004	0.000	1.351	0.000	0.916	2.122	1.348

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	36	52	0	72	0	45	75	53
N.S.	1	1.00	0.92	1.33	0.00	1.85	0.00	1.15	1.92	1.36
time (sec)	N/A	0.040	0.024	0.003	0.000	1.351	0.000	0.765	2.034	0.875

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	109	166	0	411	0	0	-1	124
N.S.	1	1.00	1.16	1.77	0.00	4.37	0.00	0.00	-0.01	1.32
time (sec)	N/A	0.068	0.128	0.008	0.000	1.405	0.000	0.000	0.000	1.842

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	138	201	0	496	0	0	-1	157
N.S.	1	1.00	0.96	1.40	0.00	3.44	0.00	0.00	-0.01	1.09
time (sec)	N/A	0.163	0.100	0.009	0.000	1.020	0.000	0.000	0.000	2.339
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	181	292	0	630	0	0	-1	216
N.S.	1	1.00	0.87	1.40	0.00	3.01	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.287	0.157	0.009	0.000	1.557	0.000	0.000	0.000	3.250
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	225	340	0	716	0	0	-1	266
N.S.	1	1.00	0.83	1.25	0.00	2.64	0.00	0.00	-0.00	0.98
time (sec)	N/A	0.452	0.203	0.010	0.000	2.084	0.000	0.000	0.000	3.605
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	272	446	0	866	0	0	-1	331
N.S.	1	1.00	0.79	1.30	0.00	2.52	0.00	0.00	-0.00	0.97
time (sec)	N/A	0.621	0.249	0.011	0.000	2.350	0.000	0.000	0.000	5.235

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	34	77	37	71	280	107	89	0
N.S.	1	1.00	0.92	2.08	1.00	1.92	7.57	2.89	2.41	0.00
time (sec)	N/A	0.013	0.026	0.003	0.446	1.265	1.176	0.485	2.084	0.307

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.008	0.002	0.002	0.428	0.994	0.061	0.382	0.029	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.007	0.002	0.002	0.428	1.231	0.061	0.391	0.031	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.00
time (sec)	N/A	0.004	0.000	0.002	0.423	1.022	0.061	0.378	0.028	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	15	16	16	20
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80	1.00
time (sec)	N/A	0.006	0.001	0.000	0.425	1.353	0.066	0.363	0.027	0.022
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	17	20	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.95	0.81	0.00
time (sec)	N/A	0.008	0.002	0.001	0.426	1.243	0.095	0.415	0.025	0.000
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	20	12	16	16	0
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89	0.00
time (sec)	N/A	0.007	0.002	0.003	0.428	1.018	0.095	0.585	0.030	0.000
Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	69	300	85	241	1377	399	271	0
N.S.	1	1.00	0.91	3.95	1.12	3.17	18.12	5.25	3.57	0.00
time (sec)	N/A	0.046	0.071	0.005	0.436	1.445	4.287	0.516	2.195	0.408

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	46	51	46	45	0
N.S.	1	1.00	1.00	0.83	0.81	0.85	0.94	0.85	0.83	0.00
time (sec)	N/A	0.036	0.007	0.002	0.424	1.057	0.079	0.405	0.034	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	48	45	44	46	46	46	45	0
N.S.	1	1.00	0.89	0.83	0.81	0.85	0.85	0.85	0.83	0.00
time (sec)	N/A	0.054	0.008	0.000	0.446	0.780	0.076	0.384	0.024	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	48	46	51	46	45	0
N.S.	1	1.00	1.00	0.83	0.89	0.85	0.94	0.85	0.83	0.00
time (sec)	N/A	0.026	0.007	0.001	0.430	1.031	0.074	0.357	0.024	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	48	45	44	44	46	46	45	50
N.S.	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83	0.93
time (sec)	N/A	0.046	0.008	0.001	0.429	0.947	0.077	0.524	0.024	0.025

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	42	41	41	48	43	42	53
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.98	0.88	0.86	1.08
time (sec)	N/A	0.027	0.005	0.001	0.430	1.283	0.080	0.507	0.023	0.024

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	93	142	0	313	391	92	842	0
N.S.	1	1.00	0.93	1.42	0.00	3.13	3.91	0.92	8.42	0.00
time (sec)	N/A	0.122	0.085	0.004	0.000	1.151	2.946	0.467	2.203	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	250	467	0	1564	194	2457	4127	0
N.S.	1	1.00	1.23	2.30	0.00	7.70	0.96	12.10	20.33	0.00
time (sec)	N/A	0.602	0.155	0.025	0.000	1.597	4.254	2.037	2.718	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	111	0	254	316	75	655	0
N.S.	1	1.00	0.96	1.37	0.00	3.14	3.90	0.93	8.09	0.00
time (sec)	N/A	0.087	0.044	0.004	0.000	1.457	1.950	0.633	2.438	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	202	343	0	1059	129	2109	3026	0
N.S.	1	1.00	1.13	1.92	0.00	5.92	0.72	11.78	16.91	0.00
time (sec)	N/A	0.227	0.108	0.014	0.000	1.270	2.177	1.500	2.584	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	62	60	0	197	223	59	118	0
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87	0.00
time (sec)	N/A	0.067	0.023	0.003	0.000	1.420	0.919	0.423	0.166	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	165	208	0	559	75	503	416	0
N.S.	1	1.00	1.10	1.39	0.00	3.73	0.50	3.35	2.77	0.00
time (sec)	N/A	0.094	0.082	0.011	0.000	1.394	0.819	1.805	2.209	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	39	36	0	129	131	35	41	0
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14	0.00
time (sec)	N/A	0.043	0.008	0.003	0.000	1.233	0.495	0.489	2.040	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	129	116	0	613	87	1026	763	0
N.S.	1	1.00	0.86	0.77	0.00	4.09	0.58	6.84	5.09	0.00
time (sec)	N/A	0.080	0.079	0.013	0.000	1.314	1.185	1.863	2.479	0.000
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	113	66	0	223	253	68	1014	0
N.S.	1	1.00	1.64	0.96	0.00	3.23	3.67	0.99	14.70	0.00
time (sec)	N/A	0.072	0.061	0.007	0.000	1.346	4.284	0.428	2.701	0.000
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	191	232	0	1116	148	1839	2997	0
N.S.	1	1.00	1.10	1.33	0.00	6.41	0.85	10.57	17.22	0.00
time (sec)	N/A	0.195	0.401	0.018	0.000	1.390	2.623	1.883	2.859	0.001
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	135	119	0	293	345	94	2033	0
N.S.	1	1.00	1.52	1.34	0.00	3.29	3.88	1.06	22.84	0.00
time (sec)	N/A	0.133	0.120	0.007	0.000	1.374	123.748	0.432	3.910	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	151	383	0	868	877	161	1473	0
N.S.	1	1.00	0.91	2.31	0.00	5.23	5.28	0.97	8.87	0.00
time (sec)	N/A	0.223	0.186	0.013	0.000	1.388	112.278	1.985	0.528	0.001
Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	331	331	327	844	0	2856	0	3339	7599	0
N.S.	1	1.00	0.99	2.55	0.00	8.63	0.00	10.09	22.96	0.00
time (sec)	N/A	0.703	0.663	0.036	0.000	1.722	0.000	3.707	3.756	0.001
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	121	222	0	663	745	152	1336	0
N.S.	1	1.00	0.92	1.68	0.00	5.02	5.64	1.15	10.12	0.00
time (sec)	N/A	0.152	0.168	0.012	0.000	1.288	19.764	1.951	2.943	0.001
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	282	602	0	2257	379	2736	6293	0
N.S.	1	1.00	1.04	2.22	0.00	8.33	1.40	10.10	23.22	0.00
time (sec)	N/A	0.526	0.513	0.029	0.000	1.502	33.307	4.025	3.858	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	93	104	0	407	282	96	187	0
N.S.	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40	0.00
time (sec)	N/A	0.072	0.086	0.010	0.000	1.385	1.517	2.000	2.197	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	235	452	0	1668	296	2132	4973	0
N.S.	1	1.00	0.99	1.91	0.00	7.04	1.25	9.00	20.98	0.00
time (sec)	N/A	0.358	0.407	0.026	0.000	0.886	4.547	3.389	3.635	0.001

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	79	77	0	360	269	82	178	0
N.S.	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37	0.00
time (sec)	N/A	0.069	0.066	0.006	0.000	1.470	1.360	2.053	0.144	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	222	342	0	1680	298	1970	4854	0
N.S.	1	1.00	1.00	1.55	0.00	7.60	1.35	8.91	21.96	0.00
time (sec)	N/A	0.240	0.434	0.069	0.000	1.742	13.168	3.080	3.363	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	79	75	0	361	267	82	172	0
N.S.	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32	0.00
time (sec)	N/A	0.065	0.080	0.006	0.000	1.259	1.277	2.132	2.160	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	243	733	0	2309	394	2682	6404	0
N.S.	1	1.00	0.96	2.91	0.00	9.16	1.56	10.64	25.41	0.00
time (sec)	N/A	0.461	0.418	0.057	0.000	1.165	165.893	3.818	3.847	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	207	253	0	813	0	166	5048	0
N.S.	1	1.00	1.70	2.07	0.00	6.66	0.00	1.36	41.38	0.00
time (sec)	N/A	0.188	0.331	0.017	0.000	2.064	0.000	2.322	6.312	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	308	308	302	712	0	2912	0	3087	7555	0
N.S.	1	1.00	0.98	2.31	0.00	9.45	0.00	10.02	24.53	0.00
time (sec)	N/A	1.352	0.616	0.034	0.000	1.645	0.000	2.433	2.639	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	248	352	0	1007	0	182	5491	0
N.S.	1	1.00	1.53	2.17	0.00	6.22	0.00	1.12	33.90	0.00
time (sec)	N/A	0.250	0.264	0.020	0.000	1.644	0.000	2.407	6.766	0.001
Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	344	913	0	3435	0	3651	8739	0
N.S.	1	1.00	0.95	2.53	0.00	9.52	0.00	10.11	24.21	0.00
time (sec)	N/A	3.077	0.715	0.038	0.000	2.117	0.000	3.916	4.907	0.001
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	328	443	0	1242	0	274	5999	0
N.S.	1	1.00	1.50	2.02	0.00	5.67	0.00	1.25	27.39	0.00
time (sec)	N/A	0.312	0.375	0.023	0.000	2.285	0.000	2.040	7.473	0.001
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	126	157	0	232	0	127	-1	126
N.S.	1	1.00	0.98	1.22	0.00	1.80	0.00	0.98	-0.01	0.98
time (sec)	N/A	0.093	0.077	0.010	0.000	1.043	0.000	0.971	0.000	0.457

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	155	136	0	666	0	0	-1	148
N.S.	1	1.00	0.80	0.70	0.00	3.43	0.00	0.00	-0.01	0.76
time (sec)	N/A	0.209	0.060	0.013	0.000	1.395	0.000	0.000	0.000	0.596
Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	192	369	0	396	0	662	-1	214
N.S.	1	1.00	0.79	1.51	0.00	1.62	0.00	2.71	-0.00	0.88
time (sec)	N/A	0.357	0.204	0.014	0.000	1.282	0.000	2.112	0.000	1.919
Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	152	295	0	332	0	518	-1	181
N.S.	1	1.00	0.86	1.67	0.00	1.88	0.00	2.93	-0.01	1.02
time (sec)	N/A	0.138	0.111	0.023	0.000	1.019	0.000	1.583	0.000	1.522
Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	72	0	135	0	60	-1	67
N.S.	1	1.00	1.00	0.88	0.00	1.65	0.00	0.73	-0.01	0.82
time (sec)	N/A	0.062	0.018	0.011	0.000	1.055	0.000	0.625	0.000	0.315

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	83	72	0	137	0	56	-1	52
N.S.	1	1.00	1.63	1.41	0.00	2.69	0.00	1.10	-0.02	1.02
time (sec)	N/A	0.029	0.019	0.018	0.000	1.171	0.000	0.545	0.000	0.363
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	126	179	0	424	0	193	-1	123
N.S.	1	1.00	1.22	1.74	0.00	4.12	0.00	1.87	-0.01	1.19
time (sec)	N/A	0.073	0.079	0.015	0.000	1.557	0.000	0.662	0.000	1.836
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	160	220	0	508	0	0	-1	159
N.S.	1	1.00	1.04	1.43	0.00	3.30	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.175	0.074	0.023	0.000	1.340	0.000	0.000	0.000	2.250
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	0	0	83	0	0	-1	73
N.S.	1	1.00	0.90	0.00	0.00	1.63	0.00	0.00	-0.02	1.43
time (sec)	N/A	0.050	0.086	0.079	0.000	1.078	0.000	0.000	0.000	0.132

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	73	58	0	55	0	60	-1	40
N.S.	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02	0.89
time (sec)	N/A	0.009	0.020	0.011	0.000	1.015	0.000	0.437	0.000	0.157

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	73	58	0	55	0	60	-1	40
N.S.	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02	0.89
time (sec)	N/A	0.012	0.003	0.007	0.000	1.227	0.000	0.725	0.000	0.036

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	73	58	0	55	0	60	-1	40
N.S.	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02	0.89
time (sec)	N/A	0.012	0.004	0.004	0.000	0.884	0.000	0.492	0.000	0.041

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	70	81	0	70	0	69	-1	73
N.S.	1	1.00	0.81	0.94	0.00	0.81	0.00	0.80	-0.01	0.85
time (sec)	N/A	0.041	0.033	0.010	0.000	1.090	0.000	0.393	0.000	0.125

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	70	81	0	70	0	69	-1	73
N.S.	1	1.00	0.81	0.94	0.00	0.81	0.00	0.80	-0.01	0.85
time (sec)	N/A	0.042	0.010	0.006	0.000	1.006	0.000	0.511	0.000	0.033
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	70	81	0	70	0	69	-1	73
N.S.	1	1.00	0.81	0.94	0.00	0.81	0.00	0.80	-0.01	0.85
time (sec)	N/A	0.041	0.003	0.004	0.000	0.971	0.000	0.381	0.000	0.034
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	37	35	0	111	0	35	34	42
N.S.	1	1.00	0.97	0.92	0.00	2.92	0.00	0.92	0.89	1.11
time (sec)	N/A	0.016	0.020	0.003	0.000	0.878	0.000	0.386	0.080	0.001
Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	70	64	0	130	0	59	-1	49
N.S.	1	1.00	1.56	1.42	0.00	2.89	0.00	1.31	-0.02	1.09
time (sec)	N/A	0.022	0.024	0.009	0.000	0.958	0.000	0.466	0.000	0.084

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	72	64	0	131	0	53	-1	53
N.S.	1	1.00	1.53	1.36	0.00	2.79	0.00	1.13	-0.02	1.13
time (sec)	N/A	0.076	0.034	0.014	0.000	1.336	0.000	0.493	0.000	0.314
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	74	66	0	139	0	53	-1	55
N.S.	1	1.00	1.51	1.35	0.00	2.84	0.00	1.08	-0.02	1.12
time (sec)	N/A	0.090	0.028	0.008	0.000	1.148	0.000	0.490	0.000	0.315
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	39	0	124	0	38	44	45
N.S.	1	1.00	1.00	0.89	0.00	2.82	0.00	0.86	1.00	1.02
time (sec)	N/A	0.036	0.007	0.006	0.000	1.368	0.000	0.449	2.226	0.001
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	81	72	0	135	0	62	-1	48
N.S.	1	1.00	1.65	1.47	0.00	2.76	0.00	1.27	-0.02	0.98
time (sec)	N/A	0.015	0.017	0.007	0.000	1.315	0.000	0.436	0.000	0.089

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	83	72	0	137	0	56	-1	52
N.S.	1	1.00	1.63	1.41	0.00	2.69	0.00	1.10	-0.02	1.02
time (sec)	N/A	0.067	0.018	0.011	0.000	1.363	0.000	0.399	0.000	0.341
Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	85	74	0	145	0	56	-1	54
N.S.	1	1.00	1.60	1.40	0.00	2.74	0.00	1.06	-0.02	1.02
time (sec)	N/A	0.077	0.019	0.010	0.000	1.194	0.000	0.551	0.000	0.354
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	31	20	47	0	55	33	38
N.S.	1	1.00	1.00	0.78	0.50	1.18	0.00	1.38	0.82	0.95
time (sec)	N/A	0.027	0.008	0.004	0.945	1.152	0.000	0.472	0.427	0.001
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	73	58	0	55	0	60	-1	40
N.S.	1	1.00	1.62	1.29	0.00	1.22	0.00	1.33	-0.02	0.89
time (sec)	N/A	0.012	0.008	0.000	0.000	1.052	0.000	0.427	0.000	0.001

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [86] had the largest ratio of [.5000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	18	0.056
3	A	1	0	1.00	16	0.000
4	A	2	1	1.00	20	0.050
5	A	2	1	1.00	20	0.050
6	A	3	2	1.00	22	0.091
7	A	3	2	1.00	20	0.100
8	A	3	2	1.00	18	0.111
9	A	3	2	1.00	22	0.091
10	A	3	2	1.00	22	0.091
11	A	7	6	1.00	22	0.273
12	A	6	6	1.00	22	0.273
13	A	5	5	1.00	22	0.227
14	A	3	3	1.00	22	0.136
15	A	7	7	1.00	20	0.350
16	A	8	7	1.00	18	0.389
17	A	8	7	1.00	22	0.318
18	A	8	7	1.00	22	0.318
19	A	8	7	1.00	22	0.318
20	A	7	7	1.00	22	0.318
21	A	4	4	1.00	22	0.182
22	A	4	4	1.00	22	0.182
23	A	4	4	1.00	22	0.182
24	A	8	7	1.00	22	0.318
25	A	8	7	1.00	22	0.318
26	A	8	7	1.00	20	0.350

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	8	7	1.00	18	0.389
28	A	8	7	1.00	22	0.318
29	A	8	6	1.00	24	0.250
30	A	7	6	1.00	22	0.273
31	A	5	5	1.00	20	0.250
32	A	4	4	1.00	24	0.167
33	A	7	6	1.00	24	0.250
34	A	7	6	1.00	24	0.250
35	A	5	5	1.00	24	0.208
36	A	6	5	1.00	24	0.208
37	A	7	5	1.00	24	0.208
38	A	10	7	1.00	22	0.318
39	A	10	7	1.00	20	0.350
40	A	8	7	1.00	24	0.292
41	A	6	5	1.00	24	0.208
42	A	5	4	1.00	24	0.167
43	A	8	7	1.00	24	0.292
44	A	8	7	1.00	24	0.292
45	A	8	7	1.00	24	0.292
46	A	9	8	1.00	24	0.333
47	A	7	6	1.00	24	0.250
48	A	8	6	1.00	24	0.250
49	A	6	6	1.00	24	0.250
50	A	4	4	1.00	24	0.167
51	A	3	3	1.00	22	0.136
52	A	2	2	1.00	20	0.100
53	A	3	3	1.00	24	0.125
54	A	5	5	1.00	24	0.208
55	A	8	6	1.00	24	0.250
56	A	7	6	1.00	24	0.250
57	A	6	6	1.00	24	0.250
58	A	1	1	1.00	24	0.042
59	A	1	1	1.00	24	0.042
60	A	3	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	22	0.227
62	A	6	5	1.00	20	0.250
63	A	7	5	1.00	24	0.208
64	A	8	5	1.00	24	0.208
65	A	2	1	1.00	18	0.056
66	A	2	1	1.00	18	0.056
67	A	2	1	1.00	16	0.062
68	A	1	0	1.00	14	0.000
69	A	2	1	1.00	18	0.056
70	A	2	1	1.00	18	0.056
71	A	2	1	1.00	18	0.056
72	A	3	2	1.00	20	0.100
73	A	3	2	1.00	20	0.100
74	A	4	3	1.00	18	0.167
75	A	3	2	1.00	16	0.125
76	A	4	3	1.00	20	0.150
77	A	3	2	1.00	20	0.100
78	A	8	7	1.00	20	0.350
79	A	6	5	1.00	20	0.250
80	A	7	7	1.00	20	0.350
81	A	5	4	1.00	20	0.200
82	A	6	6	1.00	20	0.300
83	A	4	3	1.00	20	0.150
84	A	4	4	1.00	20	0.200
85	A	4	3	1.00	18	0.167
86	A	8	8	1.00	16	0.500
87	A	5	4	1.00	20	0.200
88	A	9	8	1.00	20	0.400
89	A	9	8	1.00	20	0.400
90	A	7	5	1.00	20	0.250
91	A	8	8	1.00	20	0.400
92	A	6	5	1.00	20	0.250
93	A	5	5	1.00	20	0.250
94	A	5	4	1.00	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	5	1.00	20	0.250
96	A	5	4	1.00	20	0.200
97	A	5	5	1.00	20	0.250
98	A	5	4	1.00	20	0.200
99	A	9	8	1.00	18	0.444
100	A	6	5	1.00	16	0.312
101	A	9	8	1.00	20	0.400
102	A	7	5	1.00	20	0.250
103	A	9	8	1.00	20	0.400
104	A	5	5	1.00	24	0.208
105	A	8	7	1.00	24	0.292
106	A	8	8	1.00	24	0.333
107	A	6	5	1.00	24	0.208
108	A	4	4	1.00	24	0.167
109	A	2	2	1.00	24	0.083
110	A	3	3	1.00	24	0.125
111	A	5	5	1.00	24	0.208
112	A	1	1	1.00	34	0.029
113	A	2	2	1.00	18	0.111
114	A	3	3	1.00	18	0.167
115	A	3	3	1.00	17	0.176
116	A	5	5	1.00	18	0.278
117	A	6	6	1.00	18	0.333
118	A	6	6	1.00	17	0.353
119	A	2	2	1.00	18	0.111
120	A	3	3	1.00	18	0.167
121	A	3	3	1.00	22	0.136
122	A	3	3	1.00	24	0.125
123	A	3	3	1.00	20	0.150
124	A	3	3	1.00	20	0.150
125	A	3	3	1.00	24	0.125
126	A	3	3	1.00	26	0.115
127	A	3	3	1.00	18	0.167
128	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	3	3	1.00	20	0.150
130	A	2	2	1.00	36	0.056

Chapter 3

Listing of integrals

Local contents

3.1	$\int x^2 (ax^2 + bx^3 + cx^4) dx$	69
3.2	$\int x (ax^2 + bx^3 + cx^4) dx$	72
3.3	$\int (ax^2 + bx^3 + cx^4) dx$	75
3.4	$\int \frac{ax^2+bx^3+cx^4}{x} dx$	78
3.5	$\int \frac{ax^2+bx^3+cx^4}{x^2} dx$	81
3.6	$\int x^2 (ax^2 + bx^3 + cx^4)^2 dx$	84
3.7	$\int x (ax^2 + bx^3 + cx^4)^2 dx$	87
3.8	$\int (ax^2 + bx^3 + cx^4)^2 dx$	90
3.9	$\int \frac{(ax^2+bx^3+cx^4)^2}{x} dx$	93
3.10	$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2} dx$	97
3.11	$\int \frac{x^5}{ax^2+bx^3+cx^4} dx$	101
3.12	$\int \frac{x^4}{ax^2+bx^3+cx^4} dx$	106
3.13	$\int \frac{x^3}{ax^2+bx^3+cx^4} dx$	111
3.14	$\int \frac{x^2}{ax^2+bx^3+cx^4} dx$	115
3.15	$\int \frac{x}{ax^2+bx^3+cx^4} dx$	119
3.16	$\int \frac{1}{ax^2+bx^3+cx^4} dx$	124
3.17	$\int \frac{1}{x(ax^2+bx^3+cx^4)} dx$	129
3.18	$\int \frac{1}{x^2(ax^2+bx^3+cx^4)} dx$	134

3.19	$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2} dx$	139
3.20	$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx$	145
3.21	$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2} dx$	151
3.22	$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2} dx$	155
3.23	$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx$	159
3.24	$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2} dx$	163
3.25	$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx$	169
3.26	$\int \frac{x}{(ax^2+bx^3+cx^4)^2} dx$	175
3.27	$\int \frac{1}{(ax^2+bx^3+cx^4)^2} dx$	181
3.28	$\int \frac{1}{x(ax^2+bx^3+cx^4)^2} dx$	187
3.29	$\int x^2 \sqrt{ax^2 + bx^3 + cx^4} dx$	194
3.30	$\int x \sqrt{ax^2 + bx^3 + cx^4} dx$	199
3.31	$\int \sqrt{ax^2 + bx^3 + cx^4} dx$	204
3.32	$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x} dx$	208
3.33	$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx$	212
3.34	$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} dx$	217
3.35	$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} dx$	222
3.36	$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5} dx$	226
3.37	$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} dx$	231
3.38	$\int x (ax^2 + bx^3 + cx^4)^{3/2} dx$	236
3.39	$\int (ax^2 + bx^3 + cx^4)^{3/2} dx$	243
3.40	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x} dx$	249
3.41	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx$	255
3.42	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3} dx$	260
3.43	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx$	265

3.44	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5} dx$	270
3.45	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6} dx$	276
3.46	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7} dx$	281
3.47	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8} dx$	287
3.48	$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9} dx$	292
3.49	$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}} dx$	297
3.50	$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}} dx$	302
3.51	$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}} dx$	306
3.52	$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}} dx$	310
3.53	$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}} dx$	314
3.54	$\int \frac{1}{x^7\sqrt{ax^2+bx^3+cx^4}} dx$	318
3.55	$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}} dx$	323
3.56	$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}} dx$	329
3.57	$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}} dx$	335
3.58	$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx$	340
3.59	$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}} dx$	343
3.60	$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}} dx$	346
3.61	$\int \frac{1}{x(ax^2+bx^3+cx^4)^{3/2}} dx$	350
3.62	$\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}} dx$	355
3.63	$\int \frac{1}{x^3(ax^2+bx^3+cx^4)^{3/2}} dx$	360
3.64	$\int \frac{1}{x^4(ax^2+bx^3+cx^4)^{3/2}} dx$	366
3.65	$\int x^m (ax + bx^3 + cx^5) dx$	372
3.66	$\int x^2 (ax + bx^3 + cx^5) dx$	375
3.67	$\int x (ax + bx^3 + cx^5) dx$	378
3.68	$\int (ax + bx^3 + cx^5) dx$	381

3.69	$\int \frac{ax+bx^3+cx^5}{x} dx$	384
3.70	$\int \frac{ax+bx^3+cx^5}{x^2} dx$	387
3.71	$\int \frac{ax+bx^3+cx^5}{x^3} dx$	390
3.72	$\int x^m (ax + bx^3 + cx^5)^2 dx$	393
3.73	$\int x^2 (ax + bx^3 + cx^5)^2 dx$	398
3.74	$\int x (ax + bx^3 + cx^5)^2 dx$	401
3.75	$\int (ax + bx^3 + cx^5)^2 dx$	405
3.76	$\int \frac{(ax+bx^3+cx^5)^2}{x} dx$	408
3.77	$\int \frac{(ax+bx^3+cx^5)^2}{x^2} dx$	412
3.78	$\int \frac{x^8}{ax+bx^3+cx^5} dx$	415
3.79	$\int \frac{x^7}{ax+bx^3+cx^5} dx$	421
3.80	$\int \frac{x^6}{ax+bx^3+cx^5} dx$	429
3.81	$\int \frac{x^5}{ax+bx^3+cx^5} dx$	434
3.82	$\int \frac{x^4}{ax+bx^3+cx^5} dx$	441
3.83	$\int \frac{x^3}{ax+bx^3+cx^5} dx$	445
3.84	$\int \frac{x^2}{ax+bx^3+cx^5} dx$	450
3.85	$\int \frac{x}{ax+bx^3+cx^5} dx$	454
3.86	$\int \frac{1}{ax+bx^3+cx^5} dx$	459
3.87	$\int \frac{1}{x(ax+bx^3+cx^5)} dx$	465
3.88	$\int \frac{1}{x^2(ax+bx^3+cx^5)} dx$	472
3.89	$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2} dx$	478
3.90	$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2} dx$	485
3.91	$\int \frac{x^9}{(ax+bx^3+cx^5)^2} dx$	497
3.92	$\int \frac{x^8}{(ax+bx^3+cx^5)^2} dx$	503
3.93	$\int \frac{x^7}{(ax+bx^3+cx^5)^2} dx$	514
3.94	$\int \frac{x^6}{(ax+bx^3+cx^5)^2} dx$	519

3.95	$\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx$	528
3.96	$\int \frac{x^4}{(ax+bx^3+cx^5)^2} dx$	533
3.97	$\int \frac{x^3}{(ax+bx^3+cx^5)^2} dx$	541
3.98	$\int \frac{x^2}{(ax+bx^3+cx^5)^2} dx$	546
3.99	$\int \frac{x}{(ax+bx^3+cx^5)^2} dx$	556
3.100	$\int \frac{1}{(ax+bx^3+cx^5)^2} dx$	564
3.101	$\int \frac{1}{x(ax+bx^3+cx^5)^2} dx$	576
3.102	$\int \frac{1}{x^2(ax+bx^3+cx^5)^2} dx$	584
3.103	$\int \frac{1}{x^3(ax+bx^3+cx^5)^2} dx$	598
3.104	$\int \sqrt{x} \sqrt{ax+bx^3+cx^5} dx$	607
3.105	$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}} dx$	611
3.106	$\int x^{3/2} (ax+bx^3+cx^5)^{3/2} dx$	616
3.107	$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}} dx$	622
3.108	$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}} dx$	627
3.109	$\int \frac{1}{\sqrt{x} \sqrt{ax+bx^3+cx^5}} dx$	631
3.110	$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}} dx$	635
3.111	$\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}} dx$	639
3.112	$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}} dx$	644
3.113	$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx$	647
3.114	$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$	650
3.115	$\int \frac{1}{\sqrt{1-(1-x^2)^3}} dx$	654
3.116	$\int \sqrt{3x^2-3x^4+x^6} dx$	658
3.117	$\int \sqrt{x^2(3-3x^2+x^4)} dx$	662

3.118	$\int \sqrt{1 - (1 - x^2)^3} dx$	666
3.119	$\int \frac{1}{x\sqrt{a+bx+cx^2}} dx$	670
3.120	$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}} dx$	673
3.121	$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}} dx$	677
3.122	$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}} dx$	681
3.123	$\int \frac{1}{x\sqrt{a+bx^2+cx^4}} dx$	685
3.124	$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}} dx$	689
3.125	$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}} dx$	693
3.126	$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx$	697
3.127	$\int \frac{1}{x\sqrt{3-3x^2+x^4}} dx$	701
3.128	$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$	705
3.129	$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}} dx$	709
3.130	$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}} dx$	713

3.1 $\int x^2 (ax^2 + bx^3 + cx^4) dx$

Optimal. Leaf size=25

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 (ax^2 + bx^3 + cx^4) dx &= \int (ax^4 + bx^5 + cx^6) dx \\ &= \frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (ax^2 + bx^3 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3 + c*x^4), x]

[Out] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.89, size = 19, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{6}x^6b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2), x, algorithm="fricas")

[Out] 1/7*x^7*c + 1/6*x^6*b + 1/5*x^5*a

giac [A] time = 0.54, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2), x, algorithm="giac")

[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^3+a*x^2), x)

[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7

maxima [A] time = 0.44, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^5 (30 c x^2 + 35 b x + 42 a)}{210}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a*x^2 + b*x^3 + c*x^4),x)

[Out] (x^5*(42*a + 35*b*x + 30*c*x^2))/210

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**4+b*x**3+a*x**2),x)

[Out] a*x**5/5 + b*x**6/6 + c*x**7/7

3.2 $\int x(ax^2 + bx^3 + cx^4) dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned} \int x(ax^2 + bx^3 + cx^4) dx &= \int (ax^3 + bx^4 + cx^5) dx \\ &= \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(ax^2 + bx^3 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a*x^2 + b*x^3 + c*x^4), x]

[Out] IntegrateAlgebraic[x*(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.98, size = 19, normalized size = 0.76

$$\frac{1}{6}x^6c + \frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2), x, algorithm="fricas")

[Out] 1/6*x^6*c + 1/5*x^5*b + 1/4*x^4*a

giac [A] time = 0.45, size = 19, normalized size = 0.76

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2), x, algorithm="giac")

[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^3+a*x^2), x)

[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6

maxima [A] time = 0.44, size = 19, normalized size = 0.76

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^4 (10 c x^2 + 12 b x + 15 a)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x^2 + b*x^3 + c*x^4),x)

[Out] (x^4*(15*a + 12*b*x + 10*c*x^2))/60

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**4+b*x**3+a*x**2),x)

[Out] a*x**4/4 + b*x**5/5 + c*x**6/6

3.3 $\int (ax^2 + bx^3 + cx^4) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[a*x^2 + b*x^3 + c*x^4,x]

[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5

Rubi steps

$$\int (ax^2 + bx^3 + cx^4) dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]

[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^2 + bx^3 + cx^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a*x^2 + b*x^3 + c*x^4,x]

[Out] IntegrateAlgebraic[a*x^2 + b*x^3 + c*x^4, x]

fricas [A] time = 0.84, size = 19, normalized size = 0.76

$$\frac{1}{5}x^5c + \frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")

[Out] 1/5*x^5*c + 1/4*x^4*b + 1/3*x^3*a

giac [A] time = 0.54, size = 19, normalized size = 0.76

$$\frac{1}{5}cx^5 + \frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")

[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{5}cx^5 + \frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c*x^4+b*x^3+a*x^2,x)

[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{5}cx^5 + \frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")

[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^3 (12cx^2 + 15bx + 20a)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x^2 + b*x^3 + c*x^4,x)
```

```
[Out] (x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

$$3.4 \quad \int \frac{ax^2+bx^3+cx^4}{x} dx$$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3 + cx^4}{x} dx &= \int (ax + bx^2 + cx^3) dx \\ &= \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 0.84

$$\frac{1}{12}x^2(6a + 4bx + 3cx^2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (x^2*(6*a + 4*b*x + 3*c*x^2))/12

fricas [A] time = 0.53, size = 19, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")

[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2

giac [A] time = 0.36, size = 19, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")

[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)/x,x)

[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4

maxima [A] time = 0.45, size = 19, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^2 (3 c x^2 + 4 b x + 6 a)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)/x,x)

[Out] (x^2*(6*a + 4*b*x + 3*c*x^2))/12

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)/x,x)

[Out] a*x**2/2 + b*x**3/3 + c*x**4/4

$$3.5 \quad \int \frac{ax^2+bx^3+cx^4}{x^2} dx$$

Optimal. Leaf size=20

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {14}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3 + cx^4}{x^2} dx &= \int (a + bx + cx^2) dx \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

fricas [A] time = 0.73, size = 16, normalized size = 0.80

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x

giac [A] time = 0.49, size = 16, normalized size = 0.80

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)/x^2,x)

[Out] a*x+1/2*b*x^2+1/3*c*x^3

maxima [A] time = 0.43, size = 16, normalized size = 0.80

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x

mupad [B] time = 0.03, size = 16, normalized size = 0.80

$$\frac{cx^3}{3} + \frac{bx^2}{2} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)/x^2,x)

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

sympy [A] time = 0.06, size = 15, normalized size = 0.75

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)/x**2,x)

[Out] a*x + b*x**2/2 + c*x**3/3

3.6 $\int x^2 (ax^2 + bx^3 + cx^4)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^7}{7} + \frac{1}{9}x^9(2ac + b^2) + \frac{1}{4}abx^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^7}{7} + \frac{1}{9}x^9(2ac + b^2) + \frac{1}{4}abx^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^11)/11

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int x^2 (ax^2 + bx^3 + cx^4)^2 dx &= \int x^6 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^6 + 2abx^7 + (b^2 + 2ac)x^8 + 2bcx^9 + c^2x^{10}) dx \\ &= \frac{a^2x^7}{7} + \frac{1}{4}abx^8 + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^7}{7} + \frac{1}{9}x^9(2ac + b^2) + \frac{1}{4}abx^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (ax^2 + bx^3 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^2*(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [A] time = 0.90, size = 46, normalized size = 0.85

$$\frac{1}{11}x^{11}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{9}x^9b^2 + \frac{2}{9}x^9ca + \frac{1}{4}x^8ba + \frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] 1/11*x^11*c^2 + 1/5*x^10*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 1/4*x^8*b*a + 1/7*x^7*a^2

giac [A] time = 0.58, size = 46, normalized size = 0.85

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{9}b^2x^9 + \frac{2}{9}acx^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{11}}{11} + \frac{bcx^{10}}{5} + \frac{abx^8}{4} + \frac{a^2x^7}{7} + \frac{(2ac + b^2)x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(c*x^4+b*x^3+a*x^2)^2,x)`

[Out] $1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^{10}+1/11*c^2*x^{11}$

maxima [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{4}abx^8 + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")`

[Out] $1/11*c^2*x^{11} + 1/5*b*c*x^{10} + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^2*x^7$

mupad [B] time = 0.03, size = 45, normalized size = 0.83

$$x^9 \left(\frac{b^2}{9} + \frac{2ac}{9} \right) + \frac{a^2x^7}{7} + \frac{c^2x^{11}}{11} + \frac{abx^8}{4} + \frac{bcx^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a*x^2 + b*x^3 + c*x^4)^2,x)`

[Out] $x^9*((2*a*c)/9 + b^2/9) + (a^2*x^7)/7 + (c^2*x^{11})/11 + (a*b*x^8)/4 + (b*c*x^{10})/5$

sympy [A] time = 0.08, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{bcx^{10}}{5} + \frac{c^2x^{11}}{11} + x^9 \left(\frac{2ac}{9} + \frac{b^2}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b**2/9)$

3.7 $\int x(ax^2 + bx^3 + cx^4)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^6}{6} + \frac{1}{8}x^8(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1585, 698}

$$\frac{a^2x^6}{6} + \frac{1}{8}x^8(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*x^10)/10

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int x(ax^2 + bx^3 + cx^4)^2 dx &= \int x^5(a + bx + cx^2)^2 dx \\ &= \int (a^2x^5 + 2abx^6 + (b^2 + 2ac)x^7 + 2bcx^8 + c^2x^9) dx \\ &= \frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^6}{6} + \frac{1}{8}x^8(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*x^10)/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(ax^2 + bx^3 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x*(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [A] time = 0.77, size = 46, normalized size = 0.85

$$\frac{1}{10}x^{10}c^2 + \frac{2}{9}x^9cb + \frac{1}{8}x^8b^2 + \frac{1}{4}x^8ca + \frac{2}{7}x^7ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] 1/10*x^10*c^2 + 2/9*x^9*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 2/7*x^7*b*a + 1/6*x^6*a^2

giac [A] time = 0.48, size = 46, normalized size = 0.85

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{10}}{10} + \frac{2bcx^9}{9} + \frac{2abx^7}{7} + \frac{a^2x^6}{6} + \frac{(2ac + b^2)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c*x^4+b*x^3+a*x^2)^2,x)`

[Out] $1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^{10}$

maxima [A] time = 0.44, size = 44, normalized size = 0.81

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{2}{7}abx^7 + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")`

[Out] $1/10*c^2*x^{10} + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2*x^6$

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^8 \left(\frac{b^2}{8} + \frac{ac}{4} \right) + \frac{a^2x^6}{6} + \frac{c^2x^{10}}{10} + \frac{2abx^7}{7} + \frac{2bcx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a*x^2 + b*x^3 + c*x^4)^2,x)`

[Out] $x^8*((a*c)/4 + b^2/8) + (a^2*x^6)/6 + (c^2*x^{10})/10 + (2*a*b*x^7)/7 + (2*b*c*x^9)/9$

sympy [A] time = 0.07, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{2bcx^9}{9} + \frac{c^2x^{10}}{10} + x^8 \left(\frac{ac}{4} + \frac{b^2}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 + b**2/8)$

3.8 $\int (ax^2 + bx^3 + cx^4)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1594, 698}

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)/9

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rubi steps

$$\begin{aligned} \int (ax^2 + bx^3 + cx^4)^2 dx &= \int x^4 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^4 + 2abx^5 + (b^2 + 2ac)x^6 + 2bcx^7 + c^2x^8) dx \\ &= \frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{1}{7}(b^2 + 2ac)x^7 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^2 + bx^3 + cx^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [A] time = 0.88, size = 46, normalized size = 0.85

$$\frac{1}{9}x^9c^2 + \frac{1}{4}x^8cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{1}{3}x^6ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] 1/9*x^9*c^2 + 1/4*x^8*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 1/3*x^6*b*a + 1/5*x^5*a^2

giac [A] time = 0.40, size = 46, normalized size = 0.85

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^9}{9} + \frac{bcx^8}{4} + \frac{abx^6}{3} + \frac{a^2x^5}{5} + \frac{(2ac + b^2)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^2,x)`

[Out] $1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9$

maxima [A] time = 0.45, size = 48, normalized size = 0.89

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{1}{5}a^2x^5 + \frac{1}{21}(6cx^7 + 7bx^6)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")`

[Out] $1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*b*x^6)*a$

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^7 \left(\frac{b^2}{7} + \frac{2ac}{7} \right) + \frac{a^2x^5}{5} + \frac{c^2x^9}{9} + \frac{abx^6}{3} + \frac{bcx^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^2,x)`

[Out] $x^7*((2*a*c)/7 + b^2/7) + (a^2*x^5)/5 + (c^2*x^9)/9 + (a*b*x^6)/3 + (b*c*x^8)/4$

sympy [A] time = 0.07, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2x^9}{9} + x^7 \left(\frac{2ac}{7} + \frac{b^2}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2/7)$

$$3.9 \quad \int \frac{(ax^2+bx^3+cx^4)^2}{x} dx$$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4} + \frac{1}{6}x^6(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^4}{4} + \frac{1}{6}x^6(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x, x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*x^8)/8

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx &= \int x^3 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^3 + 2abx^4 + (b^2 + 2ac)x^5 + 2bcx^6 + c^2x^7) dx \\ &= \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{1}{6}x^6(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*x^8)/8

IntegrateAlgebraic [A] time = 0.04, size = 58, normalized size = 1.07

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{3}acx^6 + \frac{b^2x^6}{6} + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^2/x,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6 + (a*c*x^6)/3 + (2*b*c*x^7)/7 + (c^2*x^8)/8

fricas [A] time = 0.76, size = 44, normalized size = 0.81

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")

[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x^4

giac [A] time = 0.49, size = 46, normalized size = 0.85

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")

[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^8}{8} + \frac{2bcx^7}{7} + \frac{2abx^5}{5} + \frac{a^2x^4}{4} + \frac{(2ac + b^2)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^2/x,x)

[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8

maxima [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")

[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x^4

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^6 \left(\frac{b^2}{6} + \frac{ac}{3} \right) + \frac{a^2x^4}{4} + \frac{c^2x^8}{8} + \frac{2abx^5}{5} + \frac{2bcx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^2/x,x)

[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^4)/4 + (c^2*x^8)/8 + (2*a*b*x^5)/5 + (2*b*c*x^7)/7

sympy [A] time = 0.08, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6 \left(\frac{ac}{3} + \frac{b^2}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**2/x,x)

[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b**2/6)

$$3.10 \quad \int \frac{(ax^2+bx^3+cx^4)^2}{x^2} dx$$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)/7

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2} dx &= \int x^2 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^2 + 2abx^3 + (b^2 + 2ac)x^4 + 2bcx^5 + c^2x^6) dx \\ &= \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)/7

IntegrateAlgebraic [A] time = 0.04, size = 58, normalized size = 1.07

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{2}{5}acx^5 + \frac{b^2x^5}{5} + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5 + (2*a*c*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)/7

fricas [A] time = 0.96, size = 44, normalized size = 0.81

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")

[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x^3

giac [A] time = 0.48, size = 46, normalized size = 0.85

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")

[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^7}{7} + \frac{bcx^6}{3} + \frac{abx^4}{2} + \frac{a^2x^3}{3} + \frac{(2ac + b^2)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^2/x^2,x)

[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7

maxima [A] time = 0.42, size = 44, normalized size = 0.81

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")

[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x^3

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^5 \left(\frac{b^2}{5} + \frac{2ac}{5} \right) + \frac{a^2x^3}{3} + \frac{c^2x^7}{7} + \frac{abx^4}{2} + \frac{bcx^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^2/x^2,x)

[Out] x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x^6)/3

sympy [A] time = 0.08, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5 \left(\frac{2ac}{5} + \frac{b^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)

[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2/5)

$$3.11 \quad \int \frac{x^5}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=89

$$\frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{bx}{c^2} + \frac{x^2}{2c}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1585, 701, 634, 618, 206, 628}

$$\frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{bx}{c^2} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4), x]

[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 701

`Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])`

Rule 1585

`Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{ax^2 + bx^3 + cx^4} dx &= \int \frac{x^3}{a + bx + cx^2} dx \\
 &= \int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{ab + (b^2 - ac)x}{c^2(a + bx + cx^2)} \right) dx \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{\int \frac{ab + (b^2 - ac)x}{a + bx + cx^2} dx}{c^2} \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} - \frac{(b(b^2 - 3ac)) \int \frac{1}{a + bx + cx^2} dx}{2c^3} + \frac{(b^2 - ac) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^3} \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{(b(b^2 - 3ac)) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^3} \\
 &= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^3 \sqrt{b^2 - 4ac}} + \frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.94

$$\frac{(b^2 - ac) \log(a + x(b + cx)) - \frac{2b(b^2 - 3ac) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + cx(cx - 2b)}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4), x]

[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)]/(2*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3 + c*x^4), x]

[Out] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.86, size = 297, normalized size = 3.34

$$\frac{\left((b^2c^2 - 4ac^3)x^2 - (b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{c^2 + bx + a}\right) - 2(b^2c - 4abc^2)x + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^2 + bx + a) \right) \sqrt{b^2 - 4ac} \arctan\left(\frac{-\sqrt{b^2 - 4ac}(2cx + b)}{b^2 - 4ac}\right) - 2(b^2c - 4abc^2)x + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^2 + bx + a)}{2(b^2c^3 - 4ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2), x, algorithm="fricas")

[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]

giac [A] time = 0.45, size = 86, normalized size = 0.97

$$\frac{cx^2 - 2bx}{2c^2} + \frac{(b^2 - ac) \log(cx^2 + bx + a)}{2c^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2), x, algorithm="giac")

[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)

maple [A] time = 0.00, size = 132, normalized size = 1.48

$$\frac{3ab \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^2} - \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^3} + \frac{x^2}{2c} - \frac{a \ln(cx^2 + bx + a)}{2c^2} + \frac{b^2 \ln(cx^2 + bx + a)}{2c^3} - \frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^4+b*x^3+a*x^2), x)

[Out] 1/2/c*x^2-b/c^2*x-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.14, size = 112, normalized size = 1.26

$$\frac{x^2}{2c} - \frac{\ln(cx^2 + bx + a) (4a^2c^2 - 5ab^2c + b^4)}{2(4ac^4 - b^2c^3)} - \frac{bx}{c^2} + \frac{b \operatorname{atan}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) (3ac - b^2)}{c^3 \sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*x^2 + b*x^3 + c*x^4), x)

[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c - b^2))/(c^3*(4*a*c - b^2)^(1/2))

sympy [B] time = 0.84, size = 381, normalized size = 4.28

$$\frac{bx}{c^2} + \left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3} \right) \log\left(x + \frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3} \right) \log\left(x + \frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(\frac{b\sqrt{4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**4+b*x**3+a*x**2),x)

[Out]
$$\begin{aligned} & -bx/c^2 + (-b\sqrt{-4ac + b^2})(3ac - b^2)/(2c^3(4ac - b^2)) \\ & - (ac - b^2)/(2c^3) \log(x + (2a^2c - ab^2 + 4ac^3(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2c^3(4ac - b^2)) - (ac - b^2)/(2c^3)) \\ & - b^2c^2(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2c^3(4ac - b^2)) \\ & - (ac - b^2)/(2c^3)))/(3abc - b^3) + (b\sqrt{-4ac + b^2})(3ac - b^2)/(2c^3(4ac - b^2)) - (ac - b^2)/(2c^3) \log(x + (2a^2c - ab^2 + 4ac^3(b\sqrt{-4ac + b^2})(3ac - b^2)/(2c^3(4ac - b^2)) - (ac - b^2)/(2c^3)) - b^2c^2(b\sqrt{-4ac + b^2})(3ac - b^2)/(2c^3(4ac - b^2)) - (ac - b^2)/(2c^3)))/(3abc - b^3) \\ &) + x^2/(2c) \end{aligned}$$

$$3.12 \quad \int \frac{x^4}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=70

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1585, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]

[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{ax^2 + bx^3 + cx^4} dx &= \int \frac{x^2}{a + bx + cx^2} dx \\ &= \frac{x}{c} + \frac{\int \frac{-a-bx}{a+bx+cx^2} dx}{c} \\ &= \frac{x}{c} - \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} + \frac{(b^2 - 2ac) \int \frac{1}{a+bx+cx^2} dx}{2c^2} \\ &= \frac{x}{c} - \frac{b \log(a + bx + cx^2)}{2c^2} - \frac{(b^2 - 2ac) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^2} \\ &= \frac{x}{c} - \frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.04

$$\frac{(b^2 - 2ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2 \sqrt{4ac - b^2}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]

[Out] $x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3 + c*x^4),x]

[Out] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.80, size = 235, normalized size = 3.36

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - 2(b^2c - 4ac^2)x + (b^3 - 4abc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)}, \dots, \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - 2(b^2c - 4ac^2)x + (b^3 - 4abc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] $[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]$

giac [A] time = 0.55, size = 67, normalized size = 0.96

$$\frac{x}{c} - \frac{b \log(cx^2 + bx + a)}{2c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] $x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)$

maple [A] time = 0.00, size = 101, normalized size = 1.44

$$-\frac{2a \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}c} + \frac{b^2 \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}c^2} - \frac{b \ln(cx^2 + bx + a)}{2c^2} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(c*x^4+b*x^3+a*x^2),x)`

[Out] $1/c*x-1/2*b/c^2*\ln(c*x^2+b*x+a)-2/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a+1/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.03, size = 172, normalized size = 2.46

$$\frac{x}{c} + \frac{b^3 \ln(cx^2 + bx + a)}{2(4ac^3 - b^2c^2)} - \frac{2a \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}} + \frac{b^2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}} - \frac{2abc \ln(cx^2 + bx + a)}{4ac^3 - b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a*x^2 + b*x^3 + c*x^4),x)`

[Out] $x/c + (b^3*\log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))/(c*(4*a*c - b^2)^{(1/2)}) + (b^2*\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))/(c^2*(4*a*c - b^2)^{(1/2)}) - (2*a*b*c*\log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)$

sympy [B] time = 0.62, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) \log\left(x + \frac{-ab - 4ac^2\left(\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) + b^2c\left(\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right)}{2ac - b^2}\right) + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) \log\left(x + \frac{-ab - 4ac^2\left(\frac{b}{2c^2} + \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) + b^2c\left(\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right)}{2ac - b^2}\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(c*x**4+b*x**3+a*x**2),x)`

[Out] $(-b/(2*c**2) - \operatorname{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*\log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - \operatorname{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))) + b**2*c*(-b/(2*c**2) - \operatorname{sqrt}(-4*a*c + b**2)*(2*a*c$

$$\begin{aligned}
& - b^{**2}/(2*c^{**2}*(4*a*c - b^{**2}))) / (2*a*c - b^{**2}) + (-b/(2*c^{**2}) + \text{sqrt}(-4 \\
& *a*c + b^{**2})*(2*a*c - b^{**2}) / (2*c^{**2}*(4*a*c - b^{**2}))) * \log(x + (-a*b - 4*a*c* \\
& *2*(-b/(2*c^{**2}) + \text{sqrt}(-4*a*c + b^{**2})*(2*a*c - b^{**2}) / (2*c^{**2}*(4*a*c - b^{**2}))) \\
&)) + b^{**2}*c*(-b/(2*c^{**2}) + \text{sqrt}(-4*a*c + b^{**2})*(2*a*c - b^{**2}) / (2*c^{**2}*(4*a* \\
& c - b^{**2}))) / (2*a*c - b^{**2}) + x/c
\end{aligned}$$

$$3.13 \quad \int \frac{x^3}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=56

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1585, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x + c*x^2]/(2*c)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ax^2 + bx^3 + cx^4} dx &= \int \frac{x}{a + bx + cx^2} dx \\ &= \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2c} - \frac{b \int \frac{1}{a+bx+cx^2} dx}{2c} \\ &= \frac{\log(a + bx + cx^2)}{2c} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{c} \\ &= \frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a + bx + cx^2)}{2c} \end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\log(a + x(b + cx)) - \frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]

[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + x*(b + c*x)])/(2*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{ax^2 + bx^3 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3 + c*x^4), x]

[Out] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.76, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^2 - 4ac) \log(cx^2 + bx + a)}{2(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + (b^2 - 4ac) \log(cx^2 + bx + a)}{2(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c - 4*a*c^2)]

giac [A] time = 0.42, size = 55, normalized size = 0.98

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c} + \frac{\log(cx^2+bx+a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2), x, algorithm="giac")

[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(c*x^2 + b*x + a)/c

maple [A] time = 0.00, size = 56, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{\ln(cx^2+bx+a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^3+a*x^2), x)

[Out] 1/2/c*ln(c*x^2+b*x+a)-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.13, size = 112, normalized size = 2.00

$$\frac{2ac \ln(cx^2 + bx + a)}{4ac^2 - b^2c} - \frac{b \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}} - \frac{b^2 \ln(cx^2 + bx + a)}{2(4ac^2 - b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*x^2 + b*x^3 + c*x^4),x)

[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a + b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))

sympy [B] time = 0.33, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right) \log\left(x + \frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right) + 2a + b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right) \log\left(x + \frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right) + 2a + b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2),x)

[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b)

$$3.14 \quad \int \frac{x^2}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=34

$$\frac{2 \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1585, 618, 206}

$$\frac{2 \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]

[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(-n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^(-n), x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{ax^2 + bx^3 + cx^4} dx &= \int \frac{1}{a + bx + cx^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.12

$$\frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4), x]

[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3 + c*x^4), x]

[Out] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.96, size = 120, normalized size = 3.53

$$\left[\frac{\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4ac}}, -\frac{2\sqrt{-b^2+4ac} \arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)}{b^2-4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2), x, algorithm="fricas")

[Out] $\left[\log\left(\frac{(2cx^2 + 2b^2x + b^2 - 2ac - \sqrt{b^2 - 4ac})(2cx + b)}{(cx^2 + bx + a)\sqrt{b^2 - 4ac}}\right), -2\sqrt{-b^2 + 4ac}\arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right)\right]$

giac [A] time = 0.50, size = 34, normalized size = 1.00

$$\frac{2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")`

[Out] $2\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)/\sqrt{-b^2+4ac}$

maple [A] time = 0.00, size = 35, normalized size = 1.03

$$\frac{2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(c*x^4+b*x^3+a*x^2),x)`

[Out] $2/(4ac-b^2)^{1/2}\arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4ac-b^2>0)', see `assume?` for more details)Is 4ac-b^2 positive or negative?

mupad [B] time = 0.03, size = 46, normalized size = 1.35

$$\frac{2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a*x^2 + b*x^3 + c*x^4),x)`

[Out] $(2*\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)}))/(4*a*c - b^2)^{(1/2)}$

sympy [B] time = 0.22, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}} \log\left(x + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}} \log\left(x + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**4+b*x**3+a*x**2),x)`

[Out] $-\operatorname{sqrt}(-1/(4*a*c - b**2))*\log(x + (-4*a*c*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b**2*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b)/(2*c)) + \operatorname{sqrt}(-1/(4*a*c - b**2))*\log(x + (4*a*c*\operatorname{sqrt}(-1/(4*a*c - b**2)) - b**2*\operatorname{sqrt}(-1/(4*a*c - b**2)) + b)/(2*c))$

$$3.15 \quad \int \frac{x}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=62

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+bx+cx^2)}{2a} + \frac{\log(x)}{a}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1585, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+bx+cx^2)}{2a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3 + c*x^4),x]

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x + c*x^2]/(2*a)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x}{ax^2 + bx^3 + cx^4} dx &= \int \frac{1}{x(a + bx + cx^2)} dx \\
 &= \frac{\int \frac{1}{x} dx}{a} + \frac{\int \frac{-b-cx}{a+bx+cx^2} dx}{a} \\
 &= \frac{\log(x)}{a} - \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2a} - \frac{b \int \frac{1}{a+bx+cx^2} dx}{2a} \\
 &= \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{a} \\
 &= \frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.98

$$\frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \log(a + x(b + cx)) - 2 \log(x)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4), x]

[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*Log[x] + Log[a + x*(b + c*x)])/a

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{ax^2 + bx^3 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a*x^2 + b*x^3 + c*x^4), x]

[Out] IntegrateAlgebraic[x/(a*x^2 + b*x^3 + c*x^4), x]

fricas [A] time = 0.92, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - (b^2 - 4ac) \log(cx^2 + bx + a) + 2(b^2 - 4ac) \log(x)}{2(ab^2 - 4a^2c)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - (b^2 - 4ac) \log(cx^2 + bx + a) + 2(b^2 - 4ac) \log(x)}{2(ab^2 - 4a^2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

giac [A] time = 0.50, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac} a} - \frac{\log(cx^2 + bx + a)}{2a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2), x, algorithm="giac")

[Out] $-b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) / (\sqrt{-b^2+4ac} \cdot a) - \frac{1}{2} \log\left(\frac{cx^2+bx+a}{a}\right) + \log(\text{abs}(x)) / a$

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} a} + \frac{\ln(x)}{a} - \frac{\ln(cx^2+bx+a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/(c*x^4+b*x^3+a*x^2), x)$

[Out] $\frac{1}{a} \ln(x) - \frac{1}{2a} \ln(cx^2+bx+a) - \frac{1}{a} b / (4ac-b^2)^{(1/2)} \arctan((2cx+b)/(4ac-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x/(c*x^4+b*x^3+a*x^2), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.30, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{a} - \ln\left(bc - (x(6ac^2 - 2b^2c) - abc)\left(\frac{1}{2a} - \frac{b\sqrt{b^2-4ac}}{2(a^2b^2-4a^2c)}\right) + 3c^2x\right) \left(\frac{1}{2a} - \frac{b\sqrt{b^2-4ac}}{2(a^2b^2-4a^2c)}\right) - \ln\left(x(6ac^2 - 2b^2c) - abc\right) \left(\frac{1}{2a} + \frac{b\sqrt{b^2-4ac}}{2(a^2b^2-4a^2c)}\right) - bc - 3c^2x \left(\frac{1}{2a} + \frac{b\sqrt{b^2-4ac}}{2(a^2b^2-4a^2c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/(a*x^2 + b*x^3 + c*x^4), x)$

[Out] $\log(x)/a - \log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c) * (1/(2*a) - (b*(b^2 - 4*a*c)^{(1/2)})/(2*(a*b^2 - 4*a^2*c)))) + 3*c^2*x * (1/(2*a) - (b*(b^2 - 4*a*c)^{(1/2)})/(2*(a*b^2 - 4*a^2*c)))) - \log((x*(6*a*c^2 - 2*b^2*c) - a*b*c) * (1/(2*a) + (b*(b^2 - 4*a*c)^{(1/2)})/(2*(a*b^2 - 4*a^2*c)))) - b*c - 3*c^2*x * (1/(2*a) + (b*(b^2 - 4*a*c)^{(1/2)})/(2*(a*b^2 - 4*a^2*c))))$

sympy [B] time = 4.36, size = 564, normalized size = 9.10

$$\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) \log\left(x + \frac{24ac^2\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right)^2 - 14b^3b^2\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) - 12a^2c^2\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) + 2a^2b^4\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) + 3a^2b^2c\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) - 12a^2c^2 + 11ab^2c - 2a^3}{abc^2 - 2b^3}\right) \left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) \log\left(x + \frac{24ac^2\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right)^2 - 14b^3b^2\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) - 12a^2c^2\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) + 2a^2b^4\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) + 3a^2b^2c\left(\frac{b\sqrt{4ac-b^2}}{2a(4ac-b^2)} + \frac{1}{2a}\right) - 12a^2c^2 + 11ab^2c - 2a^3}{abc^2 - 2b^3}\right) + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**4+b*x**3+a*x**2),x)

[Out]
$$\begin{aligned} & (-b\sqrt{-4ac + b^2})/(2a(4ac - b^2)) - 1/(2a)) \log(x + (24a^4c^2 \\ & * (-b\sqrt{-4ac + b^2})/(2a(4ac - b^2)) - 1/(2a))^{**2} - 14a^3b^2 \\ & * (-b\sqrt{-4ac + b^2})/(2a(4ac - b^2)) - 1/(2a))^{**2} - 12a^3c^2 \\ & * (-b\sqrt{-4ac + b^2})/(2a(4ac - b^2)) - 1/(2a)) + 2a^2b^4(- \\ & b\sqrt{-4ac + b^2})/(2a(4ac - b^2)) - 1/(2a))^{**2} + 3a^2b^2c(- \\ & b\sqrt{-4ac + b^2})/(2a(4ac - b^2)) - 1/(2a)) - 12a^2c^2 + 11a \\ & * b^2c - 2b^4)/(9ab^2c^2 - 2b^3c) + (b\sqrt{-4ac + b^2})/(2a(4 \\ & ac - b^2)) - 1/(2a)) \log(x + (24a^4c^2*(b\sqrt{-4ac + b^2})/(2a \\ & (4ac - b^2)) - 1/(2a))^{**2} - 14a^3b^2c*(b\sqrt{-4ac + b^2})/(2a \\ & (4ac - b^2)) - 1/(2a))^{**2} - 12a^3c^2*(b\sqrt{-4ac + b^2})/(2a(4 \\ & ac - b^2)) - 1/(2a)) + 2a^2b^4*(b\sqrt{-4ac + b^2})/(2a(4ac - \\ & b^2)) - 1/(2a))^{**2} + 3a^2b^2c*(b\sqrt{-4ac + b^2})/(2a(4ac - \\ & b^2)) - 1/(2a)) - 12a^2c^2 + 11ab^2c - 2b^4)/(9ab^2c^2 - 2b^3 \\ & c) + \log(x)/a \end{aligned}$$

$$3.16 \quad \int \frac{1}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1594, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-1), x]

[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 709

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^{(m_.)}}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol]$
 $\text{Int}[\frac{(d + e*x)^{(m + 1)}}{(m + 1)*(c*d^2 - b*d*e + a*e^2)}, x] + \text{Dist}[1/(c*d^2 - b*d*e + a*e^2), \text{Int}[\frac{(d + e*x)^{(m + 1)*\text{Simp}[c*d - b*e - c*e*x, x]}{(a + b*x + c*x^2)}, x], x] /;$
 $\text{FreeQ}[\{a, b, c, d, e, m\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{((d_.) + (e_.)*(x_.)^{(m_.)})*((f_.) + (g_.)*(x_.)}}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol]$
 $\text{Int}[\text{ExpandIntegrand}[\frac{(d + e*x)^m*(f + g*x)}{(a + b*x + c*x^2)}, x], x] /;$
 $\text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[m]$

Rule 1594

$\text{Int}[(u_.)*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)} + (c_.)*(x_.)^{(r_.)})^{(n_.)}, x_Symbol]$
 $\text{Int}[u*x^{(n*p)}*(a + b*x^{(q - p)} + c*x^{(r - p)})^n, x] /;$
 $\text{FreeQ}[\{a, b, c, p, q, r\}, x] \&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p] \&\& \text{PosQ}[r - p]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax^2 + bx^3 + cx^4} dx &= \int \frac{1}{x^2(a + bx + cx^2)} dx \\
&= -\frac{1}{ax} + \frac{\int \frac{-b-cx}{x(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{ax} + \frac{\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx}{a} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx}{a^2} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^2} + \frac{(b^2-2ac) \int \frac{1}{a+bx+cx^2} dx}{2a^2} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{(b^2-2ac) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx \right)}{a^2} \\
&= -\frac{1}{ax} - \frac{(b^2-2ac) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{a^2 \sqrt{b^2-4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx + cx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.95

$$\frac{2(b^2-2ac) \tan^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right) + b \log(a + x(b + cx)) - \frac{2a}{x} - 2b \log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]

[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)]/(2*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^2 + bx^3 + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(-1),x]

[Out] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(-1), x]

fricas [A] time = 1.04, size = 269, normalized size = 3.32

$$\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{c^2 + bx + a}\right) + 2ab^2 - 8a^2c - (b^3 - 4abc)x \log(cx^2 + bx + a) + 2(b^3 - 4abc)x \log(x) - 2(b^2 - 2ac)\sqrt{-b^2 + 4ac} x \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + 2ab^2 - 8a^2c - (b^3 - 4abc)x \log(cx^2 + bx + a) + 2(b^3 - 4abc)x \log(x)}{2(a^2b^2 - 4a^3c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*a^3*c)*x)]

giac [A] time = 0.33, size = 79, normalized size = 0.98

$$\frac{b \log(cx^2 + bx + a)}{2a^2} - \frac{b \log(|x|)}{a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac} a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)

maple [A] time = 0.01, size = 112, normalized size = 1.38

$$-\frac{2c \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} a} + \frac{b^2 \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} a^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^2 + bx + a)}{2a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^3+a*x^2),x)

[Out] -1/a/x - 1/a^2*b*ln(x) + 1/2/a^2*b*ln(c*x^2 + b*x + a) - 2/a/(4*a*c - b^2)^(1/2)*arctan((2*c*x + b)/(4*a*c - b^2)^(1/2))*c + 1/a^2/(4*a*c - b^2)^(1/2)*arctan((2*c*x + b)/(4*a*c - b^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.50, size = 339, normalized size = 4.19

$$\frac{\ln\left(\frac{2ab^3 + 2b^4x - 2ab^2\sqrt{b^2 - 4ac} + a^2c\sqrt{b^2 - 4ac} - 2b^3x\sqrt{b^2 - 4ac} + 2a^2c^2x - 7a^2bc - 8ab^2cx + 4ab^2cx\sqrt{b^2 - 4ac}}{4a^3c - a^2b^2}\right) \left(a \left(2bc - c\sqrt{b^2 - 4ac} \right) - \frac{b^2}{a} + \frac{b^2\sqrt{4ac}}{a} \right) - \frac{1}{2a}}{\ln\left(\frac{2ab^3 + 2b^4x + 2ab^2\sqrt{b^2 - 4ac} - a^2c\sqrt{b^2 - 4ac} + 2b^3x\sqrt{b^2 - 4ac} + 2a^2c^2x - 7a^2bc - 8ab^2cx - 4ab^2cx\sqrt{b^2 - 4ac}}{4a^3c - a^2b^2}\right) \left(\frac{b^2}{a} - a \left(2bc + c\sqrt{b^2 - 4ac} \right) + \frac{b^2\sqrt{4ac}}{a} \right) - \frac{b \ln(x)}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^2 + b*x^3 + c*x^4),x)

[Out] $(\log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^{(1/2)} + a^2*c*(b^2 - 4*a*c)^{(1/2)} - 2*b^3*x*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x + 4*a*b*c*x*(b^2 - 4*a*c)^{(1/2}))* (a*(2*b*c - c*(b^2 - 4*a*c)^{(1/2})) - b^3/2 + (b^2*(b^2 - 4*a*c)^{(1/2}))/2))/ (4*a^3*c - a^2*b^2) - 1/(a*x) - (\log(2*a*b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^{(1/2)} - a^2*c*(b^2 - 4*a*c)^{(1/2)} + 2*b^3*x*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*c*x*(b^2 - 4*a*c)^{(1/2}))* (b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^{(1/2})) + (b^2*(b^2 - 4*a*c)^{(1/2}))/2))/ (4*a^3*c - a^2*b^2) - (b*\log(x))/a^2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+b*x**3+a*x**2),x)

[Out] Timed out

$$3.17 \quad \int \frac{1}{x(ax^2+bx^3+cx^4)} dx$$

Optimal. Leaf size=104

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{\log(x)(b^2-ac)}{a^3} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{\log(x)(b^2-ac)}{a^3} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)/(a + bx + cx^2)}{x} dx$; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 709

$\int \frac{(d + e*x)^m}{(a + b*x + c*x^2)} dx$; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

$\int \frac{(d + e*x)^m * (f + g*x)}{a + b*x + c*x^2} dx$; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1585

$\int u * x^m * (a + b*x^p + c*x^q)^n dx$; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax^2 + bx^3 + cx^4)} dx &= \int \frac{1}{x^3(a + bx + cx^2)} dx \\
&= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)} \right) dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2} dx}{a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac)) \int \frac{1}{a+bx+cx^2} dx}{2a^3} - \frac{(b^2-ac) \int \frac{b+2c}{a+bx+cx^2} dx}{2a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{(b(b^2-3ac)) \operatorname{Subst}\left(\int \frac{1}{u} du, a+bx+cx^2\right)}{2a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2} + 2\log(x)(b^2-ac) + (ac-b^2)\log(a+x(b+cx)) - \frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] $(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*\operatorname{ArcTan}[(b + 2*c*x)/\operatorname{Sqrt}[-b^2 + 4*a*c]])/\operatorname{Sqrt}[-b^2 + 4*a*c] + 2*(b^2 - a*c)*\operatorname{Log}[x] + (-b^2 + a*c)*\operatorname{Log}[a + x*(b + c*x)]/(2*a^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(ax^2 + bx^3 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3 + c*x^4)), x]

fricas [A] time = 0.91, size = 358, normalized size = 3.44

$$\frac{\left((b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2 + 2bx^2 - 2c\sqrt{b^2 - 4ac}x + ab}{2c^2 + 2bx^2}\right) + a^2 b^2 - 4a^2 c + (b^4 - 5ab^2c + 4a^2c^2)^2 \log(cx^2 + bx + a) - 2(b^4 - 5ab^2c + 4a^2c^2)^2 \log(x) - 2(ab^3 - 4a^2bc) \cdot 2(b^3 - 3abc)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}x}{2c^2 + 2bx^2}\right) - a^2 b^2 + 4a^2 c - (b^4 - 5ab^2c + 4a^2c^2)^2 \log(cx^2 + bx + a) + 2(b^4 - 5ab^2c + 4a^2c^2)^2 \log(x) + 2(ab^3 - 4a^2bc) \cdot 2(a^3 b^2 - 4a^4 c)^2 \right)}{2(a^3 b^2 - 4a^4 c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]

giac [A] time = 0.51, size = 105, normalized size = 1.01

$$-\frac{(b^2 - ac) \log(cx^2 + bx + a)}{2a^3} + \frac{(b^2 - ac) \log(|x|)}{a^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^3} + \frac{2abx - a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)

maple [A] time = 0.01, size = 150, normalized size = 1.44

$$\frac{3bc \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^2} - \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^3} - \frac{c \ln(x)}{a^2} + \frac{c \ln(cx^2 + bx + a)}{2a^2} + \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(cx^2 + bx + a)}{2a^3} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^3+a*x^2),x)

[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+1/a^2*b/x+1/2/a^2*c*ln(c*x^2+b*x+a)-1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.59, size = 447, normalized size = 4.30

$$\frac{\ln(2a^2 + 2b^2 + 6a^2c + 2ab\sqrt{4ac - b^2} + 2b^2\sqrt{4ac - b^2} - 9a^2c - 10ab^2c - 3ab^2\sqrt{4ac - b^2} + 9a^2c^2 + 3a^2b\sqrt{4ac - b^2} - 6ab^2c\sqrt{4ac - b^2})}{4a^2c^2} - \frac{\ln\left(\frac{bx + a}{c}\right) + \frac{bx + a}{c}}{4a^2c^2} + \frac{\ln(2a^2 + 2b^2 + 6a^2c - 2ab\sqrt{4ac - b^2} - 2b^2\sqrt{4ac - b^2} - 9a^2c - 10ab^2c + 3ab^2\sqrt{4ac - b^2} + 9a^2c^2 - 3a^2b\sqrt{4ac - b^2} + 6ab^2c\sqrt{4ac - b^2})}{4a^2c^2} - \frac{\ln\left(\frac{bx + a}{c}\right) + \frac{bx + a}{c}}{4a^2c^2} - \frac{\ln(x) + \frac{bx + a}{c}}{4a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)),x)

[Out]
$$\begin{aligned} & (\log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^{(1/2)} + 2*b^4*x*(b^2 - 4*a*c)^{(1/2)} - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^{(1/2)} + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^{(1/2)} - 6*a*b^2*c*x*(b^2 - 4*a*c)^{(1/2)}) * (b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^{(1/2}))/2) + (b^3*(b^2 - 4*a*c)^{(1/2}))/2 + 2*a^2*c^2)) / (4*a^4*c - a^3*b^2) - (\log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^{(1/2)} - 2*b^4*x*(b^2 - 4*a*c)^{(1/2)} - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^{(1/2)} + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^{(1/2)} + 6*a*b^2*c*x*(b^2 - 4*a*c)^{(1/2)}) * (a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^{(1/2}))/2) - b^4/2 + (b^3*(b^2 - 4*a*c)^{(1/2}))/2 - 2*a^2*c^2)) / (4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2) / x^2 - (\log(x)*(a*c - b^2))/a^3 \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+b*x**3+a*x**2),x)

[Out] Timed out

$$3.18 \quad \int \frac{1}{x^2(ax^2+bx^3+cx^4)} dx$$

Optimal. Leaf size=137

$$\frac{b(b^2 - 2ac) \log(a + bx + cx^2)}{2a^4} - \frac{b \log(x)(b^2 - 2ac)}{a^4} - \frac{b^2 - ac}{a^3x} + \frac{b}{2a^2x^2} - \frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{1}{3ax^3}$$

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} + \frac{b(b^2 - 2ac) \log(a + bx + cx^2)}{2a^4} - \frac{b^2 - ac}{a^3x} - \frac{b \log(x)(b^2 - 2ac)}{a^4} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax^2 + bx^3 + cx^4)} dx &= \int \frac{1}{x^4(a + bx + cx^2)} dx \\
&= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)} \right) dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2} dx}{a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac)) \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^4} + \frac{(b^4-4ab^2c+2a^2c^2)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\operatorname{atanh}^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(a+bx+cx^2)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3} + \frac{6(2a^2c^2-4ab^2c+b^4)\operatorname{atan}^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{3a^2b}{x^2} - 6\log(x)(b^3-2abc) + 3(b^3-2abc)\log(a+x(b+cx)) + \frac{6a(ac-b^2)}{x}}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)]/(6*a^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(ax^2 + bx^3 + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] IntegrateAlgebraic[1/(x^2*(a*x^2 + b*x^3 + c*x^4)), x]

fricas [A] time = 1.12, size = 445, normalized size = 3.25

$$\frac{3(b^3 - 4ab^2 + 2a^2b)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + b^2 - 2acx - \sqrt{b^2 - 4ac}(2cx + b)}{c^2x^2 + bx + a}\right) + 2a^3b^2 - 8a^4c + 3(b^5 - 6a^2b^3c + 8a^2b^2c^2)x^3 \log(c^2x^2 + bx + a) - 6(b^5 - 6a^2b^3c + 8a^2b^2c^2)x^3 \log(x) - 6(a^4b^2 - 4a^5c)x^3 \log\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right) + 2a^3b^2 - 8a^4c - 3(b^5 - 6a^2b^3c + 8a^2b^2c^2)x^3 \log(c^2x^2 + bx + a) + 6(a^4b^2 - 4a^5c)x^3 \log(x) + 6(a^4b^2 - 4a^5c)x^3 \arctan\left(\frac{-\sqrt{-b^2 + 4ac}}{2cx + b}\right) + 2a^3b^2 - 8a^4c - 3(b^5 - 6a^2b^3c + 8a^2b^2c^2)x^3 \log(c^2x^2 + bx + a) + 6(a^4b^2 - 4a^5c)x^3 \log(x) + 6(a^4b^2 - 4a^5c)x^3 \arctan\left(\frac{-\sqrt{-b^2 + 4ac}}{2cx + b}\right)}{6(a^2b^2 - 4a^3c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]

giac [A] time = 0.46, size = 136, normalized size = 0.99

$$\frac{(b^3 - 2abc) \log(cx^2 + bx + a)}{2a^4} - \frac{(b^3 - 2abc) \log(|x|)}{a^4} + \frac{(b^4 - 4ab^2c + 2a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^4} + \frac{3a^2bx - 2a^3 - 6(ab^2 - a^2c)x^2}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/ (sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/ (a^4*x^3)

maple [A] time = 0.01, size = 214, normalized size = 1.56

$$\frac{2c^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^2} - \frac{4b^2c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^3} + \frac{b^4 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}a^4} + \frac{2bc \ln(x)}{a^3} - \frac{bc \ln(cx^2 + bx + a)}{a^3} - \frac{b^3 \ln(x)}{a^4} + \frac{b^3 \ln(cx^2 + bx + a)}{2a^4} + \frac{c}{a^2x} - \frac{b^2}{a^3x} + \frac{b}{2a^2x^2} - \frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^3+a*x^2),x)

[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2/a^2*b/x^2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan

$$\frac{(2cx+b)\sqrt{4ac-b^2}}{(4ac-b^2)^{3/2}} + \frac{1}{a^2\sqrt{4ac-b^2}} \arctan\left(\frac{(2cx+b)\sqrt{4ac-b^2}}{(4ac-b^2)^{3/2}}\right)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.60, size = 524, normalized size = 3.82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x)

[Out] $\log(2ab^4(b^2 - 4ac)^{1/2} - 2b^6x - 2ab^5 + 2b^5x(b^2 - 4ac)^{1/2} + 11a^2b^3c - 13a^3bc^2 + 2a^3c^3x + a^3c^2(b^2 - 4ac)^{1/2} - 17a^2b^2c^2x + 12ab^4cx - 5a^2b^2c(b^2 - 4ac)^{1/2} - 8ab^3cx(b^2 - 4ac)^{1/2} + 7a^2b^2c^2x(b^2 - 4ac)^{1/2}) \cdot (b^3/(2a^4) - (b^2(b^2 - 4ac)^{1/2})/(2a^4) - (bc)/a^3 + (a^2c^2(b^2 - 4ac)^{1/2})/(4a^5c - a^4b^2)) + \log(2ab^5 + 2b^6x + 2ab^4(b^2 - 4ac)^{1/2} + 2b^5x(b^2 - 4ac)^{1/2} - 11a^2b^3c + 13a^3bc^2 - 2a^3c^3x + a^3c^2(b^2 - 4ac)^{1/2} + 17a^2b^2c^2x - 12ab^4cx - 5a^2b^2c(b^2 - 4ac)^{1/2} - 8ab^3cx(b^2 - 4ac)^{1/2} + 7a^2b^2c^2x(b^2 - 4ac)^{1/2}) \cdot (b^3/(2a^4) + (b^2(b^2 - 4ac)^{1/2})/(2a^4) - (bc)/a^3 - (a^2c^2(b^2 - 4ac)^{1/2})/(4a^5c - a^4b^2)) + ((x^2(ac - b^2))/a^3 - 1/(3a) + (bx)/(2a^2))/x^3 + (b \log(x)(2ac - b^2))/a^4$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)

[Out] Timed out

$$3.19 \quad \int \frac{x^8}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=150

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{b \log(a+bx+cx^2)}{c^3}$$

Rubi [A] time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 738, 800, 634, 618, 206, 628}

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{bx^2}{c(b^2-4ac)} - \frac{b \log(a+bx+cx^2)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int((((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x^4}{(a + bx + cx^2)^2} dx \\
&= \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)} \right) dx}{-b^2 + 4ac} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2 \int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2} dx}{c^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{c^3} + \frac{(b^4 - 6ab^2c + 6a^2c^2)}{c^3} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{b \log(a + bx + cx^2)}{c^3} - \frac{2(b^4 - 6ab^2c + 6a^2c^2)}{c^3}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 132, normalized size = 0.88

$$\frac{-\frac{2(6a^2c^2-6ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{(b^2-4ac)(a+x(b+cx))} - b \log(a + x(b + cx)) + cx}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (c*x + (-b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)]/c^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^8/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 0.89, size = 837, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) \\ &) * x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3) * x^2 + (a*b^4 - 6*a^2*b^2*c + 6 \\ & * a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3) * x^2 + (b^5 - 6*a*b^3*c + 6*a^2 \\ & * b*c^2) * x) * \sqrt{b^2 - 4*a*c} * \log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c}) \\ & * (2*c*x + b)) / (c*x^2 + b*x + a) + (b^6 - 9*a*b^4*c + 26*a^2*b^2 \\ & * c^2 - 24*a^3*c^3) * x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a \\ & * b^3*c^2 + 16*a^2*b*c^3) * x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2) * x) * \log(c*x \\ & ^2 + b*x + a) / (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2 \\ & * c^5 + 16*a^2*c^6) * x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5) * x), -(a*b^5 \\ & - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) * x^3 - \\ & (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3) * x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c \\ & ^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3) * x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 \\ &) * x) * \sqrt{-b^2 + 4*a*c} * \arctan(-\sqrt{-b^2 + 4*a*c} * (2*c*x + b) / (b^2 - 4*a*c \\ &)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3) * x + (a*b^5 - 8*a^2*b^3 \\ & * c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3) * x^2 + (b^6 - 8*a*b \\ & ^4*c + 16*a^2*b^2*c^2) * x) * \log(c*x^2 + b*x + a) / (a*b^4*c^3 - 8*a^2*b^2*c^4 \\ & + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6) * x^2 + (b^5*c^3 - 8*a*b^3 \\ & * c^4 + 16*a^2*b*c^5) * x)] \end{aligned}$$

giac [A] time = 0.57, size = 161, normalized size = 1.07

$$\frac{2(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^3 - 4ac^4)\sqrt{-b^2+4ac}} + \frac{x}{c^2} - \frac{b \log(cx^2 + bx + a)}{c^3} - \frac{\frac{(b^4 - 4ab^2c + 2a^2c^2)x}{c} + \frac{ab^3 - 3a^2bc}{c}}{(cx^2 + bx + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2) * \arctan((2*c*x + b) / \sqrt{-b^2 + 4*a*c}) / ((b^2 \\ & * c^3 - 4*a*c^4) * \sqrt{-b^2 + 4*a*c}) + x/c^2 - b * \log(c*x^2 + b*x + a) / c^3 - \\ & ((b^4 - 4*a*b^2*c + 2*a^2*c^2) * x / c + (a*b^3 - 3*a^2*b*c) / c) / ((c*x^2 + b*x \\ & + a) * (b^2 - 4*a*c) * c^2) \end{aligned}$$

maple [B] time = 0.01, size = 352, normalized size = 2.35

$$\frac{2a^2x}{(cx^2+bx+a)(4ac-b^2)c} - \frac{12a^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c} - \frac{4ab^2x}{(cx^2+bx+a)(4ac-b^2)c^2} + \frac{12ab^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^2} + \frac{b^4x}{(cx^2+bx+a)(4ac-b^2)c^3} - \frac{2b^4 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^3} - \frac{3a^2b}{(cx^2+bx+a)(4ac-b^2)c^2} + \frac{ab^3}{(cx^2+bx+a)(4ac-b^2)c^3} - \frac{4ab \ln(cx^2+bx+a)}{(4ac-b^2)c^2} + \frac{b^3 \ln(cx^2+bx+a)}{(4ac-b^2)c^3} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.46, size = 261, normalized size = 1.74

$$\frac{x}{c^2} + \frac{a(b^3-3abc)}{c^3x^2+bc^2x+ac^2} + \frac{x(2a^2c^2-4ab^2c+b^4)}{c(4ac-b^2)} + \frac{\ln(cx^2+bx+a)(-128a^3bc^3+96a^2b^3c^2-24ab^5c+2b^7)}{2(64a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3)} - \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{c^3(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))

sympy [B] time = 1.79, size = 842, normalized size = 5.61

$$\left(\frac{1}{2} \sqrt{\frac{4ac-b^2}{16a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3}}\right) \left[\frac{-16a^3bc^3-16a^2b^3c^2-24ab^5c+2b^7}{12a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3} + 2ab^2 \left(\frac{1}{2} \sqrt{\frac{4ac-b^2}{16a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3}} \right) - b^2 \left(\frac{1}{2} \sqrt{\frac{4ac-b^2}{16a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3}} \right) \right] + \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{12a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3} + \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{12a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3} + \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{12a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3} + \frac{2 \operatorname{atan}\left(\frac{2cx}{\sqrt{4ac-b^2}} - \frac{b^3c^2-4abc^3}{c^2(4ac-b^2)^{3/2}}\right)(6a^2c^2-6ab^2c+b^4)}{12a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)

[Out]
$$\begin{aligned} & (-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3} \\ & *(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))*\log(x + (-10*a^{**2}*b*c - 16*a^{**2}*c^{**4} \\ & *(-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & + 2*a*b^{**3} + 8*a*b^{**2}*c^{**3}*(-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & - b^{**4}*c^{**2}*(-b/c^{**3} - \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})))) \\ &)/(12*a^{**2}*c^{**2} - 12*a*b^{**2}*c + 2*b^{**4})) + (-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & *\log(x + (-10*a^{**2}*b*c - 16*a^{**2}*c^{**4}*(-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & + 2*a*b^{**3} + 8*a*b^{**2}*c^{**3}*(-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) \\ & - b^{**4}*c^{**2}*(-b/c^{**3} + \sqrt{-(4*a*c - b^{**2})^{**3}}*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})))) \\ &)/(12*a^{**2}*c^{**2} - 12*a*b^{**2}*c + 2*b^{**4})) + (-3*a^{**2}*b*c + a*b^{**3} + x*(2*a^{**2}*c^{**2} - 4*a*b^{**2}*c + b^{**4}))/ \\ & (4*a^{**2}*c^{**4} - a*b^{**2}*c^{**3} + x^{**2}*(4*a*c^{**5} - b^{**2}*c^{**4}) + x*(4*a*b*c^{**4} - b^{**3}*c^{**3})) + x/c^{**2} \end{aligned}$$

$$3.20 \quad \int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=114

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx}{c(b^2 - 4ac)} + \frac{\log(a + bx + cx^2)}{2c^2}$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 738, 773, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx}{c(b^2 - 4ac)} + \frac{\log(a + bx + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1585

```
Int[(u_.)*(x_)^m*((a_.)*(x_)^p + (b_.)*(x_)^q + (c_.)*(x_)^r)^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x^3}{(a + bx + cx^2)^2} dx \\
&= \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2} dx}{c(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} - \frac{(b(b^2 - 6ac)) \int \frac{1}{a+bx+cx^2} dx}{2c^2(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(a + bx + cx^2)}{2c^2} + \frac{(b(b^2 - 6ac)) \operatorname{Su}}{2c^2(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{\log(a + bx + cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 109, normalized size = 0.96

$$\frac{2(-2a^2c+ab(b-3cx)+b^3x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \log(a + x(b + cx))}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + x*(b + c*x)]/(2*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^7/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 0.87, size = 635, normalized size = 5.57

$$\frac{2a^4 - 12a^3b + 16a^2b^2 - (a^2 - 4ab + b^2)\sqrt{-b^2 + 4ac} \log\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right) - 2(b^3 - 6abc) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right) - 2(b^5 - 7a^2b^3c + 12a^2b^2c^2)x - (a^2b^3 - 6a^2b^2c + (b^3c - 6a^2b^2c^2))x^2 + (b^4 - 6a^2b^2c)x \sqrt{b^2 - 4ac} \log((2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac})(2cx + b)/(cx^2 + bx + a)) + 2(b^5 - 7a^2b^3c + 12a^2b^2c^2)x + (a^2b^4 - 8a^2b^2c^2 + 16a^3c^2 + (b^4c - 8a^2b^2c^2 + 16a^2c^3))x^2 + (b^5 - 8a^2b^3c + 16a^2b^2c^2)x \log(cx^2 + bx + a)/(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4 + (b^4c^3 - 8a^2b^2c^4 + 16a^2c^5))x^2 + (b^5c^2 - 8a^2b^3c^3 + 16a^2b^2c^4)x, 1/2(2a^2b^4 - 12a^2b^2c + 16a^3c^2 + 2(a^2b^3 - 6a^2b^2c + (b^3c - 6a^2b^2c^2))x^2 + (b^4 - 6a^2b^2c)x) \sqrt{-b^2 + 4ac} \arctan(-\sqrt{-b^2 + 4ac}(2cx + b)/(b^2 - 4ac)) + 2(b^5 - 7a^2b^3c + 12a^2b^2c^2)x + (a^2b^4 - 8a^2b^2c^2 + 16a^3c^2 + (b^4c - 8a^2b^2c^2 + 16a^2c^3))x^2 + (b^5 - 8a^2b^3c + 16a^2b^2c^2)x \log(cx^2 + bx + a)/(a^2b^4c^2 - 8a^2b^2c^3 + 16a^3c^4 + (b^4c^3 - 8a^2b^2c^4 + 16a^2c^5))x^2 + (b^5c^2 - 8a^2b^3c^3 + 16a^2b^2c^4)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2))*x^2 + (b^4 - 6*a*b^2*c)*x)*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \text{sqrt}(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b^2*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b^2*c^2)*x) * \\ & \log(c*x^2 + b*x + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b^2*c^4)*x), \\ & 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2))*x^2 + (b^4 - 6*a*b^2*c)*x)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b^2*c^2)*x + \\ & (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b^2*c^2)*x)*\log(c*x^2 + b*x + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b^2*c^4)*x] \end{aligned}$$

giac [A] time = 0.60, size = 125, normalized size = 1.10

$$-\frac{(b^3 - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^2 - 4ac^3)\sqrt{-b^2+4ac}} + \frac{\log(cx^2 + bx + a)}{2c^2} + \frac{ab^2 - 2a^2c + (b^3 - 3abc)x}{(cx^2 + bx + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out]
$$-(b^3 - 6a*b*c)*\arctan((2*c*x + b)/\text{sqrt}(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*\text{sqrt}(-b^2 + 4*a*c)) + 1/2*\log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)$$

maple [A] time = 0.01, size = 209, normalized size = 1.83

$$-\frac{6ab \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}c} + \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}c^2} + \frac{2a \ln(cx^2 + bx + a)}{(4ac-b^2)c} - \frac{b^2 \ln(cx^2 + bx + a)}{2(4ac-b^2)c^2} + \frac{\frac{(3ac-b^2)bx}{(4ac-b^2)c^2} + \frac{(2ac-b^2)a}{(4ac-b^2)c^2}}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^7/(c*x^4+b*x^3+a*x^2)^2,x)$

[Out] $((3*a*c-b^2)/(4*a*c-b^2)*b/c^2*x+(2*a*c-b^2)/(4*a*c-b^2)*a/c^2)/(c*x^2+b*x+a)+2/c/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^2-6/c/(4*a*c-b^2)^{3/2}*\arctan((2*c*x+b)/(4*a*c-b^2)^{1/2})*a*b+1/c^2/(4*a*c-b^2)^{3/2}*\arctan((2*c*x+b)/(4*a*c-b^2)^{1/2})*b^3$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^7/(c*x^4+b*x^3+a*x^2)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.49, size = 279, normalized size = 2.45

$$\frac{a(2ac-b^2)}{c^2(4ac-b^2)} + \frac{bx(3ac-b^2)}{c^2(4ac-b^2)} - \frac{\ln(cx^2+bx+a)(-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6)}{2(64a^3c^5-48a^2b^2c^4+12ab^4c^3-b^6c^2)} + \frac{b \operatorname{atan}\left(\frac{c^2(4ac-b^2)^{3/2}\left(\frac{2bx(6ac-b^2)}{c(4ac-b^2)^3} + \frac{b^2(4ac^2-b^2c)(6ac-b^2)}{c^3(4ac-b^2)^4}\right)}{b^3-6abc}\right)}{c^2(4ac-b^2)^{3/2}}(6ac-b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^7/(a*x^2 + b*x^3 + c*x^4)^2,x)$

[Out] $((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (\log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c^2))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (b*\operatorname{atan}((c^2*(4*a*c - b^2))^{5/2}*((2*b*x*(6*a*c - b^2))/(c*(4*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/(b^3 - 6*a*b*c)*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^{3/2})$

sympy [B] time = 1.37, size = 729, normalized size = 6.39

$$\frac{\sqrt{a^2-4ac}}{2\sqrt{a^2-4ac-12ab^2}} \frac{1}{2\sqrt{a^2-4ac}} \ln\left(\frac{-18a^2\sqrt{a^2-4ac} \left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right) + 6a^2\sqrt{a^2-4ac} \left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right) + a^2 - \sqrt{a^2-4ac} \left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right)}{4a^2-4a^2\sqrt{a^2-4ac} + 12a^2b^2}\right) + \frac{\sqrt{a^2-4ac} \sqrt{a^2-4ac} \operatorname{atan}\left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right)}{2\sqrt{a^2-4ac} \sqrt{a^2-4ac} + 12a^2b^2} \sqrt{a^2-4ac} \operatorname{atan}\left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right) + \frac{1}{2\sqrt{a^2-4ac}} \operatorname{atan}\left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right)}{4a^2-4a^2\sqrt{a^2-4ac} + 12a^2b^2} \sqrt{a^2-4ac} \operatorname{atan}\left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right) + \frac{1}{2\sqrt{a^2-4ac}} \operatorname{atan}\left(\frac{\sqrt{a^2-4ac}(4ac-4a^2)}{2\sqrt{a^2-4ac-12ab^2}} + \frac{1}{2\sqrt{a^2-4ac}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**7/(c*x**4+b*x**3+a*x**2)**2,x)$

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

$$3.21 \quad \int \frac{x^6}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=67

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1585, 722, 618, 206}

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 722

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 - b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,

0] && LtQ[p, -1]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x^2}{(a + bx + cx^2)^2} dx \\ &= \frac{x(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2a) \int \frac{1}{a+bx+cx^2} dx}{b^2 - 4ac} \\ &= \frac{x(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(4a) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\ &= \frac{x(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{4a \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 1.21

$$\frac{a(b - 2cx) + b^2x}{c(4ac - b^2)(a + x(b + cx))} + \frac{4a \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 1.04, size = 387, normalized size = 5.78

$$\left[\frac{ab^3 - 4a^2bc + 2(ac^2x^2 + abcx + a^2c)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^4 - 6ab^2c + 8a^2c^2)x}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x}, -\frac{ab^3 - 4a^2bc - 4(ac^2x^2 + abcx + a^2c)\sqrt{-b^2 + 4ac} \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + (b^4 - 6ab^2c + 8a^2c^2)x}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $[-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*\text{sqrt}(b^2 - 4*a*c))*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \text{sqrt}(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]$

giac [A] time = 0.49, size = 88, normalized size = 1.31

$$-\frac{4a \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} - \frac{b^2x-2acx+ab}{(b^2c-4ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $-4*a*\arctan((2*c*x + b)/\text{sqrt}(-b^2 + 4*a*c))/((b^2 - 4*a*c)*\text{sqrt}(-b^2 + 4*a*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))$

maple [A] time = 0.01, size = 97, normalized size = 1.45

$$\frac{4a \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{\frac{ab}{(4ac-b^2)c} - \frac{(2ac-b^2)x}{(4ac-b^2)c}}{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] $(-(2ac-b^2)/(4ac-b^2)/cx+1/(4ac-b^2)*ab/c)/(cx^2+bx+a)+4a/(4ac-b^2)^{3/2}*\arctan((2cx+b)/(4ac-b^2)^{1/2})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4ac-b^2>0)', see `assume?` for more details) Is 4ac-b^2 positive or negative?

mupad [B] time = 2.13, size = 135, normalized size = 2.01

$$\frac{\frac{x(2ac-b^2)}{c(4ac-b^2)} - \frac{ab}{c(4ac-b^2)}}{cx^2+bx+a} - \frac{4a \operatorname{atan}\left(\frac{\left(\frac{2a(b^3-4abc)}{(4ac-b^2)^{5/2}} - \frac{4acx}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2a}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(a*x^2 + b*x^3 + c*x^4)^2,x)`

[Out] $-\left(\frac{x(2ac-b^2)}{c(4ac-b^2)} - \frac{ab}{c(4ac-b^2)}\right) - \frac{(ab)/(c(4ac-b^2))}{(a+bx+c*x^2) - (4a*\operatorname{atan}\left(\frac{(2a*(b^3-4a*b*c))}{(4ac-b^2)^{5/2}} - (4a*c*x)/(4ac-b^2)^{3/2}\right)*(4ac-b^2)/(2a))}{(4ac-b^2)^{3/2}}$

sympy [B] time = 0.60, size = 280, normalized size = 4.18

$$-2a\sqrt{\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)+2a\sqrt{\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)+\frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $-2a*\sqrt{-1/(4ac-b^2)^3}*\log(x+(-32a^3c^2*\sqrt{-1/(4ac-b^2)^3}+16a^2b^2c*\sqrt{-1/(4ac-b^2)^3}-2a*b^4*\sqrt{-1/(4ac-b^2)^3}+2a*b)/(4ac)) + 2a*\sqrt{-1/(4ac-b^2)^3}*\log(x+(32a^3c^2*\sqrt{-1/(4ac-b^2)^3}-16a^2b^2c*\sqrt{-1/(4ac-b^2)^3}+2a*b^4*\sqrt{-1/(4ac-b^2)^3}+2a*b)/(4ac)) + (ab+x*(-2ac+b^2))/(4a^2c^2-ab^2c+x^2(4ac^3-b^2c^2)+x(4abc^2-b^3c)) + x(4a*b*c^2-b^3c)$

$$3.22 \quad \int \frac{x^5}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=66

$$\frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1585, 638, 618, 206}

$$\frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x}{(a + bx + cx^2)^2} dx \\ &= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b \int \frac{1}{a+bx+cx^2} dx}{b^2 - 4ac} \\ &= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2b) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\ &= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 1.05

$$\frac{2a + bx}{(b^2 - 4ac)(a + x(b + cx))} - \frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 0.86, size = 338, normalized size = 5.12

$$\left[\frac{2ab^2 - 8a^2c - (bcx^2 + b^2x + ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^3 - 4abc)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x}, \frac{2ab^2 - 8a^2c - 2(bc^2 + b^2x + ab)\sqrt{-b^2 + 4ac} \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + (b^3 - 4abc)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $[(2ab^2 - 8a^2c - (bcx^2 + b^2x + a*b)*\sqrt{b^2 - 4ac})*\log((2c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4ac})*(2*c*x + b))/(c*x^2 + b*x + a) + (b^3 - 4*a*b*c)*x]/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*\sqrt{-b^2 + 4ac})*\arctan(-\sqrt{-b^2 + 4ac})*(2*c*x + b)/(b^2 - 4*a*c) + (b^3 - 4*a*b*c)*x]/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]$

giac [A] time = 0.38, size = 76, normalized size = 1.15

$$\frac{2b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} + \frac{bx+2a}{(cx^2+bx+a)(b^2-4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $2*b*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2 - 4*a*c)*\sqrt{-b^2 + 4*a*c}) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))$

maple [A] time = 0.00, size = 70, normalized size = 1.06

$$-\frac{2b \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{-bx-2a}{(4ac-b^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] $(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.18, size = 110, normalized size = 1.67

$$-\frac{\frac{2a}{4ac-b^2} + \frac{bx}{4ac-b^2}}{cx^2 + bx + a} - \frac{2b \operatorname{atan}\left(\frac{\left(\frac{b^2}{(4ac-b^2)^{3/2}} + \frac{2bcx}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{b}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/b))/(4*a*c - b^2)^(3/2)

sympy [B] time = 0.56, size = 253, normalized size = 3.83

$$b \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{-16a^2bc^2\sqrt{\frac{1}{(4ac-b^2)^3}} + 8ab^3c\sqrt{\frac{1}{(4ac-b^2)^3}} - b^5\sqrt{\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right) - b \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{16a^2bc^2\sqrt{\frac{1}{(4ac-b^2)^3}} - 8ab^3c\sqrt{\frac{1}{(4ac-b^2)^3}} + b^5\sqrt{\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right) + \frac{-2a - bx}{4a^2c - ab^2 + x^2(4ac^2 - b^2c) + x(4abc - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))

$$3.23 \quad \int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=66

$$\frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1585, 614, 618, 206}

$$\frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{(a + bx + cx^2)^2} dx \\ &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2c) \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\ &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(4c) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\ &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{4c \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 1.06

$$\frac{\frac{4c \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + \frac{b + 2cx}{a + x(b + cx)}}{b^2 - 4ac}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 0.92, size = 341, normalized size = 5.17

$$\left[\frac{b^3 - 4abc + 2(c^2x^2 + bcx + ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2(b^2c - 4ac^2)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x}, -\frac{b^3 - 4abc - 4(c^2x^2 + bcx + ac)\sqrt{-b^2 + 4ac} \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + 2(b^2c - 4ac^2)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] $[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c))*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \text{sqrt}(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c))*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)]$

giac [A] time = 0.46, size = 76, normalized size = 1.15

$$-\frac{4c \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} - \frac{2cx+b}{(cx^2+bx+a)(b^2-4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $-4*c*\arctan((2*c*x + b)/\text{sqrt}(-b^2 + 4*a*c))/((b^2 - 4*a*c)*\text{sqrt}(-b^2 + 4*a*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))$

maple [A] time = 0.00, size = 68, normalized size = 1.03

$$\frac{4c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{2cx+b}{(4ac-b^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] $(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.08, size = 119, normalized size = 1.80

$$\frac{b}{4ac-b^2} + \frac{2cx}{4ac-b^2} - \frac{4c \operatorname{atan}\left(\frac{\left(\frac{2c(b^3-4abc)}{(4ac-b^2)^{5/2}} - \frac{4c^2x}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2c}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))* (4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)

sympy [B] time = 0.59, size = 265, normalized size = 4.02

$$-2c \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{-32a^2c^3 \sqrt{-\frac{1}{(4ac-b^2)^3}} + 16ab^2c^2 \sqrt{-\frac{1}{(4ac-b^2)^3}} - 2b^4c \sqrt{-\frac{1}{(4ac-b^2)^3}} + 2bc}{4c^2}\right) + 2c \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x + \frac{32a^2c^3 \sqrt{-\frac{1}{(4ac-b^2)^3}} - 16ab^2c^2 \sqrt{-\frac{1}{(4ac-b^2)^3}} + 2b^4c \sqrt{-\frac{1}{(4ac-b^2)^3}} + 2bc}{4c^2}\right) + \frac{b+2cx}{4a^2c-ab^2+x^2(4ac^2-b^2c)+x(4abc-b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))

$$3.24 \quad \int \frac{x^3}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=108

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx}{a(b^2 - 4ac)(a + bx + cx^2)}$$

Rubi [A] time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx}{a(b^2 - 4ac)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x(a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \frac{-b^2 + 4ac - bcx}{x(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \left(\frac{-b^2 + 4ac}{ax} + \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a(a + bx + cx^2)} \right) dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a + bx + cx^2} dx}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b + 2cx}{a + bx + cx^2} dx}{2a^2} - \frac{(b(b^2 - 6ac)) \int \frac{1}{a + bx + cx^2} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{(b(b^2 - 6ac)) \text{Subst} \left(\frac{1}{a + bx + cx^2} \right)}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{b(b^2 - 6ac) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{a^2(b^2 - 4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a + bx + cx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 0.99

$$\frac{\frac{2a(-2ac + b^2 + bcx)}{(b^2 - 4ac)(a + x(b + cx))} + \frac{2b(b^2 - 6ac) \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}} - \log(a + x(b + cx)) + 2 \log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 1.40, size = 781, normalized size = 7.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2))*x^2 + (b^4 - 6*a*b^2*c)*x)*\sqrt{b^2 - 4*a*c}*\log((2*c*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2))*x^2 + (b^4 - 6*a*b^2*c)*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)] \end{aligned}$$

giac [A] time = 0.51, size = 126, normalized size = 1.17

$$\frac{(b^3 - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(a^2b^2 - 4a^3c)\sqrt{-b^2+4ac}} - \frac{\log(cx^2 + bx + a)}{2a^2} + \frac{\log(|x|)}{a^2} + \frac{abcx + ab^2 - 2a^2c}{(cx^2 + bx + a)(b^2 - 4ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out]
$$-(b^3 - 6*a*b*c)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((a^2*b^2 - 4*a^3*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*\log(c*x^2 + b*x + a)/a^2 + \log(\text{abs}(x))/a^2 + (a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)$$

maple [B] time = 0.01, size = 237, normalized size = 2.19

$$\frac{bcx}{(cx^2+bx+a)(4ac-b^2)a} - \frac{6bc \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a} + \frac{b^3 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^2} - \frac{b^2}{(cx^2+bx+a)(4ac-b^2)a} - \frac{2c \ln(cx^2+bx+a)}{(4ac-b^2)a} + \frac{b^2 \ln(cx^2+bx+a)}{2(4ac-b^2)a^2} + \frac{2c}{(cx^2+bx+a)(4ac-b^2)} + \frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] 1/a^2*ln(x)-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)*c-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/a^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.87, size = 620, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2))/(2*a^2*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*

$$x*(-(4*a*c - b^2)^3)^{(1/2)}*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^{(1/2)})/(2*a^2*(4*a*c - b^2)^3)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] Timed out

$$3.25 \quad \int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=148

$$\frac{b \log(a + bx + cx^2)}{a^3} - \frac{2b \log(x)}{a^3} - \frac{2(b^2 - 3ac)}{a^2 x (b^2 - 4ac)} - \frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3 (b^2 - 4ac)^{3/2}} + \frac{-2ac + b^2 + b}{ax (b^2 - 4ac) (a + b)}$$

Rubi [A] time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$-\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3 (b^2 - 4ac)^{3/2}} - \frac{2(b^2 - 3ac)}{a^2 x (b^2 - 4ac)} + \frac{b \log(a + bx + cx^2)}{a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac + b^2 + bcx}{ax (b^2 - 4ac) (a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^2 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{\int \frac{-2(b^2 - 3ac) - 2bcx}{x^2(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{\int \left(\frac{2(-b^2 + 3ac)}{ax^2} - \frac{2b(-b^2 + 4ac)}{a^2x} + \frac{2(-b^4 + 5ab^2c - 3a^2c^2 - bc(b^2 - 4ac))}{a^2(a + bx + cx^2)} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} - \frac{2 \int \frac{-b^4 + 5ab^2c - 3a^2c^2 - bc(b^2 - 4ac)}{a + bx + cx^2} dx}{a^3(b^2 - 4ac)} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \int \frac{b + 2cx}{a + bx + cx^2} dx}{a^3} + \frac{b \log(a + bx + cx^2)}{a^3} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \log(a + bx + cx^2)}{a^3} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1}\left(\frac{b + 2cx}{a + bx + cx^2}\right)}{a^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 131, normalized size = 0.89

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right) + \frac{a(-3abc - 2ac^2x + b^3 + b^2cx)}{(b^2 - 4ac)(a + x(b + cx))} - b \log(a + x(b + cx)) + \frac{a}{x} + 2b \log(x)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 1.36, size = 975, normalized size = 6.59

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-(a^2b^4 - 8a^3b^2c + 16a^4c^2 + 2*(ab^4c - 7a^2b^2c^2 + 12a^3c^3)*x^2 + ((b^4c - 6a*b^2c^2 + 6a^2c^3)*x^3 + (b^5 - 6a*b^3c + 6a^2b*c^2)*x^2 + (a*b^4 - 6a^2b^2c + 6a^3c^2)*x)*\sqrt{b^2 - 4ac}*\log((2c^2*x^2 + 2b*c*x + b^2 - 2ac + \sqrt{b^2 - 4ac})*(2cx + b))/(cx^2 + bx + a) + (2ab^5 - 15a^2b^3c + 28a^3b*c^2)*x - ((b^5c - 8a*b^3c^2 + 16a^2b*c^3)*x^3 + (b^6 - 8a*b^4c + 16a^2b^2c^2)*x^2 + (ab^5 - 8a^2b^3c + 16a^3b*c^2)*x)*\log(cx^2 + bx + a) + 2*((b^5c - 8a*b^3c^2 + 16a^2b*c^3)*x^3 + (b^6 - 8a*b^4c + 16a^2b^2c^2)*x^2 + (ab^5 - 8a^2b^3c + 16a^3b*c^2)*x)*\log(x)]/((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)*x^3 + (a^3b^5 - 8a^4b^3c + 16a^5b*c^2)*x^2 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)*x), -(a^2b^4 - 8a^3b^2c + 16a^4c^2 + 2*(ab^4c - 7a^2b^2c^2 + 12a^3c^3)*x^2 + 2*((b^4c - 6a*b^2c^2 + 6a^2c^3)*x^3 + (b^5 - 6a*b^3c + 6a^2b*c^2)*x^2 + (a*b^4 - 6a^2b^2c + 6a^3c^2)*x)*\sqrt{-b^2 + 4ac}*\arctan(-\sqrt{-b^2 + 4ac}*(2cx + b)/(b^2 - 4ac)) + (2ab^5 - 15a^2b^3c + 28a^3b*c^2)*x - ((b^5c - 8a*b^3c^2 + 16a^2b*c^3)*x^3 + (b^6 - 8a*b^4c + 16a^2b^2c^2)*x^2 + (ab^5 - 8a^2b^3c + 16a^3b*c^2)*x)*\log(cx^2 + bx + a) + 2*((b^5c - 8a*b^3c^2 + 16a^2b*c^3)*x^3 + (b^6 - 8a*b^4c + 16a^2b^2c^2)*x^2 + (ab^5 - 8a^2b^3c + 16a^3b*c^2)*x)*\log(x)]/((a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)*x^3 + (a^3b^5 - 8a^4b^3c + 16a^5b*c^2)*x^2 + (a^4b^4 - 8a^5b^2c + 16a^6c^2)*x)] \end{aligned}$$

giac [A] time = 0.45, size = 171, normalized size = 1.16

$$\frac{2(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(a^3b^2 - 4a^4c)\sqrt{-b^2+4ac}} - \frac{2b^2cx^2 - 6ac^2x^2 + 2b^3x - 7abcx + ab^2 - 4a^2c}{(a^2b^2 - 4a^3c)(cx^3 + bx^2 + ax)} + \frac{b \log(cx^2 + bx + a)}{a^3} - \frac{2b \log(|x|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out]
$$2*(b^4 - 6a*b^2c + 6a^2c^2)*\arctan((2cx + b)/\sqrt{-b^2 + 4ac})/((a^3b^2 - 4a^4c)*\sqrt{-b^2 + 4ac}) - (2b^2c*x^2 - 6a*c^2*x^2 + 2b^3*x$$

$$- 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x)) + b*\log(c*x^2 + b*x + a)/a^3 - 2*b*\log(\text{abs}(x))/a^3$$

maple [B] time = 0.02, size = 328, normalized size = 2.22

$$\frac{2c^2x}{(cx^2+bx+a)(4ac-b^2)a} - \frac{12c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a} + \frac{b^2cx}{(cx^2+bx+a)(4ac-b^2)a^2} + \frac{12b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^2} - \frac{2b^4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^3} - \frac{3bc}{(cx^2+bx+a)(4ac-b^2)a} + \frac{b^3}{(cx^2+bx+a)(4ac-b^2)a^2} + \frac{4bc\ln(cx^2+bx+a)}{(4ac-b^2)a^2} - \frac{b^3\ln(cx^2+bx+a)}{(4ac-b^2)a^3} - \frac{2b\ln(x)}{a^3} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] $-1/a^2/x - 2/a^3*b*\ln(x) - 2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x + 1/a^2/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^2 - 3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c + 1/a^2/(c*x^2+b*x+a)*b^3/(4*a*c-b^2) + 4/a^2/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b - 1/a^3/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^3 - 12/a/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*c^2 + 12/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*c - 2/a^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^4$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.83, size = 775, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] $\log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^{(1/2)} - 23*a^2*b^5*c - 108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^{(1/2)} + 87*a^3*b^3*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)} + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)^3)^{(1/2)} + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)})*((b^4*(-(4*a*c - b^2)^3)^{(1/2)} + 6*a^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*(-(4*a*c - b^2)^3)^{(1/2)})/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c)))/(a^2*(4*a*c - b^2)) + (2*c*x^2*$

$$\frac{(3ac - b^2)}{(a^2(4ac - b^2))} \frac{1}{(ax + bx^2 + cx^3)} - \log(2ab^4(-4ac - b^2)^{3/2} - 2b^8x - 2ab^7 + 23a^2b^5c + 108a^4b^3c^3 - 24a^4c^4x + 2b^5x(-4ac - b^2)^{3/2} - 87a^3b^3c^2 + 3a^3c^2(-4ac - b^2)^{3/2} - 9a^2b^2c(-4ac - b^2)^{3/2} - 97a^2b^4c^2x + 138a^3b^2c^3x + 24ab^6cx - 12ab^3cx(-4ac - b^2)^{3/2} + 15a^2b^2cx(-4ac - b^2)^{3/2}) * ((b^4(-4ac - b^2)^{3/2} + 6a^2c^2(-4ac - b^2)^{3/2} - 6ab^2c(-4ac - b^2)^{3/2})^{1/2}) / (a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2) - b/a^3 - (2b \log(x))/a^3$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] Timed out

$$3.26 \quad \int \frac{x}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=202

$$-\frac{(3b^2 - 2ac) \log(a + bx + cx^2)}{2a^4} + \frac{\log(x)(3b^2 - 2ac)}{a^4} + \frac{b(3b^2 - 11ac)}{a^3x(b^2 - 4ac)} - \frac{3b^2 - 8ac}{2a^2x^2(b^2 - 4ac)} + \frac{b(30a^2c^2 - 20ab^2c + 3b^4)}{a^4(b^2 - 4ac)}$$

Rubi [A] time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} - \frac{3b^2-8ac}{2a^2x^2(b^2-4ac)} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4} + \frac{b(3b^2-11ac)}{a^3x(b^2-4ac)} + \frac{\log(x)(3b^2-2ac)}{a^4} + \frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] $-(3b^2 - 8ac)/(2a^2(b^2 - 4ac)x^2) + (b(3b^2 - 11ac))/(a^3(b^2 - 4ac)x) + (b^2 - 2ac + bcx)/(a(b^2 - 4ac)x^2(a + bx + cx^2)) + (b(3b^4 - 20ab^2c + 30a^2c^2) \operatorname{ArcTanh}[(b + 2cx)/\sqrt{b^2 - 4ac}])/(a^4(b^2 - 4ac)^{3/2}) + ((3b^2 - 2ac) \log(x))/a^4 - ((3b^2 - 2ac) \log[a + bx + cx^2])/(2a^4)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4ac - x^2, x], x], x, b + 2cx, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + bx + cx^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^3 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2 (a + bx + cx^2)} - \frac{\int \frac{-3b^2 + 8ac - 3bcx}{x^3(a+bx+cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2 (a + bx + cx^2)} - \frac{\int \left(\frac{-3b^2 + 8ac}{ax^3} + \frac{3b^3 - 11abc}{a^2x^2} + \frac{(b^2 - 4ac)(-3b^2 + 2ac)}{a^3x} + \frac{b(3b^4 - 11ac)}{a^3} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{b(3b^4 - 20ac^2)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 175, normalized size = 0.87

$$\frac{2b(30a^2c^2 - 20ab^2c + 3b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{2a(2a^2c^2 - 4ab^2c - 3abc^2x + b^4 + b^3cx)}{(b^2 - 4ac)(a + x(b + cx))} - \frac{a^2}{x^2} + 2 \log(x)(3b^2 - 2ac) + (2ac - 3b^2) \log(a + x(b + cx)) + \frac{4ab}{x}}{2a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] $(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)]/(2*a^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] IntegrateAlgebraic[x/(a*x^2 + b*x^3 + c*x^4)^2, x]

fricas [B] time = 1.69, size = 1226, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + \\ & 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3) \\ &)x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c \\ & + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*\sqrt{b^2 - 4*a*c} \\ &)\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a) \\ & - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 \\ & - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 \\ & - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*\log(c*x^2 + b*x + a) \\ & - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c \\ & + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3) \\ &)x^2)*\log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c \\ & + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2), \\ & -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3) \\ &)x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 \\ & + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c \\ & + 30*a^3*b*c^2)*x^2)*\sqrt{-b^2 + 4*a*c} \\ &)\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c \\ & + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 \\ & - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 \\ & - 32*a^4*c^3)*x^2)*\log(c*x^2 + b*x + a) \\ & - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c \\ & + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3) \\ &)x^2)*\log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c \\ & + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)] \end{aligned}$$

giac [A] time = 0.44, size = 229, normalized size = 1.13

$$\frac{(3b^5 - 20ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (3b^2 - 2ac) \log(cx^2 + bx + a) + \frac{(3b^2 - 2ac) \log(|x|)}{a^4} - \frac{a^3b^2 - 4a^4c - 2(3ab^3c - 11a^2bc^2)x^3 - (6ab^4 - 25a^2b^2c + 8a^3c^2)x^2 - 3(a^2b^3 - 4a^3bc)x}{2(cx^2 + bx + a)(b^2 - 4ac)a^4x^2}}{(a^4b^2 - 4a^5c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c}) / ((a^4*b^2 - 4*a^5*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*(3*b^2 - 2*a*c)*\log(c*x^2 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*\log(\text{abs}(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x) / ((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)$

maple [B] time = 0.02, size = 418, normalized size = 2.07

$$\frac{3b^2c}{(c^2+bx+a)(4ac-b^2)^{3/2}} + \frac{30b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} - \frac{b^2cx}{(c^2+bx+a)(4ac-b^2)^{3/2}} + \frac{20b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \frac{3b^2c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} - \frac{2c^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{4b^2c}{(c^2+bx+a)(4ac-b^2)^2} - \frac{4c^2\ln(c^2+bx+a)}{(4ac-b^2)^2} - \frac{b^4}{(c^2+bx+a)(4ac-b^2)^2} - \frac{7b^2\ln(c^2+bx+a)}{(4ac-b^2)^2} + \frac{3b^4\ln(c^2+bx+a)}{2(4ac-b^2)^2} - \frac{2c\ln(c)}{a^2} + \frac{3b^2\ln(c)}{a^2} + \frac{2b}{a^2} - \frac{1}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] $-1/2/a^2/x^2 - 2/a^3*\ln(x)*c + 3/a^4*\ln(x)*b^2 + 2/a^3*b/x + 3/a^2/(c*x^2+b*x+a)*b*c^2/(4*a*c-b^2)*x - 1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x - 2/a/(c*x^2+b*x+a)/(4*a*c-b^2)*c^2 + 4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c - 1/a^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4 + 4/a^2/(4*a*c-b^2)*c^2*\ln(c*x^2+b*x+a) - 7/a^3/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b^2 + 3/2/a^4/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^4 + 30/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*c^2 - 20/a^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*c + 3/a^4/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^5$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.96, size = 914, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] $(\log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^{(1/2)} - 7*3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^{(1/2)} + 307*a^3*b^4*c^2 - 492*a^4*$

$$\begin{aligned}
& b^2c^3 + 31a^2b^3c*(-(4ac - b^2)^3)^{1/2} - 27a^3b^2c^2*(-(4ac - b^2)^3)^{1/2} + 339a^2b^5c^2x - 602a^3b^3c^3x + 24a^3c^3x*(-(4ac - b^2)^3)^{1/2} - 76ab^7cx + 312a^4b^2c^4x + 40ab^4c^2x*(-(4ac - b^2)^3)^{1/2} - 69a^2b^2c^2x*(-(4ac - b^2)^3)^{1/2}*(3b^8 + 128a^4c^4 - 3b^5*(-(4ac - b^2)^3)^{1/2} + 168a^2b^4c^2 - 288a^3b^2c^3 - 38ab^6c - 30a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} + 20ab^3c*(-(4ac - b^2)^3)^{1/2}))/2a^4(4ac - b^2)^3 - (\log(x)(2ac - 3b^2))/a^4 - (1/(2a) - (3bx)/(2a^2) + (x^2(6b^4 + 8a^2c^2 - 25ab^2c)))/(2a^3(4ac - b^2)) - (bcx^3(11ac - 3b^2))/(a^3(4ac - b^2))/(ax^2 + bx^3 + cx^4) + (\log(6ab^8 + 6b^9x + 192a^5c^4 + 6ab^5*(-(4ac - b^2)^3)^{1/2} - 73a^2b^6c + 6b^6x*(-(4ac - b^2)^3)^{1/2} + 307a^3b^4c^2 - 492a^4b^2c^3 - 31a^2b^3c*(-(4ac - b^2)^3)^{1/2} + 27a^3b^2c^2*(-(4ac - b^2)^3)^{1/2} + 339a^2b^5c^2x - 602a^3b^3c^3x - 24a^3c^3x*(-(4ac - b^2)^3)^{1/2} - 76ab^7cx + 312a^4b^2c^4x - 40ab^4c^2x*(-(4ac - b^2)^3)^{1/2} + 69a^2b^2c^2x*(-(4ac - b^2)^3)^{1/2}*(3b^8 + 128a^4c^4 + 3b^5*(-(4ac - b^2)^3)^{1/2} + 168a^2b^4c^2 - 288a^3b^2c^3 - 38ab^6c + 30a^2b^2c^2*(-(4ac - b^2)^3)^{1/2} - 20ab^3c*(-(4ac - b^2)^3)^{1/2}))/2a^4(4ac - b^2)^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] Timed out

$$3.27 \quad \int \frac{1}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=252

$$\frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5} - \frac{2b\log(x)(2b^2-3ac)}{a^5} + \frac{b(2b^2-7ac)}{a^3x^2(b^2-4ac)} - \frac{2(2b^2-5ac)}{3a^2x^3(b^2-4ac)} - \frac{2(5a^2c^2-9ab^2c+2b^4)}{a^4x(b^2-4ac)}$$

Rubi [A] time = 0.32, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1594, 740, 800, 634, 618, 206, 628}

$$\frac{2(5a^2c^2-9ab^2c+2b^4)}{a^4x(b^2-4ac)} - \frac{2(30a^2b^2c^2-10a^3c^3-15ab^4c+2b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}} + \frac{b(2b^2-7ac)}{a^3x^2(b^2-4ac)} - \frac{2(2b^2-5ac)}{3a^2x^3(b^2-4ac)} + \frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5} - \frac{2b\log(x)(2b^2-3ac)}{a^5} + \frac{-2ac+b^2+bcx}{a^3(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-2), x]

[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 + (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^p + (b_.)*(x_)^q + (c_.)*(x_)^r)^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^4 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} - \frac{\int \frac{-2(2b^2 - 5ac) - 4bcx}{x^4 (a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} - \frac{\int \left(\frac{2(-2b^2 + 5ac)}{ax^4} - \frac{2(-2b^3 + 7abc)}{a^2 x^3} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^3 x^2} + \frac{2b^2 - 2ac}{a^4 x} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2 (b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3 (b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4 (b^2 - 4ac)x} + \frac{b^2 - 2ac}{a(b^2 - 4ac)x^3} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2 (b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3 (b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4 (b^2 - 4ac)x} + \frac{b^2 - 2ac}{a(b^2 - 4ac)x^3} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2 (b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3 (b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4 (b^2 - 4ac)x} + \frac{b^2 - 2ac}{a(b^2 - 4ac)x^3} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2 (b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3 (b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4 (b^2 - 4ac)x} + \frac{b^2 - 2ac}{a(b^2 - 4ac)x^3}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 218, normalized size = 0.87

$$\frac{-\frac{a^3}{x^3} - \frac{3a(5a^2b^2c^2 + 2a^2c^3x - 5ab^3c - 4ab^2c^2x + b^5 + b^4cx)}{(b^2 - 4ac)(a + x(b + cx))} + \frac{3a^2b}{x^2} - \frac{6(-10a^3c^3 + 30a^2b^2c^2 - 15ab^4c + 2b^6) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + 6 \log(x)(3abc - 2b^3) + 3(2b^3 - 3abc) \log(a + x(b + cx)) + \frac{3a(2ac - 3b^2)}{x}}{3a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2), x]

[Out] $(-a^3/x^3) + (3a^2b)/x^2 + (3a(-3b^2 + 2ac))/x - (3a(b^5 - 5a^2b^3c + 5a^2b^2c^2 + b^4c^2x - 4a^2b^2c^2x + 2a^2c^3x))/((b^2 - 4ac)(a + x(b + cx))) - (6(2b^6 - 15a^2b^4c + 30a^2b^2c^2 - 10a^3c^3) \operatorname{ArcTan}[(b + 2cx)/\sqrt{-b^2 + 4ac}])/(-b^2 + 4ac)^{3/2} + 6(-2b^3 + 3a^2b^2c) \operatorname{Log}[x] + 3(2b^3 - 3a^2b^2c) \operatorname{Log}[a + x(b + cx)]/(3a^5)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(-2), x]
```

fricas [B] time = 2.47, size = 1407, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3), -1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3)]
```


giac [A] time = 0.55, size = 282, normalized size = 1.12

$$\frac{2(2b^3 - 15ab^2c + 30a^2b^2c^2 - 10a^3c^3) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + \frac{(2b^3 - 3abc) \log(cx^2 + bx + a)}{a^5} - \frac{2(2b^3 - 3abc) \log(|x|)}{a^5} - \frac{a^4b^2 - 4a^2c + 6(2ab^2c - 9a^2b^2c^2 + 5a^3c^3)x^4 + 3(4ab^5 - 20a^2b^3c + 17a^3bc^2)x^3 + (6a^2b^4 - 29a^3b^2c + 20a^4c^2)x^2 - 2(a^3b^3 - 4a^4bc)x}{3(cx^2 + bx + a)(b^2 - 4ac)a^5x^3}}{(a^2b^2 - 4a^2c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((a^5*b^2 - 4*a^6*c)*\sqrt{-b^2 + 4*a*c}) + (2*b^3 - 3*a*b*c)*\log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*\log(\text{abs}(x))/a^5 - 1/3*(a^4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x^3)$

maple [B] time = 0.02, size = 515, normalized size = 2.04

$$\frac{2c^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{2b^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{4bc^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{4b^2c^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{bc^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{3b^2c \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{b^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{9c^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{9bc}{(c^2+bx+a)(4ac-b^2)^2} + \frac{12b^2 \ln(c^2+bx+a)}{(4ac-b^2)^2} + \frac{b^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{11b^2 \ln(c^2+bx+a)}{(4ac-b^2)^2} + \frac{2b^2 \ln(c^2+bx+a)}{(4ac-b^2)^2} + \frac{6bc \ln(c)}{a^2} + \frac{6b^2 \ln(c)}{a^2} + \frac{2}{21} + \frac{3b^2}{21a^2} + \frac{1}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] $-1/3/a^2/x^3+2/a^3/x*c-3/a^4/x*b^2+1/a^3*b/x^2+6*b/a^4*\ln(x)*c-4*b^3/a^5*\ln(x)+2/a^2/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x-4/a^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*b^2+1/a^4/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^4+5/a^2/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c^2-5/a^3/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)*c+1/a^4/(c*x^2+b*x+a)*b^5/(4*a*c-b^2)-12/a^3/(4*a*c-b^2)*c^2*\ln(c*x^2+b*x+a)*b+11/a^4/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b^3-2/a^5/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^5+20/a^2/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3-60/a^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2+30/a^4/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*c-4/a^5/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 3.06, size = 1120, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a*x^2 + b*x^3 + c*x^4)^2, x)$

[Out]
$$\begin{aligned} & ((x^2*(5*a*c - 6*b^2))/(3*a^3) - 1/(3*a) + (2*b*x)/(3*a^2) + (x^3*(4*b^5 + 17*a^2*b*c^2 - 20*a*b^3*c))/(a^4*(4*a*c - b^2)) + (2*c*x^4*(2*b^4 + 5*a^2*c^2 - 9*a*b^2*c))/(a^4*(4*a*c - b^2)))/(a*x^3 + b*x^4 + c*x^5) + (\log(4*a*b^9 + 4*b^10*x - 4*a*b^6*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x - 4*b^7*x*(-(4*a*c - b^2)^3)^{(1/2)} + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 + 5*a^4*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^{(1/2)} + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x - 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^{(1/2)} + 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^{(1/2)} - 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)})*(a^2*(132*b^5*c^2 - 30*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)}) - a^3*(272*b^3*c^3 - 10*c^3*(-(4*a*c - b^2)^3)^{(1/2)}) + 2*b^9 - 2*b^6*(-(4*a*c - b^2)^3)^{(1/2)} - a*(27*b^7*c - 15*b^4*c*(-(4*a*c - b^2)^3)^{(1/2)}) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2) + (\log(4*a*b^9 + 4*b^10*x + 4*a*b^6*(-(4*a*c - b^2)^3)^{(1/2)} - 52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x + 4*b^7*x*(-(4*a*c - b^2)^3)^{(1/2)} + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 - 5*a^4*c^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^{(1/2)} + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x + 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)} - 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^{(1/2)} - 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^{(1/2)} + 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^{(1/2)})*(a^2*(132*b^5*c^2 + 30*b^2*c^2*(-(4*a*c - b^2)^3)^{(1/2)}) - a^3*(272*b^3*c^3 + 10*c^3*(-(4*a*c - b^2)^3)^{(1/2)}) + 2*b^9 + 2*b^6*(-(4*a*c - b^2)^3)^{(1/2)} - a*(27*b^7*c + 15*b^4*c*(-(4*a*c - b^2)^3)^{(1/2)}) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2) + (2*b*log(x)*(3*a*c - 2*b^2))/a^5 \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(c*x**4+b*x**3+a*x**2)**2, x)$

[Out] Timed out

$$3.28 \quad \int \frac{1}{x(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=318

$$\frac{b(5b^2 - 17ac)}{3a^3x^3(b^2 - 4ac)} - \frac{5b^2 - 12ac}{4a^2x^4(b^2 - 4ac)} - \frac{(3a^2c^2 - 12ab^2c + 5b^4) \log(a + bx + cx^2)}{2a^6} + \frac{\log(x)(3a^2c^2 - 12ab^2c + 5b^4)}{a^6}$$

Rubi [A] time = 0.39, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{12a^2c^2 - 22ab^2c + 5b^4}{2a^3x^3(b^2 - 4ac)} - \frac{(3a^2c^2 - 12ab^2c + 5b^4) \log(a + bx + cx^2)}{2a^6} + \frac{b(29a^2c^2 - 27ab^2c + 5b^4)}{a^3x(b^2 - 4ac)} + \frac{\log(x)(3a^2c^2 - 12ab^2c + 5b^4)}{a^6} + \frac{b(105a^2b^2c^2 - 70a^3c^3 - 42ab^4c + 5b^6) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6(b^2-4ac)^{3/2}} + \frac{b(5b^2-17ac)}{3a^3x^3(b^2-4ac)} - \frac{5b^2-12ac}{4a^2x^4(b^2-4ac)} + \frac{-2ac+b^2+bcx}{ax^4(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2), x]

[Out] $-(5*b^2 - 12*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^3*(b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(2*a^4*(b^2 - 4*a*c)*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^5*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/(a^6*(b^2 - 4*a*c)^{(3/2)}) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*\text{Log}[x])/a^6 - ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*\text{Log}[a + b*x + c*x^2])/(2*a^6)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_)^(m_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int((((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_) + (b_.)*(x_)^(q_) + (c_.)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^5(a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^4(a + bx + cx^2)} - \frac{\int \frac{-5b^2 + 12ac - 5bcx}{x^5(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^4(a + bx + cx^2)} - \frac{\int \left(\frac{-5b^2 + 12ac}{ax^5} + \frac{5b^3 - 17abc}{a^2x^4} + \frac{-5b^4 + 22ab^2c - 12a^2c^2}{a^3x^3} + \frac{5b^5 - 17ab^3c + 12a^2c^2}{a^4x^2} + \frac{b(5b^4 - 27ab^2c + 27a^2c^2)}{a^5x} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 27a^2c^2)}{a^5(b^2 - 4ac)} \\
&= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 27a^2c^2)}{a^5(b^2 - 4ac)} \\
&= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 27a^2c^2)}{a^5(b^2 - 4ac)} \\
&= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 27a^2c^2)}{a^5(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 272, normalized size = 0.86

$$\frac{-\frac{3a^4}{x^4} + \frac{8a^3b}{x^3} + \frac{6a^2(2ac-3b^2)}{x^2} + 12\log(x)(3a^2c^2 - 12ab^2c + 5b^4) - 6(3a^2c^2 - 12ab^2c + 5b^4)\log(a + x(b + cx)) + \frac{12b(-70a^3c^3 + 105a^2b^2c^2 - 42ab^4c + 5b^6)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) - 12a(2a^2c^3 - 9a^2b^2c^2 - 5a^2bc^3x + 6ab^4c + 5ab^3c^2x - b^6 - b^5cx) - 24ab(3ac-2b^2)}{(4ac-b^2)^{3/2}(b^2-4ac)(a+x(b+cx))}}{12a^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2), x]

[Out] $\left((-3a^4)/x^4 + (8a^3b)/x^3 + (6a^2(-3b^2 + 2ac))/x^2 - (24ab(-2b^2 + 3ac))/x - (12a(-b^6 + 6ab^4c - 9a^2b^2c^2 + 2a^3c^3 - b^5cx + 5ab^3c^2x - 5a^2b^3cx))/(b^2 - 4ac)(a + x(b + cx)) + (12b(5b^6 - 42ab^4c + 105a^2b^2c^2 - 70a^3c^3) \operatorname{ArcTan}[(b + 2cx)/\sqrt{-b^2 + 4ac}]) / (-b^2 + 4ac)^{3/2} + 12(5b^4 - 12ab^2c + 3a^2c^2) \operatorname{Log}[x] - 6(5b^4 - 12ab^2c + 3a^2c^2) \operatorname{Log}[a + x(b + cx)] \right) / (12a^6)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(ax^2 + bx^3 + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3 + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3 + c*x^4)^2), x]

fricas [B] time = 3.18, size = 1640, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4), -1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2

$$\begin{aligned} & *b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + \\ & 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*\log(c*x^2 + b*x + a) - \\ & 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c \\ & ^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4* \\ & b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + \\ & 48*a^5*c^4)*x^4)*\log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a \\ & ^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9* \\ & c^2)*x^4)] \end{aligned}$$

giac [A] time = 0.44, size = 347, normalized size = 1.09

$$\frac{(5b^7 - 42ab^5c + 105a^2b^3c^2 - 70a^3b*c^3) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (5b^8 - 12ab^6c + 3a^2c^2) \log(cx^2 + bx + a) + (5b^8 - 12ab^6c + 3a^2c^2) \log(0) - 3a^2b^2 - 12a^2c - 12(5ab^5c - 27a^2b^3c^2 + 29a^3bc^3)^2 - 6(10ab^6 - 59a^2b^4c + 80a^3b^2c^2 - 12a^4c^3)^2 - 2(5a^2b^5 - 86a^3b^3c + 104a^4b*c^2)^2 + (10a^3b^4 - 49a^4b^2c + 36a^5c^2)^2 - 5(a^4b^3 - 4a^5bc)^2}{(a^6b^4 - 4a^7b^2c^2 + 16a^8c^3)^2 - 4a^6b^5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & -(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((a^6*b^4 - 4*a^7*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*(5*b^4 - \\ & 12*a*b^2*c + 3*a^2*c^2)*\log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c + 3* \\ & a^2*c^2)*\log(\text{abs}(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27* \\ & a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c \\ & ^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 + \\ & (10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x)/ \\ & (c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4) \end{aligned}$$

maple [B] time = 0.02, size = 619, normalized size = 1.95

$$\frac{b^7c}{(c^2+bx+a)(4ac-b^2)^2} - \frac{7b^6c \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{5b^5c^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{105b^4c \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(4ac-b^2)^2} + \frac{b^4c}{(c^2+bx+a)(4ac-b^2)^2} - \frac{42b^3c^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{105b^2c^3}{(c^2+bx+a)(4ac-b^2)^2} - \frac{70b^2c^3}{(c^2+bx+a)(4ac-b^2)^2} + \frac{3a^2c^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{12a^2c^2 \log\left(\frac{c^2x^2+bx+a}{|x|}\right)}{(c^2+bx+a)(4ac-b^2)^2} - \frac{12a^2c^2 \log(0)}{(c^2+bx+a)(4ac-b^2)^2} - \frac{3a^2b^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{12a^2c}{(c^2+bx+a)(4ac-b^2)^2} - \frac{12(5ab^5c - 27a^2b^3c^2 + 29a^3bc^3)^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{6(10ab^6 - 59a^2b^4c + 80a^3b^2c^2 - 12a^4c^3)^2}{(c^2+bx+a)(4ac-b^2)^2} - \frac{2(5a^2b^5 - 86a^3b^3c + 104a^4b*c^2)^2}{(c^2+bx+a)(4ac-b^2)^2} + \frac{5(a^4b^3 - 4a^5bc)^2}{(c^2+bx+a)(4ac-b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^3+a*x^2)^2,x)

$$\begin{aligned} \text{[Out]} & -1/4/a^2/x^4+1/a^3/x^2*c-3/2/a^4/x^2*b^2+3/a^4*\ln(x)*c^2-12/a^5*\ln(x)*b^2*c \\ & +5/a^6*\ln(x)*b^4+2/3/a^3*b/x^3-6*b/a^4/x*c+4*b^3/a^5/x-5/a^3/(c*x^2+b*x+a)* \\ & b*c^3/(4*a*c-b^2)*x+5/a^4/(c*x^2+b*x+a)*b^3*c^2/(4*a*c-b^2)*x-1/a^5/(c*x^2+ \\ & b*x+a)*b^5*c/(4*a*c-b^2)*x+2/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*c^3-9/a^3/(c*x^2 \\ & +b*x+a)/(4*a*c-b^2)*b^2*c^2+6/a^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*c-1/a^5/(c* \\ & x^2+b*x+a)/(4*a*c-b^2)*b^6-6/a^3/(4*a*c-b^2)*c^3*\ln(c*x^2+b*x+a)+51/2/a^4/(\\ & 4*a*c-b^2)*c^2*\ln(c*x^2+b*x+a)*b^2-16/a^5/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b^4 \\ & +5/2/a^6/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^6-70/a^3/(4*a*c-b^2)^(3/2)*\arctan((2 \\ & *c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3+105/a^4/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b) \\ & / (4*a*c-b^2)^(1/2))*b^3*c^2-42/a^5/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a* \\ & c-b^2)^(1/2))*b^5*c+5/a^6/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1 \\ & /2))*b^7 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 3.14, size = 1260, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x)

[Out]
$$\begin{aligned} & (\log(x) \cdot (5b^4 + 3a^2c^2 - 12ab^2c)) / a^6 - (1/(4a) - (x^2(9ac - 10b^2)) / (12a^3) - (5bx) / (12a^2) + (x^4(10b^6 - 12a^3c^3 + 80a^2b^2c^2 - 59ab^4c)) / (2a^5(4ac - b^2)) + (bx^3(26ac - 15b^2)) / (6a^4) + (bcx^5(5b^4 + 29a^2c^2 - 27ab^2c)) / (a^5(4ac - b^2))) / (ax^4 + bx^5 + cx^6) + (\log(288a^6c^5 - 10b^{11}x - 10ab^{10} + 10ab^7(-4ac - b^2)^3)^{1/2} + 139a^2b^8c + 10b^8x(-4ac - b^2)^3)^{1/2} \\ & - 717a^3b^6c^2 + 1643a^4b^4c^3 - 1508a^5b^2c^4 - 69a^2b^5c(-4ac - b^2)^3)^{1/2} - 53a^4b^3c^3(-4ac - b^2)^3)^{1/2} - 779a^2b^7c^2x + 1916a^3b^5c^3x - 1998a^4b^3c^4x + 36a^4c^4x(-4ac - b^2)^3)^{1/2} + 144ab^9cx + 129a^3b^3c^2(-4ac - b^2)^3)^{1/2} + 568a^5b^5c^5x - 84ab^6cx(-4ac - b^2)^3)^{1/2} + 225a^2b^4c^2x(-4ac - b^2)^3)^{1/2} - 206a^3b^2c^3x(-4ac - b^2)^3)^{1/2}) \cdot (a^3(466b^4c^3 - 35b^3c^3(-4ac - b^2)^3)^{1/2}) - a^2((387b^6c^2)/2 - (105b^3c^2(-4ac - b^2)^3)^{1/2})/2) - (5b^{10})/2 + 96a^5c^5 + (5b^7(-4ac - b^2)^3)^{1/2})/2 + a(36b^8c - 21b^5c(-4ac - b^2)^3)^{1/2}) - 456a^4b^2c^4) / (a^6b^6 - 64a^9c^3 - 12a^7b^4c + 48a^8b^2c^2) - (\log(10ab^{10} + 10b^{11}x - 288a^6c^5 + 10ab^7(-4ac - b^2)^3)^{1/2} - 139a^2b^8c + 10b^8x(-4ac - b^2)^3)^{1/2} + 717a^3b^6c^2 - 1643a^4b^4c^3 + 1508a^5b^2c^4 - 69a^2b^5c(-4ac - b^2)^3)^{1/2} - 53a^4b^3c^3(-4ac - b^2)^3)^{1/2} + 779a^2b^7c^2x - 1916a^3b^5c^3x + 1998a^4b^3c^4x + 36a^4c^4x(-4ac - b^2)^3)^{1/2} - 144ab^9cx + 129a^3b^3c^2(-4ac - b^2)^3)^{1/2} - 568a^5b^5c^5x - 84ab^6cx(-4ac - b^2)^3)^{1/2} + 225a^2b^4c^2x(-4ac - b^2)^3)^{1/2} - 206a^3b^2c^3x(-4ac - b^2)^3)^{1/2}) \cdot (a^2((387b^6c^2)/2 + (105b^3c^2(-4ac - b^2)^3)^{1/2})/2) - a^3(466b^4c^3 + 35b^3c^3(-4ac - b^2)^3)^{1/2}) \end{aligned}$$

$$*c^3*(-(4*a*c - b^2)^3)^{(1/2)} + (5*b^{10})/2 - 96*a^5*c^5 + (5*b^7*(-(4*a*c - b^2)^3)^{(1/2)})/2 - a*(36*b^8*c + 21*b^5*c*(-(4*a*c - b^2)^3)^{(1/2)} + 456*a^4*b^2*c^4)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] Timed out

3.29 $\int x^2 \sqrt{ax^2 + bx^3 + cx^4} dx$

Optimal. Leaf size=257

$$\frac{(256a^2c^2 - 460ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} + \frac{bx(7b^2 - 12ac)(b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

Rubi [A] time = 0.59, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$\frac{(256a^2c^2 - 460ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{x(7b^2 - 16ac) \sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{b(35b^2 - 116ac) \sqrt{ax^2 + bx^3 + cx^4}}{960c^3} + \frac{bx(7b^2 - 12ac)(b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{x^2(b + 8cx) \sqrt{ax^2 + bx^3 + cx^4}}{40c}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))]*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
)*(A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1)
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{ax^2 + bx^3 + cx^4} dx &= \frac{x^2(b + 8cx)\sqrt{ax^2 + bx^3 + cx^4}}{40c} + \int \frac{x^3(-3ab - \frac{1}{2}(7b^2 - 16ac)x)}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{x^2(b + 8cx)\sqrt{ax^2 + bx^3 + cx^4}}{40c} - \int \frac{x^2(-a(7b^2 - 16ac))}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{x^2(b + 8cx)\sqrt{ax^2 + bx^3 + cx^4}}{40c} \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} \\
&= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 180, normalized size = 0.70

$$\frac{15x(48a^2bc^2 - 40ab^3c + 7b^5)\sqrt{a+x(b+cx)} \log(2\sqrt{c}\sqrt{a+x(b+cx)} + b + 2cx) + 2\sqrt{c}x(a+x(b+cx))(128c^2(-2a^2+acx^2+3c^2x^4) + 4b^2c(115a-14cx^2) + 8b^2cx(6cx^2-29a) - 105b^4 + 70b^3cx)}{3840c^{9/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 14*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2*x^4)) + 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]/(3840*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.84, size = 211, normalized size = 0.82

$$\frac{\log(x)(48a^2bc^2 - 40ab^3c + 7b^5)}{256c^{9/2}} + \frac{(-48a^2bc^2 + 40ab^3c - 7b^5) \log\left(\frac{-2c^{9/2}\sqrt{ax^2 + bx^3 + cx^4} + bc^4x + 2c^5x^2}{256c^{9/2}}\right)}{256c^{9/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-256a^2c^2 + 460ab^2c - 232abc^2x + 128ac^3x^2 - 105b^4 + 70b^3cx - 56b^2c^2x^2 + 48bc^3x^3 + 384c^4x^4)}{1920c^4x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (Sqrt[a*x^2 + b*x^3 + c*x^4]*(-105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 70*b^3*c*x - 232*a*b*c^2*x - 56*b^2*c^2*x^2 + 128*a*c^3*x^2 + 48*b*c^3*x^3 + 384*c^4*x^4))/(1920*c^4*x) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[x])/(256*c^(9/2)) + ((-7*b^5 + 40*a*b^3*c - 48*a^2*b*c^2)*Log[b*c^4*x + 2*c^5*x^2 - 2*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(256*c^(9/2))

fricas [A] time = 1.23, size = 390, normalized size = 1.52

$$\frac{15(7b^5 - 40ab^3c + 48a^2b^2c^2)\sqrt{c}\log\left(\frac{b^4b^3b^2a + \sqrt{(4b^3c^2 + 48a^2b^2c^2)(b^3c^2 + 48a^2b^2c^2)}}{7680c^5}\right) + 4(384b^5c^4 + 48b^4a^3c - 105b^4c^3 - 256a^2b^2c^2 - 8(7b^5c^3 - 16a^2c^4))\sqrt{c^4 + b^3c + a^2} + 15(7b^5 - 40ab^3c + 48a^2b^2c^2)\sqrt{c}\arctan\left(\frac{\sqrt{(4b^3c^2 + 48a^2b^2c^2)(b^3c^2 + 48a^2b^2c^2)}}{2191 + 197b^2c}\right) - 2(384b^5c^4 + 48b^4a^3c - 105b^4c^3 - 256a^2b^2c^2 - 8(7b^5c^3 - 16a^2c^4))\sqrt{c^4 + b^3c + a^2}}{3840c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="fricas")

[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4))*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c))/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4))*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]

giac [A] time = 0.87, size = 283, normalized size = 1.10

$$\frac{\frac{1}{1920}\sqrt{c^2 + bx + a}\left(\left(\frac{1}{2}\left(\frac{1}{\sqrt{c}}\operatorname{sgn}(x) + \frac{\operatorname{sgn}(x)}{\sqrt{c}}\right)\right) + \frac{7b^2c\operatorname{sgn}(x) - 16ac^2\operatorname{sgn}(x)}{2c^2}\right) + \frac{35b^3c\operatorname{sgn}(x) - 116ab^2c\operatorname{sgn}(x)}{2c^2} - \frac{105b^4c\operatorname{sgn}(x) - 460ab^2c\operatorname{sgn}(x) + 256a^2c\operatorname{sgn}(x)}{256c^2}\right)\frac{[7b^5\operatorname{sgn}(x) - 40ab^3\operatorname{sgn}(x) + 48a^2b^2\operatorname{sgn}(x)]\log\left(\frac{2(\sqrt{c}x - \sqrt{c^2 + bx + a})}{\sqrt{c}}\right)}{256c^4} + \frac{[105b^4\log(|-b + 2\sqrt{c}\sqrt{c}|) - 600ab^3\log(|-b + 2\sqrt{c}\sqrt{c}|) + 720b^2c^2\log(|-b + 2\sqrt{c}\sqrt{c}|) + 210\sqrt{c}\sqrt{c} - 920a^{3/2}b^2 + 512a^{5/2}]\operatorname{sgn}(x)}{3840c^4}}{1920}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="giac")

[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2*c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn(x)))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c^4) - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2) + 1/3840*(105*b^5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 720*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)

maple [A] time = 0.01, size = 310, normalized size = 1.21

$$\frac{\sqrt{c^2 + bx + a}\left(\frac{1}{1920}b^2c^2\ln\left(\frac{2(105b^4\sqrt{c^2 + bx + a}\sqrt{c})}{2c^2}\right) - 600ab^3c^2\ln\left(\frac{2(105b^4\sqrt{c^2 + bx + a}\sqrt{c})}{2c^2}\right) + 105b^4c^2\ln\left(\frac{2(105b^4\sqrt{c^2 + bx + a}\sqrt{c})}{2c^2}\right) + 720\sqrt{c^2 + bx + a}ab^3c^2 - 420\sqrt{c^2 + bx + a}b^3c^2 + 768(c^2 + bx + a)^{3/2}c^2 + 360\sqrt{c^2 + bx + a}ab^3c^2 - 210\sqrt{c^2 + bx + a}b^3c^2 - 672(c^2 + bx + a)^{3/2}b^2c^2 - 512(c^2 + bx + a)^{3/2}ac^2 + 560(c^2 + bx + a)^{3/2}b^2c^2\right)}{3840\sqrt{c^2 + bx + a}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)`

[Out] $1/3840*(c*x^4+b*x^3+a*x^2)^{(1/2)}*(768*x^2*(c*x^2+b*x+a)^{(3/2)}*c^{(9/2)}-672*c^{(7/2)}*(c*x^2+b*x+a)^{(3/2)}*x*b-512*c^{(7/2)}*(c*x^2+b*x+a)^{(3/2)}*a+560*c^{(5/2)}*(c*x^2+b*x+a)^{(3/2)}*b^2+720*c^{(7/2)}*(c*x^2+b*x+a)^{(1/2)}*x*a*b-420*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*x*b^3+360*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*a*b^2-210*c^{(3/2)}*(c*x^2+b*x+a)^{(1/2)}*b^4+720*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*a^2*b*c^3-600*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*a*b^3*c^2+105*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*b^5*c)/x/(c*x^2+b*x+a)^{(1/2)}/c^{(11/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^3 + ax^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{cx^4 + bx^3 + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)`

[Out] `int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{x^2 (a + bx + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)`

3.30 $\int x\sqrt{ax^2 + bx^3 + cx^4} dx$

Optimal. Leaf size=205

$$\frac{x(b^2 - 4ac)(5b^2 - 4ac)\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} - \frac{x(b^2 - 4ac)(5b^2 - 4ac)\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c}$$

Rubi [A] time = 0.37, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} - \frac{x(b^2 - 4ac)(5b^2 - 4ac)\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] -((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*c^2) + (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b + 6*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_) + (a_.)*(x_)^(q_) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[

$a*x^q + b*x^n + c*x^{(2*n - q)}$], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1919

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1) + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]

Rule 1949

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_)*(A_) + (B_)*(x_)^(r_)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] - Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int x\sqrt{ax^2 + bx^3 + cx^4} dx &= \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} + \int \frac{x^2(-2ab - \frac{1}{2}(5b^2 - 12ac)x)}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} - \int \frac{x(-\frac{1}{2}a(5b^2 - 12ac))}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 150, normalized size = 0.73

$$\frac{2\sqrt{c}x(a + x(b + cx))(b(8c^2x^2 - 52ac) + 24c^2x(a + 2cx^2) + 15b^3 - 10b^2cx) - 3x(16a^2c^2 - 24ab^2c + 5b^4)\sqrt{a + x(b + cx)} \log(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx)}{384c^{7/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/(384*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.58, size = 173, normalized size = 0.84

$$\frac{(16a^2c^2 - 24ab^2c + 5b^4) \log\left(\frac{-2\sqrt{c}\sqrt{ax^2 + bx^3 + cx^4} + bx + 2cx^2}{128c^{7/2}}\right) + \log(x)\left(\frac{-16a^2c^2 + 24ab^2c - 5b^4}{128c^{7/2}}\right) + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-52abc + 24ac^2x + 15b^3 - 10b^2cx + 8bc^2x^2 + 48c^3x^3)}{192c^3x}}{192c^3x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] $((15*b^3 - 52*a*b*c - 10*b^2*c*x + 24*a*c^2*x + 8*b*c^2*x^2 + 48*c^3*x^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + ((-5*b^4 + 24*a*b^2*c - 16*a^2*c^2)*\text{Log}[x])/(128*c^{(7/2)}) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*\text{Log}[b*x + 2*c*x^2 - 2*\text{Sqrt}[c]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]])/(128*c^{(7/2)})$

fricas [A] time = 0.68, size = 326, normalized size = 1.59

$$\frac{3(5b^3 - 24ab^2c + 16a^2c^2)\sqrt{c}x \log\left(\frac{b^2x^2 + 8b^2cx^2 + \sqrt{c^2 + 4b^2c}x + a + b^2x^2}{768c^3x}\right) + 4(48c^4b^3 + 8b^3c^2 + 15b^2c - 52ab^2 - 2(5b^2c^2 - 12ac^2))\sqrt{c}x^2 + b^3 + a^2}{768c^3x} + \frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{-c}x \arctan\left(\frac{\sqrt{c^2 + 4b^2c}x + a}{2(b^2x^2 + b^2cx + a)}\right) + 2(48c^4b^3 + 8b^3c^2 + 15b^2c - 52ab^2 - 2(5b^2c^2 - 12ac^2))\sqrt{c^2 + b^2c + a^2}}{384c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $[1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*\text{sqrt}(c)*x*\log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*\text{sqrt}(c) + (b^2 + 4*a*c)*x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*\text{sqrt}(-c)*x*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*\text{sqrt}(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]$

giac [A] time = 0.75, size = 230, normalized size = 1.12

$$\frac{1}{192}\sqrt{c^2 + b^2 + a}\left(2\left(4\left(6*\text{sgn}(x) + \frac{\log(\text{sgn}(x))}{c}\right)x - 5b^2*\text{sgn}(x) - 12a^2*\text{sgn}(x)\right)\frac{1}{c^3} + \frac{15b^3*\text{sgn}(x) - 52ab^2*\text{sgn}(x)}{c^3}\right) + \frac{(5b^4*\text{sgn}(x) - 24ab^2*\text{sgn}(x) + 16a^2*\text{sgn}(x))\log\left(-2\left(\sqrt{c}x - \sqrt{c^2 + b^2 + a}\right)\sqrt{c} - b\right)}{128c^3} - \frac{(15b^4\log(-b + 2\sqrt{c}\sqrt{c}) - 72ab^2\log(-b + 2\sqrt{c}\sqrt{c}) + 48a^2\log(-b + 2\sqrt{c}\sqrt{c}) + 30\sqrt{c}b^3\sqrt{c} - 104a^2bc^3)\text{sgn}(x)}{384c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] $1/192*\text{sqrt}(c*x^2 + b*x + a)*(2*(4*(6*x*\text{sgn}(x) + b*\text{sgn}(x)/c)*x - (5*b^2*c*\text{sgn}(x) - 12*a*c^2*\text{sgn}(x))/c^3)*x + (15*b^3*\text{sgn}(x) - 52*a*b*c*\text{sgn}(x))/c^3) + 1/128*(5*b^4*\text{sgn}(x) - 24*a*b^2*c*\text{sgn}(x) + 16*a^2*c^2*\text{sgn}(x))*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(7/2)} - 1/384*(15*b^4*\log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) - 72*a*b^2*c*\log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) + 48*a^2*c^2*\log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) + 30*\text{sqrt}(a)*b^3*\text{sqrt}(c) - 104*a^{(3/2)}*b*c^{(3/2)})*\text{sgn}(x)/c^{(7/2)}$

maple [A] time = 0.01, size = 265, normalized size = 1.29

$$\frac{\sqrt{c^2 + b^2 + a}x^2 \ln\left(\frac{-48a^2c^3 \ln\left(\frac{2ax + b + 2\sqrt{c^2 + b^2 + a}\sqrt{c}}{2\sqrt{c}}\right) + 72ab^2c^2 \ln\left(\frac{2ax + b + 2\sqrt{c^2 + b^2 + a}\sqrt{c}}{2\sqrt{c}}\right) - 15b^4c \ln\left(\frac{2ax + b + 2\sqrt{c^2 + b^2 + a}\sqrt{c}}{2\sqrt{c}}\right) - 48\sqrt{c}x^2 + bx + a}{384\sqrt{c}x^2 + bx + a}\right) + 60\sqrt{c}x^2 + bx + a}{384\sqrt{c}x^2 + bx + a} + \frac{b^2c^2x^5 - 24\sqrt{c}x^2 + a}{384\sqrt{c}x^2 + bx + a} + \frac{30\sqrt{c}x^2 + bx + a}{384\sqrt{c}x^2 + bx + a} + \frac{b^3c^2 + 96(c^2x + bx + a)^{\frac{3}{2}}c^2x - 80(c^2x + bx + a)^{\frac{3}{2}}bc^3}{384\sqrt{c}x^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(96*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)-80*c^(5/2)
)*(c*x^2+b*x+a)^(3/2)*b-48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a+60*c^(5/2)*(c*x^
2+b*x+a)^(1/2)*x*b^2-24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b+30*c^(3/2)*(c*x^2+b
*x+a)^(1/2)*b^3-48*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*
a^2*c^3+72*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*a*b^2*c^
2-15*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))*b^4*c)/x/(c*x^
2+b*x+a)^(1/2)/c^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^3 + ax^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{cx^4 + bx^3 + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{x^2 (a + bx + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

3.31 $\int \sqrt{ax^2 + bx^3 + cx^4} dx$

Optimal. Leaf size=163

$$\frac{b(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}} - \frac{b(b+2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2 + bx^3}}{3cx}$$

Rubi [A] time = 0.06, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1903, 640, 612, 621, 206}

$$\frac{b(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}} - \frac{b(b+2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] -(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*x + c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*x*Sqrt[a + b*x + c*x^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
:> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\begin{aligned} \int \sqrt{ax^2 + bx^3 + cx^4} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4} \int x\sqrt{a + bx + cx^2} dx}{x\sqrt{a + bx + cx^2}} \\ &= \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} - \frac{(b\sqrt{ax^2 + bx^3 + cx^4}) \int \sqrt{a + bx + cx^2} dx}{2cx\sqrt{a + bx + cx^2}} \\ &= -\frac{b(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} + \frac{(b(b^2 - 4ac)\sqrt{a + bx + cx^2})}{1} \\ &= -\frac{b(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} + \frac{(b(b^2 - 4ac)\sqrt{a + bx + cx^2})}{1} \\ &= -\frac{b(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} + \frac{b(b^2 - 4ac)\sqrt{a + bx + cx^2}}{1} \end{aligned}$$

Mathematica [A] time = 0.21, size = 119, normalized size = 0.73

$$\frac{2\sqrt{c}x(a + x(b + cx))(8c(a + cx^2) - 3b^2 + 2bcx) + 3bx(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{48c^{5/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4], x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) + 3*b*(
b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

IntegrateAlgebraic [A] time = 0.43, size = 137, normalized size = 0.84

$$\frac{\log(x)(b^3 - 4abc)}{16c^{5/2}} + \frac{(4abc - b^3)\log\left(-2c^{5/2}\sqrt{ax^2 + bx^3 + cx^4} + bc^2x + 2c^3x^2\right)}{16c^{5/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(8ac - 3b^2 + 2bcx + 8c^2x^2)}{24c^2x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] $\left(\frac{(-3b^2 + 8ac + 2b^2cx + 8c^2x^2)\sqrt{ax^2 + bx^3 + cx^4}}{(24c^2x)} + \frac{(b^3 - 4abc)\log(x)}{(16c^{5/2})} + \frac{((-b^3 + 4abc)\log[b^2cx^2 + 2c^3x^2 - 2c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}])}{(16c^{5/2})}\right)$

fricas [A] time = 1.25, size = 260, normalized size = 1.60

$$\left[\frac{3(b^3 - 4abc)\sqrt{c}x \log\left(\frac{-8c^2x^2 + 8bcx^2 - 4\sqrt{c^4 + b^3 + ax^2}(2cx + b)\sqrt{c} + (b^2 + 4ac)x}{x}\right) - 4(8c^3x^2 + 2bc^2x - 3b^2c + 8ac^2)\sqrt{cx^4 + bx^3 + ax^2}}{96c^3x}, \frac{3(b^3 - 4abc)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}}{2(x^2 + bcx + acx)}\right) - 2(8c^3x^2 + 2bc^2x - 3b^2c + 8ac^2)\sqrt{cx^4 + bx^3 + ax^2}}{48c^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $\left[-\frac{1}{96}(3(b^3 - 4abc)\sqrt{c})x \log\left(\frac{-(8c^2x^3 + 8b^2cx^2 - 4\sqrt{c}(cx^4 + bx^3 + ax^2))(2cx + b)\sqrt{c} + (b^2 + 4ac)x}{x}\right) - 4(8c^3x^2 + 2b^2c^2x - 3b^2c + 8ac^2)\sqrt{c}(cx^4 + bx^3 + ax^2)}{(c^3x)}, -\frac{1}{48}(3(b^3 - 4abc)\sqrt{-c})x \arctan\left(\frac{1}{2}\sqrt{\frac{cx^4 + bx^3 + ax^2}{2cx + b}}\right) - 2(8c^3x^2 + 2b^2c^2x - 3b^2c + 8ac^2)\sqrt{-c}(cx^4 + bx^3 + ax^2)}{(c^3x)}\right]$

giac [A] time = 0.88, size = 166, normalized size = 1.02

$$\frac{1}{24}\sqrt{cx^2 + bx + a}\left(2\left(4x\operatorname{sgn}(x) + \frac{b\operatorname{sgn}(x)}{c}\right)x - \frac{3b^2\operatorname{sgn}(x) - 8ac\operatorname{sgn}(x)}{c^2}\right) - \frac{(b^3\operatorname{sgn}(x) - 4abc\operatorname{sgn}(x))\log\left(\frac{-2(\sqrt{c}x - \sqrt{cx^2 + bx + a})\sqrt{c} - b}{16c^2}\right)}{16c^2} + \frac{(3b^3\log(|-b + 2\sqrt{a}\sqrt{c}|) - 12abc\log(|-b + 2\sqrt{a}\sqrt{c}|) + 6\sqrt{a}b^2\sqrt{c} - 16a^{\frac{3}{2}}c^{\frac{3}{2}})\operatorname{sgn}(x)}{48c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{24}\sqrt{cx^2 + bx + a}(2(4x\operatorname{sgn}(x) + b\operatorname{sgn}(x)/c)x - (3b^2\operatorname{sgn}(x) - 8ac\operatorname{sgn}(x))/c^2) - \frac{1}{16}(b^3\operatorname{sgn}(x) - 4abc\operatorname{sgn}(x))\log(\operatorname{abs}(-2(\sqrt{c}x - \sqrt{cx^2 + bx + a})\sqrt{c} - b)/c^{5/2}) + \frac{1}{48}(3b^3\log(\operatorname{abs}(-b + 2\sqrt{a}\sqrt{c})) - 12abc\log(\operatorname{abs}(-b + 2\sqrt{a}\sqrt{c})) + 6\sqrt{a}b^2\sqrt{c} - 16a^{3/2}c^{3/2})\operatorname{sgn}(x)/c^{5/2})$

maple [A] time = 0.01, size = 167, normalized size = 1.02

$$\frac{\sqrt{cx^4 + bx^3 + ax^2}\left(-12abc^2\ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a}\sqrt{c}}{2\sqrt{c}}\right) + 3b^3c\ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a}\sqrt{c}}{2\sqrt{c}}\right) - 12\sqrt{cx^2 + bx + a}bc^2x - 6\sqrt{cx^2 + bx + a}b^2c^{\frac{3}{2}} + 16(cx^2 + bx + a)^{\frac{3}{2}}c^{\frac{5}{2}}\right)}{48\sqrt{cx^2 + bx + a}c^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(1/2),x)`

[Out] $\frac{1}{48}(cx^4+bx^3+ax^2)^{1/2}(16(c^2x^2+bx+a)^{3/2}c^{5/2}-12c^{5/2}(cx^2+bx+a)^{1/2}xb-6c^{3/2}(cx^2+bx+a)^{1/2}b^2-12\ln(1/2(2cx+b+2(cx^2+bx+a)^{1/2}c^{1/2}))/c^{1/2})ab^2c^2+3\ln(1/2(2cx+b+2(cx^2+bx+a)^{1/2}c^{1/2}))/c^{1/2})b^3c)/x/(cx^2+bx+a)^{1/2}/c^{7/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + bx^3 + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{cx^4 + bx^3 + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(1/2),x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + bx^3 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)`

$$3.32 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x} dx$$

Optimal. Leaf size=119

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{x(b^2-4ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1918, 1914, 621, 206}

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{x(b^2-4ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]

[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, n - q]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{(b^2 - 4ac) \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c} \\ &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{\left((b^2 - 4ac)x\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{8c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{\left((b^2 - 4ac)x\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b}{\sqrt{a + bx + cx^2}}\right)}{4c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{(b^2 - 4ac)x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}\sqrt{ax^2 + bx^3 + cx^4}} \end{aligned}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.84

$$\frac{x(2\sqrt{c}(b + 2cx)(a + x(b + cx)) - (b^2 - 4ac)\sqrt{a + x(b + cx)} \log(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx))}{8c^{3/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x, x]

[Out] (x*(2*Sqrt[c]*(b + 2*c*x)*(a + x*(b + c*x)) - (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(8*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.12, size = 116, normalized size = 0.97

$$\frac{\log(x)(4ac - b^2)}{8c^{3/2}} + \frac{(b^2 - 4ac) \log\left(-2c^{3/2}\sqrt{ax^2 + bx^3 + cx^4} + bcx + 2c^2x^2\right)}{8c^{3/2}} + \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]

[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) + ((-b^2 + 4*a*c)*Log[x])/(8*c^(3/2)) + ((b^2 - 4*a*c)*Log[b*c*x + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(8*c^(3/2))

fricas [A] time = 1.11, size = 220, normalized size = 1.85

$$\left[\frac{(b^2 - 4ac)\sqrt{c}x \log\left(\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{cx^4 + bx^3 + ax^2}(2cx+b)\sqrt{c} + (b^2 + 4ac)x}{x}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}(2c^2x + bc)}{16c^2x}, \frac{(b^2 - 4ac)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(2cx+b)\sqrt{-c}}{2(z^2x^3 + bc^2 + acx)}\right) + 2\sqrt{cx^4 + bx^3 + ax^2}(2c^2x + bc)}{8c^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")

[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]

giac [A] time = 0.95, size = 125, normalized size = 1.05

$$\frac{1}{8} \left(2\sqrt{cx^2 + bx + a} \left(2x + \frac{b}{c} \right) + \frac{(b^2 - 4ac) \log\left(\left| -2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b \right|\right)}{c^{\frac{3}{2}}} \right) \operatorname{sgn}(x) - \frac{(b^2 \log\left(\left| -b + 2\sqrt{a}\sqrt{c} \right|\right) - 4ac \log\left(\left| -b + 2\sqrt{a}\sqrt{c} \right|\right) + 2\sqrt{a}b\sqrt{c}) \operatorname{sgn}(x)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c)))) + 2*sqrt(a)*b*sqrt(c))*sgn(x)/c^(3/2)

maple [A] time = 0.00, size = 146, normalized size = 1.23

$$\frac{\sqrt{cx^4 + bx^3 + ax^2} \left(4ac^2 \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) - b^2c \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) + 4\sqrt{cx^2 + bx + a} \frac{5}{c^{\frac{5}{2}}}x + 2\sqrt{cx^2 + bx + a} \frac{3}{bc^{\frac{3}{2}}}\right)}{8\sqrt{cx^2 + bx + a} \frac{5}{c^{\frac{5}{2}}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x)

[Out] $\frac{1}{8}(cx^4+bx^3+ax^2)^{1/2}(4(cx^2+bx+a)^{1/2}c^{5/2}x+2(cx^2+bx+a)^{1/2}c^{3/2}b+4\ln(1/2(2cx+b+2(cx^2+bx+a)^{1/2}c^{1/2}))/c^{1/2}))a^2-\ln(1/2(2cx+b+2(cx^2+bx+a)^{1/2}c^{1/2}))/c^{1/2})b^2c)/(cx^2+bx+a)^{1/2}/c^{5/2}/x$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)`

[Out] `Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)`

$$3.33 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx$$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1921, 1933, 843, 621, 206, 724}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] + (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/((2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q)))^p/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} + \frac{1}{2} \int \frac{2a + bx}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} + \frac{\left(x\sqrt{a + bx + cx^2}\right) \int \frac{2a+bx}{x\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} + \frac{\left(ax\sqrt{a + bx + cx^2}\right) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(bx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} - \frac{\left(2ax\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(bx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} - \frac{\sqrt{a} x \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{bx\sqrt{a + bx + cx^2}}{2\sqrt{c} \sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 0.77

$$\frac{x\sqrt{a + x(b + cx)} \left(2\sqrt{c} \sqrt{a + x(b + cx)} - 2\sqrt{a} \sqrt{c} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+x(b+cx)}}\right) + b \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c} \sqrt{a+x(b+cx)}}\right)\right)}{2\sqrt{c} \sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] - 2*Sqrt[a]*Sqrt[c]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.42, size = 134, normalized size = 0.77

$$\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} - \frac{b \log\left(-2\sqrt{c} \sqrt{ax^2 + bx^3 + cx^4} + bx + 2cx^2\right)}{2\sqrt{c}} + 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} x}{\sqrt{c} x^2 - \sqrt{ax^2 + bx^3 + cx^4}}\right) + \frac{b \log(x)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x + 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^2 - Sqrt[a*x^2 + b*x^3 + c*x^4])] + (b*Log[x])/(2*Sqrt[c]) - (b*Log[b*x + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(2*Sqrt[c])

fricas [A] time = 1.41, size = 638, normalized size = 3.69

$$\frac{\sqrt{c} \log\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) - 2\sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) + 4\sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) - \sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) - 2\sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) + 4\sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) - \sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right) - 2\sqrt{c} \arctan\left(\frac{\sqrt{c} \sqrt{bx^2 + bx + a}}{2\sqrt{c}}\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))
*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)
*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-c)*x
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
- 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqrt(c)*x*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(
c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/2*(2*s
qrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a
*c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*c)/(c*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 126, normalized size = 0.73

$$\frac{\sqrt{cx^4 + bx^3 + ax^2} \left(2\sqrt{a} \sqrt{c} \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - b \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) - 2\sqrt{cx^2+bx+a}\sqrt{c} \right)}{2\sqrt{cx^2+bx+a}\sqrt{c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x)

[Out] $-1/2*(c*x^4+b*x^3+a*x^2)^{(1/2)}*(2*a^{(1/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*c^{(1/2)}-2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}-b*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)}))/x/(c*x^2+b*x+a)^{(1/2)}/c^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)`

[Out] `Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)`

$$3.34 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} dx$$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{c}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1920, 1933, 843, 621, 206, 724}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{c}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]

[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} + \frac{1}{2} \int \frac{b + 2cx}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} + \frac{\left(x\sqrt{a + bx + cx^2}\right) \int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} + \frac{\left(bx\sqrt{a + bx + cx^2}\right) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(cx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} - \frac{\left(bx\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(2cx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} - \frac{bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\sqrt{c}x\sqrt{a + bx + cx^2}}{\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 131, normalized size = 0.76

$$\frac{\sqrt{a + x(b + cx)} \left(bx \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) + 2\sqrt{a} \left(\sqrt{a + x(b + cx)} - \sqrt{c} x \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) \right) \right)}{2\sqrt{a}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3, x]

[Out] $-\frac{1}{2} \left(\text{Sqrt}[a + x(b + cx)] \left(b x \text{ArcTanh}\left[\frac{2a + bx}{2 \text{Sqrt}[a] \text{Sqrt}[a + x(b + cx)]}\right] + 2 \text{Sqrt}[a] \left(\text{Sqrt}[a + x(b + cx)] - \text{Sqrt}[c] x \text{ArcTanh}\left[\frac{b + 2cx}{2 \text{Sqrt}[c] \text{Sqrt}[a + x(b + cx)]}\right] \right) \right) \right) / \left(\text{Sqrt}[a] \text{Sqrt}[x^2(a + x(b + cx))] \right)$

IntegrateAlgebraic [A] time = 0.41, size = 128, normalized size = 0.74

$$-\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} - \sqrt{c} \log\left(-2\sqrt{c}\sqrt{ax^2 + bx^3 + cx^4} + bx + 2cx^2\right) + \frac{b \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}x^2 - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{\sqrt{a}} + \sqrt{c} \log(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3, x]

[Out] $-\left(\text{Sqrt}[a x^2 + b x^3 + c x^4] / x^2 \right) + \left(b \text{ArcTanh}\left[\text{Sqrt}[a] x / \left(\text{Sqrt}[c] x^2 - \text{Sqrt}[a x^2 + b x^3 + c x^4] \right) \right] \right) / \text{Sqrt}[a] + \text{Sqrt}[c] \text{Log}[x] - \text{Sqrt}[c] \text{Log}[b x + 2 c x^2 - 2 \text{Sqrt}[c] \text{Sqrt}[a x^2 + b x^3 + c x^4]]$

fricas [A] time = 1.43, size = 653, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(2*a*\sqrt{c}*x^2*\log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*\sqrt{c*x^4 + b*x^3 + a*x^2} + \\ & a*x^2)*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x) + \sqrt{a}*b*x^2*\log(-(8*a* \\ & b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + \\ & 2*a)*\sqrt{a})/x^3) - 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*a/(a*x^2), -1/4*(4*a*\sqrt{ \\ & -c}*x^2*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{-c}/(c^2 \\ & *x^3 + b*c*x^2 + a*c*x)) - \sqrt{a}*b*x^2*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 \\ & + 8*a^2*x - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3) + 4*\sqrt{ \\ & c*x^4 + b*x^3 + a*x^2}*a/(a*x^2), 1/2*(\sqrt{-a}*b*x^2*\arctan(1/2*\sqrt{ \\ & c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) + \\ & a*\sqrt{c}*x^2*\log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(\\ & 2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x) - 2*\sqrt{c*x^4 + b*x^3 + a*x^2}*a) \\ & / (a*x^2), 1/2*(\sqrt{-a}*b*x^2*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + \\ & 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*\sqrt{-c}*x^2*\arctan(1/2*\sqrt{ \\ & c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x) \\ &) - 2*\sqrt{c*x^4 + b*x^3 + a*x^2}*a)/(a*x^2)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(x)]
 Warning, choosing root of [1,0,%%{-2,[1,0,0,2]%%}+%%{-2,[0,1,0,1]%%}+%%
 %{-4,[0,0,1,0]%%},0,%%{1,[2,0,0,4]%%}+%%{2,[1,1,0,3]%%}+%%{1,[0,2,0,2
]%%}] at parameters values [-97,-82,63.4443001123,-27]Warning, choosing ro
 ot of [1,0,%%{-2,[1,0,0,2]%%}+%%{-2,[0,1,0,1]%%}+%%{-4,[0,0,1,0]%%},0
 ,%%{1,[2,0,0,4]%%}+%%{2,[1,1,0,3]%%}+%%{1,[0,2,0,2]%%}] at parameters
 values [63,-49,35.2935628123,-64]Warning, choosing root of [1,0,%%{-2,[2,
 1,0,0]%%}+%%{-2,[1,0,1,0]%%}+%%{-4,[0,0,0,1]%%},0,%%{1,[4,2,0,0]%%}+
 %%{2,[3,1,1,0]%%}+%%{1,[2,0,2,0]%%}] at parameters values [22,42,56,43.
 9628838282]Sign error (%%{b-2*sqrt(a)*sqrt(c),0%%}+%%{-(-2*a*c+b*sqrt(a)
 *sqrt(c))/a,1%%}+%%{-4*a*c*sqrt(a)*sqrt(c)-b^2*sqrt(a)*sqrt(c)/(4*a^2),
 2%%}+%%{undef,3%%})Limit: Max order reached or unable to make series exp
 ansion Error: Bad Argument Value

maple [A] time = 0.01, size = 173, normalized size = 1.00

$$\frac{\sqrt{cx^4 + bx^3 + ax^2} \left(-2ac^2x \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) + \sqrt{a}bc^{\frac{3}{2}}x \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - 2\sqrt{cx^2+bx+a}c^{\frac{5}{2}}x^2 - 2\sqrt{cx^2+bx+a}bc^{\frac{3}{2}}x + 2(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{3}{2}} \right)}{2\sqrt{cx^2+bx+a}ac^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x)

[Out] $-1/2*(c*x^4+b*x^3+a*x^2)^{(1/2)}*(-2*(c*x^2+b*x+a)^{(1/2)}*c^{(5/2)}*x^2+c^{(3/2)}*a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*x*b+2*(c*x^2+b*x+a)^{(3/2)}*c^{(3/2)}-2*(c*x^2+b*x+a)^{(1/2)}*c^{(3/2)}*x*b-2*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)})*x*a*c^2)/x^2/(c*x^2+b*x+a)^{(1/2)}/a/c^{(3/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)

[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)

$$3.35 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} dx$$

Optimal. Leaf size=114

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Rubi [A] time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1920

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^p, x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q

+ 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1951

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} + \frac{1}{4} \int \frac{b + 2cx}{x\sqrt{ax^2 + bx^3 + cx^4}} dx \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} - \frac{\int \frac{b^2 - 4ac}{2\sqrt{ax^2 + bx^3 + cx^4}} dx}{4a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} - \frac{(b^2 - 4ac) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} + \frac{(b^2 - 4ac) \text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x(2a + bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{4a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} + \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a + bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{8a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.98

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(x^2 (b^2 - 4ac) \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) - 2\sqrt{a}(2a + bx)\sqrt{a + x(b + cx)} \right)}{8a^{3/2}x^3\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(8*a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 0.53, size = 100, normalized size = 0.88

$$\frac{(4ac - b^2) \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}x^2 - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{4a^{3/2}} + \frac{(-2a - bx)\sqrt{ax^2 + bx^3 + cx^4}}{4ax^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]

[Out] ((-2*a - b*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a*x^3) + ((-b^2 + 4*a*c)*ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^2 - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(4*a^(3/2))

fricas [A] time = 1.27, size = 226, normalized size = 1.98

$$\left[\frac{(b^2 - 4ac)\sqrt{a}x^3 \log\left(\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x - 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) + 4\sqrt{cx^4 + bx^3 + ax^2}(abx + 2a^2)}{16a^2x^3}, \frac{(b^2 - 4ac)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{-a}}{2(acx^3 + abx^2 + a^2x)}\right) + 2\sqrt{cx^4 + bx^3 + ax^2}(abx + 2a^2)}{8a^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 207, normalized size = 1.82

$$\frac{\sqrt{cx^4 + bx^3 + ax^2} \left(4a^{\frac{3}{2}}c^2 \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a}\sqrt{a}}{x}\right) - \sqrt{a}b^2x^2 \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a}\sqrt{a}}{x}\right) + 2\sqrt{cx^2 + bx + a}bcx^3 - 4\sqrt{cx^2 + bx + a}acx^2 + 2\sqrt{cx^2 + bx + a}b^2x^2 - 2(c^2 + bx + a)^{\frac{3}{2}}bx + 4(c^2 + bx + a)^{\frac{3}{2}}a \right)}{8\sqrt{cx^2 + bx + a}a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x)`

[Out]
$$-1/8*(c*x^4+b*x^3+a*x^2)^{(1/2)}*(4*c*a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)})*a^{(1/2)})/x)*x^2+2*c*(c*x^2+b*x+a)^{(1/2)}*x^3*b-4*c*(c*x^2+b*x+a)^{(1/2)}*x^2*a-a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)})*a^{(1/2)})/x)*x^2*b^2-2*(c*x^2+b*x+a)^{(3/2)}*x*b+2*(c*x^2+b*x+a)^{(1/2)}*x^2*b^2+4*(c*x^2+b*x+a)^{(3/2)}*a/x^3/(c*x^2+b*x+a)^{(1/2)}/a^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)`

[Out] `Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)`

$$3.36 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5} dx$$

Optimal. Leaf size=155

$$-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} - \frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

Rubi [A] time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} - \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3} - \frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(5/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1920

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
+ 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))* (a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_.) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} + \frac{1}{6} \int \frac{b + 2cx}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} - \frac{\int \frac{\frac{1}{2}(3b^2 - 8ac) + bcx}{x \sqrt{ax^2 + bx^3 + cx^4}} dx}{12a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} + \frac{\int \frac{3}{4\sqrt{a}}}{4\sqrt{a}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} + \frac{b(b^2)}{4\sqrt{a}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} - \frac{b(b^2)}{4\sqrt{a}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} - \frac{b(b^2)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 131, normalized size = 0.85

$$\frac{\sqrt{x^2(a+x(b+cx))} \left(-2\sqrt{a} \sqrt{a+x(b+cx)} (8a^2 + 2ax(b+4cx) - 3b^2x^2) - 3bx^3(b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+x(b+cx)}} \right) \right)}{48a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5, x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(8*a^2 - 3*b^2*x^2 + 2*a*x*(b + 4*c*x)) - 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(48*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 0.79, size = 117, normalized size = 0.75

$$\frac{(b^3 - 4abc) \tanh^{-1} \left(\frac{\sqrt{a} x}{\sqrt{c} x^2 - \sqrt{ax^2 + bx^3 + cx^4}} \right)}{8a^{5/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4} (-8a^2 - 2abx - 8acx^2 + 3b^2x^2)}{24a^2x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5, x]

[Out] ((-8*a^2 - 2*a*b*x + 3*b^2*x^2 - 8*a*c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^4) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^2 - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(5/2))

fricas [A] time = 1.33, size = 272, normalized size = 1.75

$$\left[\frac{3(b^3 - 4abc)\sqrt{a}x^4 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 4\sqrt{cx^4 + bx^3 + ax^2}(2a^2bx + 8a^3 - (3ab^2 - 8a^2c)x^2)}{96a^3x^4}\right) + 4\sqrt{cx^4 + bx^3 + ax^2}(2a^2bx + 8a^3 - (3ab^2 - 8a^2c)x^2)}{96a^3x^4}, \frac{3(b^3 - 4abc)\sqrt{-a}x^4 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{-a}}{2(acx^2 + abx^3 + a^2x^4)}\right) - 2\sqrt{cx^4 + bx^3 + ax^2}(2a^2bx + 8a^3 - (3ab^2 - 8a^2c)x^2)}{48a^3x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5, x, algorithm="fricas")

[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.01, size = 234, normalized size = 1.51

$$\frac{\sqrt{cx^4+bx^3+ax^2} \left(12a^{\frac{3}{2}}bcx^3 \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - 3\sqrt{a}b^{\frac{3}{2}}x^3 \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) + 6\sqrt{cx^2+bx+a}b^2cx^4 - 12\sqrt{cx^2+bx+a}abcx^3 + 6\sqrt{cx^2+bx+a}b^3x^3 - 6(cx^2+bx+a)^{\frac{3}{2}}b^2x^2 + 12(cx^2+bx+a)^{\frac{3}{2}}abx - 16(cx^2+bx+a)^{\frac{3}{2}}a^2 \right)}{48\sqrt{cx^2+bx+a}a^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x)

[Out] $\frac{1}{48}(cx^4+bx^3+ax^2)^{\frac{1}{2}} \left((12ca^{\frac{3}{2}} \ln((bx+2a+2\sqrt{cx^2+bx+a})^{\frac{1}{2}}a^{\frac{1}{2}})/x) x^3b+6c(cx^2+bx+a)^{\frac{1}{2}}x^4b^2-12c(cx^2+bx+a)^{\frac{1}{2}}x^3ab-3a^{\frac{3}{2}} \ln((bx+2a+2\sqrt{cx^2+bx+a})^{\frac{1}{2}}a^{\frac{1}{2}})/x) x^3b^3-6(cx^2+bx+a)^{\frac{3}{2}}x^2b^2+6(cx^2+bx+a)^{\frac{1}{2}}x^3b^3+12(cx^2+bx+a)^{\frac{3}{2}}x^2ab-16(cx^2+bx+a)^{\frac{3}{2}}a^2 \right) / x^4 / (cx^2+bx+a)^{\frac{1}{2}} / a^{\frac{3}{2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(a+bx+cx^2)}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

$$3.37 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} dx$$

Optimal. Leaf size=205

$$\frac{(b^2 - 4ac)(5b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}} - \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192a^3x^2} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3}}{96a^2x^3}$$

Rubi [A] time = 0.39, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$-\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192a^3x^2} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} + \frac{(b^2 - 4ac)(5b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} - \frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]

[Out] $-\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(24*a*x^4) + ((5*b^2 - 12*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(15*b^2 - 52*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])])/(128*a^{(7/2)})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1920

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
+ 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^p)/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^6} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} + \frac{1}{8} \int \frac{b + 2cx}{x^3 \sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} - \frac{\int \frac{\frac{1}{2}(5b^2 - 12ac) + 2bcx}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx}{24a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} + \int \frac{\frac{1}{4}b}{x^3 \sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15\sqrt{ax^2 + bx^3 + cx^4})}{384a^2x^3} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15\sqrt{ax^2 + bx^3 + cx^4})}{384a^2x^3} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15\sqrt{ax^2 + bx^3 + cx^4})}{384a^2x^3} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15\sqrt{ax^2 + bx^3 + cx^4})}{384a^2x^3}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 160, normalized size = 0.78

$$\frac{\sqrt{x^2(a+x(b+cx))} \left(3x^4(16a^2c^2 - 24ab^2c + 5b^4) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}\sqrt{a+x(b+cx)}(48a^3 + 8a^2x(b+3cx) - 2abx^2(5b+26cx) + 15b^3x^3) \right)}{384a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3 + 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))]/(384*a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 1.03, size = 150, normalized size = 0.73

$$\frac{(-16a^2c^2 + 24ab^2c - 5b^4) \tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{cx^2 - \sqrt{ax^2 + bx^3 + cx^4}}}\right) + \sqrt{ax^2 + bx^3 + cx^4}(-48a^3 - 8a^2bx - 24a^2cx^2 + 10ab^2x^2 + 52abcx^3 - 15b^3x^3)}{64a^{7/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-48a^3 - 8a^2bx - 24a^2cx^2 + 10ab^2x^2 + 52abcx^3 - 15b^3x^3)}{192a^3x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]

[Out]
$$\frac{((-48a^3 - 8a^2bx + 10ab^2x^2 - 24a^2cx^2 - 15b^3x^3 + 52ab^2c)x^3) \sqrt{ax^2 + bx^3 + cx^4}}{(192a^3x^5) + ((-5b^4 + 24ab^2c - 16a^2c^2) \operatorname{ArcTanh}[\frac{\sqrt{a}x}{\sqrt{c}x^2 - \sqrt{ax^2 + bx^3 + cx^4}}])}{(64a^{7/2})}$$

fricas [A] time = 1.35, size = 336, normalized size = 1.64

$$\frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{a}x^5 \log\left(\frac{8ab^2x^2 + 16a^2c^2}{3}\right) - 4(8a^3bx + 48a^4 + (15ab^3 - 52a^2bc)x^3 - 2(5a^2b^2 - 12a^2c^2)\sqrt{cx^4 + bx^3 + ax^2})}{768a^4x^5} + \frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{-a}x^5 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}}{2(ax^2 + abx^2 + a^2)}\right) + 2(8a^3bx + 48a^4 + (15ab^3 - 52a^2bc)x^3 - 2(5a^2b^2 - 12a^2c^2)\sqrt{cx^4 + bx^3 + ax^2})}{384a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")

[Out]
$$\frac{1}{768} \frac{(3(5b^4 - 24ab^2c + 16a^2c^2) \sqrt{a} x^5 \log(-8ab^2x^2 + b^2 + 4a^2c) x^3 + 8a^2bx + 4 \sqrt{cx^4 + bx^3 + ax^2} (bx + 2a) \sqrt{a})}{x^3} - \frac{4(8a^3bx + 48a^4 + (15ab^3 - 52a^2bc) x^3 - 2(5a^2b^2 - 12a^2c^2) \sqrt{cx^4 + bx^3 + ax^2})}{a^4 x^5}, -\frac{1}{384} \frac{(3(5b^4 - 24ab^2c + 16a^2c^2) \sqrt{-a} x^5 \arctan(1/2 \sqrt{cx^4 + bx^3 + ax^2}) (bx + 2a) \sqrt{-a})}{(a^2 c x^3 + ab^2 x^2 + a^2 x)} + \frac{2(8a^3bx + 48a^4 + (15ab^3 - 52a^2bc) x^3 - 2(5a^2b^2 - 12a^2c^2) \sqrt{cx^4 + bx^3 + ax^2})}{a^4 x^5}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 387, normalized size = 1.89

$$\frac{\sqrt{c^2 + 3b^2 + 3a^2} \left(8a^2 b^2 x^4 \ln\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}}{\sqrt{c^2 + 3b^2 + 3a^2}}\right) - 72a^2 b^2 x^4 \ln\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}}{\sqrt{c^2 + 3b^2 + 3a^2}}\right) + 15\sqrt{c^2 + 3b^2 + 3a^2} \ln\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}}{\sqrt{c^2 + 3b^2 + 3a^2}}\right) + 24\sqrt{c^2 + 3b^2 + 3a^2} ab^2 x^3 - 30\sqrt{c^2 + 3b^2 + 3a^2} b^2 x^3 - 48\sqrt{c^2 + 3b^2 + 3a^2} a^2 x^3 + 84\sqrt{c^2 + 3b^2 + 3a^2} ab^2 x^2 - 30\sqrt{c^2 + 3b^2 + 3a^2} b^2 x^2 - 24(c^2 + 3a + a^2) \sqrt{cx^4 + bx^3 + ax^2} + 48(c^2 + 3a + a^2) \sqrt{cx^4 + bx^3 + ax^2} - 48(c^2 + 3a + a^2) \sqrt{cx^4 + bx^3 + ax^2} + 80(c^2 + 3a + a^2) \sqrt{cx^4 + bx^3 + ax^2} - 96(c^2 + 3a + a^2) \sqrt{cx^4 + bx^3 + ax^2} \right)}{384\sqrt{c^2 + 3b^2 + 3a^2} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x)

[Out]
$$\frac{1}{384} \frac{(c^2 x^4 + b^2 x^3 + a^2 x^2)^{1/2} (48c^2 a^{5/2} \ln((bx + 2a + 2(c^2 x^2 + bx + a)^{1/2}) a^{1/2})/x) x^4 + 24c^2 (c^2 x^2 + bx + a)^{1/2} x^5 a^2 b - 72c^2 a^{3/2} \ln((bx + 2a + 2(c^2 x^2 + bx + a)^{1/2}) a^{1/2})/x) x^4 b^2 - 48c^2 (c^2 x^2 + bx + a)^{1/2} x^4 a^2 - 30c^2 (c^2 x^2 + bx + a)^{1/2} x^5 b^3 - 24c^2 (c^2 x^2 + bx + a)^{3/2} x^3 a^2 b + 84c^2 (c^2 x^2 + bx + a)^{1/2} x^4 a^2 b^2 + 15a^{1/2} \ln((bx + 2a + 2(c^2 x^2 + bx + a)^{1/2}) a^{1/2})}{x^5}$$

$(1/2)*a^{(1/2)}/x)*x^4*b^4+48*c*(c*x^2+b*x+a)^{(3/2)}*x^2*a^2+30*(c*x^2+b*x+a)^{(3/2)}*x^3*b^3-30*(c*x^2+b*x+a)^{(1/2)}*x^4*b^4-60*(c*x^2+b*x+a)^{(3/2)}*x^2*a*b^2+80*(c*x^2+b*x+a)^{(3/2)}*x*a^2*b-96*(c*x^2+b*x+a)^{(3/2)}*a^3)/x^5/(c*x^2+b*x+a)^{(1/2)}/a^4$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)

[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)

$$3.38 \quad \int x (ax^2 + bx^3 + cx^4)^{3/2} dx$$

Optimal. Leaf size=422

$$\frac{3x(b^2 - 4ac)^2 (16a^2c^2 - 72ab^2c + 33b^4) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) + bx(2416a^2c^2 - 1560ab^2c + 231b^4)}{32768c^{13/2}\sqrt{ax^2 + bx^3 + cx^4}} \quad 71680c^4$$

Rubi [A] time = 1.20, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1919, 1945, 1949, 12, 1914, 621, 206}

$$\frac{x^2(560a^2c^2 - 568ab^2c + 99b^3)\sqrt{a+bx+cx^2}}{3584c^3} - \frac{bx(2416a^2c^2 - 1560ab^2c + 231b^4)\sqrt{a+bx+cx^2}}{71680c^4} - \frac{(18896a^2c^2 - 6720a^3c^3 - 8988a^4c + 1155a^5)\sqrt{a+bx+cx^2}}{286720c^5} - \frac{b(3465a^2c^2 - 30660a^3c^3 - 58816a^4c^4 - 58816a^5c^5 - 30660a^6c^6 + 3465a^7)\sqrt{a+bx+cx^2}}{573440c^6x} - \frac{3x(b^2 - 4ac)^2(16a^2c^2 - 72ab^2c + 33b^4)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{x^2(10a(11b^2 - 28a) + b(6a + 11b^2))\sqrt{a+bx+cx^2}}{4480c^2} - \frac{c(3b + 14cx)(a^2 + bx + cx^2)^{3/2}}{112c} + \frac{c(3b + 14cx)(a^2 + bx + cx^2)^{3/2}}{112c}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,

b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2])) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1919

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
)))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]

Rule 1945

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
.)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
q + (n - q)(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
+ (n - q)*(2*p + 1) + 1, 0]

Rule 1949

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
.)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b

```

*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int x(ax^2 + bx^3 + cx^4)^{3/2} dx &= \frac{x(3b + 14cx)(ax^2 + bx^3 + cx^4)^{3/2}}{112c} + \frac{3 \int x^2 \left(-4ab - \frac{1}{2}(11b^2 - 28ac)x\right) \sqrt{ax^2 + bx^3 + cx^4}}{112c} \\
&= -\frac{x^3(b(11b^2 + 68ac) + 10c(11b^2 - 28ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{4480c^2} + \frac{x(3b + 14cx)(ax^2 + bx^3 + cx^4)^{3/2}}{112c} \\
&= \frac{(99b^4 - 568ab^2c + 560a^2c^2)x^2 \sqrt{ax^2 + bx^3 + cx^4}}{35840c^3} - \frac{x^3(b(11b^2 + 68ac) + 10c(11b^2 - 28ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{4480c^2} \\
&= -\frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)x \sqrt{ax^2 + bx^3 + cx^4}}{71680c^4} + \frac{(99b^4 - 568ab^2c + 560a^2c^2)x^2 \sqrt{ax^2 + bx^3 + cx^4}}{35840c^3} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)x \sqrt{ax^2 + bx^3 + cx^4}}{71680c^4} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 2856ab^4c + 7168a^2b^2c^2 - 2240a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 2856ab^4c + 7168a^2b^2c^2 - 2240a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 2856ab^4c + 7168a^2b^2c^2 - 2240a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 2856ab^4c + 7168a^2b^2c^2 - 2240a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 2856ab^4c + 7168a^2b^2c^2 - 2240a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 236, normalized size = 0.56

$$\frac{(x^2(a + x(b + cx)))^{3/2} \left(\frac{(16a^2c^2 - 72ab^2c + 33b^4) \left(2\sqrt{c} (b + 2cx) \sqrt{a + x(b + cx)} (4c(5a + 2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + x(b + cx)}} \right) \right)}{4096c^{11/2} x^3 (a + x(b + cx))^{3/2}} + \frac{(372abc - 280a^2c^2x - 231b^3 + 330b^2cx)(a + x(b + cx))}{560c^3 x^3} - \frac{11b(a + x(b + cx))}{14cx} + a + bx + cx^2 \right)}{8c}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(a + b*x + c*x^2 - (11*b*(a + x*(b + c*x)))/
(14*c*x) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x - 280*a*c^2*x)*(a + x*(b +
c*x)))/(560*c^3*x^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(
b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/(40
96*c^(11/2)*x^3*(a + x*(b + c*x))^(3/2)))/(8*c)
```

IntegrateAlgebraic [A] time = 5.16, size = 323, normalized size = 0.77

$$\frac{\sqrt{a^2 + b^2 + c^2} (58816a^3c^2 - 13440a^2b^2c - 8160a^2b^2c^2 + 37792a^2b^2c^3 - 19328a^2b^2c^4 + 8960a^2b^2c^5 - 17976a^2b^2c^6 + 12480a^2b^2c^7 - 9280a^2b^2c^8 + 6656a^2b^2c^9 + 107520a^2b^2c^{10} - 3468c^2 + 2310c^3 - 1848c^4 + 1584c^5 - 1408c^6 + 1280c^7 + 87040a^2b^2c^8 + 71680a^2b^2c^9)}{57440c^4} - \frac{3(256a^4 - 1280a^2b^2c + 1120b^4c^2 - 336a^2c^3 + 336c^4) \operatorname{arctanh}\left(\frac{c^2}{\sqrt{c^2 - 4a^2 - 4b^2 - 4c^2}}\right)}{16384c^{13}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a*x^2 + b*x^3 + c*x^4]*(-3465*b^7 + 30660*a*b^5*c - 81648*a^2*b^3*c^2
+ 58816*a^3*b*c^3 + 2310*b^6*c*x - 17976*a*b^4*c^2*x + 37792*a^2*b^2*c^3*x
- 13440*a^3*c^4*x - 1848*b^5*c^2*x^2 + 12480*a*b^3*c^3*x^2 - 19328*a^2*b*c
^4*x^2 + 1584*b^4*c^3*x^3 - 9088*a*b^2*c^4*x^3 + 8960*a^2*c^5*x^3 - 1408*b^
3*c^4*x^4 + 6656*a*b*c^5*x^4 + 1280*b^2*c^5*x^5 + 107520*a*c^6*x^5 + 87040*
b*c^6*x^6 + 71680*c^7*x^7))/(573440*c^6*x) - (3*(33*b^8 - 336*a*b^6*c + 112
0*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*ArcTanh[(Sqrt[c]*x^2)/(Sqrt
[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16384*c^(13/2))
```

fricas [A] time = 1.38, size = 664, normalized size = 1.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
+ 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7 + 8
7040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a
^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^
4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 15
60*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x), -1/
1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 +
256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040*b*c^7*x^6
- 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280
*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*
c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 +
```


$$2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]$$

giac [A] time = 1.46, size = 521, normalized size = 1.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] $\frac{1}{573440} \sqrt{c x^2 + b x + a} (2 (4 (2 (8 (10 (4 (14 c x \operatorname{sgn}(x) + 17 b \operatorname{sgn}(x)) x + (b^2 c^6 \operatorname{sgn}(x) + 84 a c^7 \operatorname{sgn}(x)) / c^7) x - (11 b^3 c^5 \operatorname{sgn}(x) - 52 a b c^6 \operatorname{sgn}(x)) / c^7) x + (99 b^4 c^4 \operatorname{sgn}(x) - 568 a b^2 c^5 \operatorname{sgn}(x) + 560 a^2 c^6 \operatorname{sgn}(x)) / c^7) x - (231 b^5 c^3 \operatorname{sgn}(x) - 1560 a b^3 c^4 \operatorname{sgn}(x) + 2416 a^2 b c^5 \operatorname{sgn}(x)) / c^7) x + (1155 b^6 c^2 \operatorname{sgn}(x) - 8988 a b^4 c^3 \operatorname{sgn}(x) + 18896 a^2 b^2 c^4 \operatorname{sgn}(x) - 6720 a^3 c^5 \operatorname{sgn}(x)) / c^7) x - (3465 b^7 c \operatorname{sgn}(x) - 30660 a b^5 c^2 \operatorname{sgn}(x) + 81648 a^2 b^3 c^3 \operatorname{sgn}(x) - 58816 a^3 b c^4 \operatorname{sgn}(x)) / c^7) - \frac{3}{32768} (33 b^8 \operatorname{sgn}(x) - 336 a b^6 c \operatorname{sgn}(x) + 1120 a^2 b^4 c^2 \operatorname{sgn}(x) - 1280 a^3 b^2 c^3 \operatorname{sgn}(x) + 256 a^4 c^4 \operatorname{sgn}(x)) \log(\operatorname{abs}(-2 \sqrt{c} x - \sqrt{c x^2 + b x + a}) \sqrt{c} - b) / c^{13/2} + \frac{1}{1146880} (3465 b^8 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) - 35280 a b^6 c \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) + 117600 a^2 b^4 c^2 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) - 134400 a^3 b^2 c^3 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) + 26880 a^4 c^4 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) + 6930 \sqrt{a} b^7 \sqrt{c} - 61320 a^{3/2} b^5 c^{3/2} + 163296 a^{5/2} b^3 c^{5/2} - 117632 a^{7/2} b c^{7/2}) \operatorname{sgn}(x) / c^{13/2}$

maple [A] time = 0.01, size = 649, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] $\frac{1}{1146880} (c x^4 + b x^3 + a x^2)^{3/2} (26880 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2}) c^{1/2}) / c^{1/2}) a^4 c^5 + 3465 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2}) c^{1/2}) / c^{1/2}) b^8 c + 143360 x^3 (c x^2 + b x + a)^{5/2} c^{13/2} - 59136 (c x^2 + b x + a)^{5/2} c^{7/2} b^3 + 18480 (c x^2 + b x + a)^{3/2} c^{5/2} b^5 - 6930 (c x^2 + b x + a)^{1/2} c^{3/2} b^7 + 42840 (c x^2 + b x + a)^{1/2} c^{5/2} a b^5 + 8960 (c x^2 + b x + a)^{3/2} c^{9/2} a^2 b - 40320 (c x^2 + b x + a)^{3/2} c^{7/2} a b^3 - 134400 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2}) c^{1/2}) / c^{1/2}) a^3 b^2 c^4 + 117600 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2}) c^{1/2}) / c^{1/2}) a^2 b^4 c^3 - 35280 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2}) c^{1/2}) / c^{1/2}) a b^6 c^2 - 13860 (c x^2 + b x + a)^{1/2} c^{5/2} x b^6 + 13440 (c x^2 + b x + a)^{1/2} c^{9/2} a^3 b - 63840 (c x^2 + b x + a)^{1/2} c^{7/2} a^2 b^3 - 112640 (c x^2 + b x + a)^{5/2} c^{11/2}$

$*x^2*b-71680*(c*x^2+b*x+a)^{(5/2)}*c^{(11/2)}*x*a+84480*(c*x^2+b*x+a)^{(5/2)}*c^{(9/2)}*x*b^2+95232*(c*x^2+b*x+a)^{(5/2)}*c^{(9/2)}*a*b+17920*(c*x^2+b*x+a)^{(3/2)}*c^{(11/2)}*x*a^2+36960*(c*x^2+b*x+a)^{(3/2)}*c^{(7/2)}*x*b^4+26880*(c*x^2+b*x+a)^{(1/2)}*c^{(11/2)}*x*a^3-80640*(c*x^2+b*x+a)^{(3/2)}*c^{(9/2)}*x*a*b^2-127680*(c*x^2+b*x+a)^{(1/2)}*c^{(9/2)}*x*a^2*b^2+85680*(c*x^2+b*x+a)^{(1/2)}*c^{(7/2)}*x*a*b^4/x^3/(c*x^2+b*x+a)^{(3/2)}/c^{(15/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (cx^4 + bx^3 + ax^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x)

[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (x^2 (a + bx + cx^2))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)

$$3.39 \quad \int (ax^2 + bx^3 + cx^4)^{3/2} dx$$

Optimal. Leaf size=364

$$\frac{b(1168a^2c^2 - 728ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(-2048a^3c^3 + 5488a^2b^2c^2 - 2520ab^4c + 315b^6)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}$$

Rubi [A] time = 1.04, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1906, 1945, 1949, 12, 1914, 621, 206}

$$\frac{b(1168a^2c^2 - 728ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(5488a^2b^2c^2 - 2048a^3c^3 - 2520ab^4c + 315b^6)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x} - \frac{b^2(9b^2 - 44ac)\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} - \frac{x(7b^2 - 32ac)(3b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3} - \frac{36x(b^2 - 4ac)^2(b^2 - 4ac)\sqrt{a + bx + cx^2}\operatorname{tanh}^{-1}\left(\frac{b-2cx}{2\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{x^3(24ac + b^2 + 10bc)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{2}(ax^2 + bx^3 + cx^4)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $-(b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4})/(17920c^4) + ((315b^6 - 2520a^2b^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4})/(35840c^5x) + ((7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4})/(4480c^3) - (b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4})/(2240c^2) + (x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4})/(280c) + (x(ax^2 + bx^3 + cx^4)^{3/2})/7 - (3b(b^2 - 4ac)^2(3b^2 - 4ac)x\sqrt{a + bx + cx^2}\operatorname{ArcTanh}[(b + 2cx)/(2\sqrt{c}\sqrt{a + bx + cx^2})])/(2048c^{11/2}\sqrt{ax^2 + bx^3 + cx^4})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1906

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol]
:= Simp[(x*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(p*(2*n - q) + 1), x] + Dist[
((n - q)*p)/(p*(2*n - q) + 1), Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*
n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
&& NeQ[p*(2*n - q) + 1, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
+ (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
```

) + 1, 0]

Rubi steps

$$\begin{aligned}
\int (ax^2 + bx^3 + cx^4)^{3/2} dx &= \frac{1}{7}x(ax^2 + bx^3 + cx^4)^{3/2} + \frac{3}{14} \int x^2(2a + bx)\sqrt{ax^2 + bx^3 + cx^4} dx \\
&= \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{7}x(ax^2 + bx^3 + cx^4)^{3/2} + \frac{\int x^4(-4a(b^2 - 4ac) - 4bx^2(2a + bx))\sqrt{ax^2 + bx^3 + cx^4} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} + \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} \\
&= \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3} - \frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 197, normalized size = 0.54

$$\frac{(x^2(a + x(b + cx)))^{3/2} \left(\frac{(-16ac + 21b^2 - 30bcx)(a + x(b + cx))}{40c^2} + \frac{7(4abc - 3b^3) \left(2\sqrt{c} \sqrt{a + x(b + cx)} (4c(5a + 2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + x(b + cx)}} \right) \right)}{2048c^{9/2}(a + x(b + cx))^{3/2}} + x^2(a + x(b + cx)) \right)}{7cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(x^2*(a + x*(b + c*x)) + ((21*b^2 - 16*a*c - 30*b*c*x)*(a + x*(b + c*x)))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*sqrt[c]*(b + 2*c*x))*sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])]))/(2048*c^(9/2)*(a + x*(b + c*x))^(3/2)))/(7*c*x^3)

IntegrateAlgebraic [A] time = 4.08, size = 266, normalized size = 0.73

$$\frac{3(-64b^3bc^3 + 80a^2b^3c^2 - 28ab^5c + 3b^7) \operatorname{tanh}^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + bx^2 + cx^2}}\right) + \sqrt{ax^2 + bx^2 + cx^2}(-2048a^3c^3 + 5488a^2b^2c^2 - 2336ab^3c^2 + 1024a^2c^4x^2 - 2520ab^4c + 1456a^2c^2x - 992a^2c^2x^2 + 704abc^3x^3 + 8192ac^5x^4 + 315b^6 - 210b^5cx + 168b^4c^2x^2 - 144b^3c^3x^3 + 128b^2c^4x^4 + 6400b^5x^5 + 5120a^6x^6)}{1024c^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (sqrt[a*x^2 + b*x^3 + c*x^4]*(315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3 - 210*b^5*c*x + 1456*a*b^3*c^2*x - 2336*a^2*b*c^3*x + 168*b^4*c^2*x^2 - 992*a*b^2*c^3*x^2 + 1024*a^2*c^4*x^2 - 144*b^3*c^3*x^3 + 704*a*b*c^4*x^3 + 128*b^2*c^4*x^4 + 8192*a*c^5*x^4 + 6400*b*c^5*x^5 + 5120*c^6*x^6))/(35840*c^5*x) + (3*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*ArcTanh[(sqrt[c]*x^2)/(sqrt[a]*x - sqrt[a*x^2 + b*x^3 + c*x^4])])/(1024*c^(11/2))

fricas [A] time = 1.33, size = 558, normalized size = 1.53

$$\frac{-1}{143360} \left(105(3b^7 - 28ab^5c + 80a^2b^3c^2 - 64a^3b^2c^3) \sqrt{c} x \log\left(-\frac{(8c^2x^3 + 8b^2cx^2 + 4\sqrt{c}(c^2x^4 + b^2x^3 + a^2x^2))(2cx + b)\sqrt{c}}{(b^2 + 4ac)x}\right) - 4(5120c^7x^6 + 6400b^6c^6x^5 + 315b^6c^6 - 2520a^2b^4c^2 + 5488a^2b^2c^3 - 2048a^3c^4 + 128(b^2c^5 + 64ac^6)x^4 - 16(9b^3c^4 - 44ab^2c^5)x^3 + 8(21b^4c^3 - 124a^2b^2c^4 + 128a^2c^5)x^2 - 2(105b^5c^2 - 728ab^3c^3 + 1168a^2b^2c^4)x\right) \sqrt{c^2x^4 + b^2x^3 + a^2x^2} \right) / (c^6x) + \frac{1}{71680} \left(105(3b^7 - 28ab^5c + 80a^2b^3c^2 - 64a^3b^2c^3) \sqrt{-c} x \arctan\left(\frac{1}{2}\sqrt{c^2x^4 + b^2x^3 + a^2x^2}\right) + 2(5120c^7x^6 + 6400b^6c^6x^5 + 315b^6c^6 - 2520a^2b^4c^2 + 5488a^2b^2c^3 - 2048a^3c^4 + 128(b^2c^5 + 64ac^6)x^4 - 16(9b^3c^4 - 44ab^2c^5)x^3 + 8(21b^4c^3 - 124a^2b^2c^4 + 128a^2c^5)x^2 - 2(105b^5c^2 - 728ab^3c^3 + 1168a^2b^2c^4)x\right) \sqrt{c^2x^4 + b^2x^3 + a^2x^2} \right) / (c^6x) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b^2*c^3)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b^2*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b^6*c^6*x^5 + 315*b^6*c^6 - 2520*a^2*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b^2*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a^2*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b^2*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x) + 2*(5120*c^7*x^6 + 6400*b^6*c^6*x^5 + 315*b^6*c^6 - 2520*a^2*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b^2*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]

giac [A] time = 1.35, size = 429, normalized size = 1.18

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] $\frac{1}{35840} \sqrt{c x^2 + b x + a} (2 (4 (2 (8 (10 (4 c x \operatorname{sgn}(x) + 5 b \operatorname{sgn}(x)) x + (b^2 c^5 \operatorname{sgn}(x) + 64 a c^6 \operatorname{sgn}(x)) / c^6) x - (9 b^3 c^4 \operatorname{sgn}(x) - 44 a b c^5 \operatorname{sgn}(x)) / c^6) x + (21 b^4 c^3 \operatorname{sgn}(x) - 124 a b^2 c^4 \operatorname{sgn}(x) + 128 a^2 c^5 \operatorname{sgn}(x)) / c^6) x - (105 b^5 c^2 \operatorname{sgn}(x) - 728 a b^3 c^3 \operatorname{sgn}(x) + 1168 a^2 b c^4 \operatorname{sgn}(x)) / c^6) x + (315 b^6 c \operatorname{sgn}(x) - 2520 a b^4 c^2 \operatorname{sgn}(x) + 5488 a^2 b^2 c^3 \operatorname{sgn}(x) - 2048 a^3 c^4 \operatorname{sgn}(x)) / c^6) + \frac{3}{2048} (3 b^7 \operatorname{sgn}(x) - 28 a b^5 c \operatorname{sgn}(x) + 80 a^2 b^3 c^2 \operatorname{sgn}(x) - 64 a^3 b c^3 \operatorname{sgn}(x)) \log(\operatorname{abs}(-2(\sqrt{c} x - \sqrt{c x^2 + b x + a}) \sqrt{c} - b)) / c^{11/2} - \frac{1}{71680} (315 b^7 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) - 2940 a b^5 c \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) + 8400 a^2 b^3 c^2 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) - 6720 a^3 b c^3 \log(\operatorname{abs}(-b + 2 \sqrt{a} \sqrt{c})) + 630 \sqrt{a} b^6 \sqrt{c} - 5040 a^{3/2} b^4 c^{3/2} + 10976 a^{5/2} b^2 c^{5/2} - 4096 a^{7/2} c^{7/2}) \operatorname{sgn}(x) / c^{11/2}$

maple [A] time = 0.01, size = 479, normalized size = 1.32

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] $\frac{1}{71680} (c x^4 + b x^3 + a x^2)^{3/2} (10240 x^2 (c x^2 + b x + a)^{5/2} c^{11/2} - 680 c^{9/2} (c x^2 + b x + a)^{5/2} x b - 4096 c^{9/2} (c x^2 + b x + a)^{5/2} a + 5376 c^{7/2} (c x^2 + b x + a)^{5/2} b^2 + 4480 c^{9/2} (c x^2 + b x + a)^{3/2} x a b - 3360 c^{7/2} (c x^2 + b x + a)^{3/2} x b^3 + 2240 c^{7/2} (c x^2 + b x + a)^{3/2} a b^2 - 1680 c^{5/2} (c x^2 + b x + a)^{3/2} b^4 + 6720 c^{9/2} (c x^2 + b x + a)^{1/2} x a^2 b - 6720 c^{7/2} (c x^2 + b x + a)^{1/2} x a b^3 + 1260 c^{5/2} (c x^2 + b x + a)^{1/2} x b^5 + 3360 c^{7/2} (c x^2 + b x + a)^{1/2} a^2 b^2 - 3360 c^{5/2} (c x^2 + b x + a)^{1/2} a b^4 + 630 c^{3/2} (c x^2 + b x + a)^{1/2} b^6 + 6720 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2} c^{1/2}) / c^{1/2}) a^3 b c^4 - 8400 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2} c^{1/2}) / c^{1/2}) a^2 b^3 c^3 + 2940 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2} c^{1/2}) / c^{1/2}) a b^5 c^2 - 315 \ln(1/2 (2 c x + b + 2 (c x^2 + b x + a)^{1/2} c^{1/2}) / c^{1/2}) b^7 c) / x^3 (c x^2 + b x + a)^{3/2} / c^{13/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c x^4 + b x^3 + a x^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c x^4 + b x^3 + a x^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2),x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a x^2 + b x^3 + c x^4)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)

$$3.40 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x} dx$$

Optimal. Leaf size=288

$$\frac{b(1296a^2c^2 - 760ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} + \frac{(240a^2c^2 - 216ab^2c + 35b^4)\sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} + \frac{x(7b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}}{1024c^{9/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(3b + 10cx)(ax^2 + bx^3 + cx^4)^{3/2}}{60cx}$$

Rubi [A] time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1919, 1934, 1949, 12, 1914, 621, 206}

$$\frac{(240a^2c^2 - 216ab^2c + 35b^4)\sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(1296a^2c^2 - 760ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} - \frac{x(6cx(7b^2 - 20ac) + b(12ac + 7b^2))\sqrt{ax^2 + bx^3 + cx^4}}{960c^2} + \frac{x(7b^2 - 4ac)(b^2 - 4ac)^2\sqrt{a + bx + cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(3b + 10cx)(ax^2 + bx^3 + cx^4)^{3/2}}{60cx}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]

[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2])) && EqQ[n, 3] && EqQ[q, 1])
```

Rule 1919

```
Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1934

```
Int[((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_)*((A_) + (
B_)*(x_)^(r_)), x_Symbol] := Simp[(x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*
(2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
- q))^p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((
n - q)*p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), Int[x^q*(2*a
*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n - q
) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1))
*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b,
c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] &&
NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q + (
n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
```

ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} dx &= \frac{(3b + 10cx)(ax^2 + bx^3 + cx^4)^{3/2}}{60cx} + \frac{\int \left(-2ab + \frac{1}{2}(-7b^2 + 20ac)x\right) \sqrt{ax^2 + bx^3 + cx^4}}{20c} \\
 &= -\frac{x(b(7b^2 + 12ac) + 6c(7b^2 - 20ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{960c^2} + \frac{(3b + 10cx)(ax^2 + bx^3 + cx^4)^{3/2}}{60cx} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{x(b(7b^2 + 12ac) + 6c(7b^2 - 20ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{960c^2} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2)}{7680c^4x} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2)}{7680c^4x} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2)}{7680c^4x} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2)}{7680c^4x} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2)}{7680c^4x} \\
 &= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2)}{7680c^4x}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 180, normalized size = 0.62

$$\frac{(x^2(a + x(b + cx)))^{3/2} \left(\frac{(7b^2 - 4ac) \left(2\sqrt{c} \sqrt{a + x(b + cx)} (4c(5a + 2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + x(b + cx)}} \right) \right)}{512c^{7/2}(a + x(b + cx))^{3/2}} + x(a + x(b + cx)) - \frac{7b(a + x(b + cx))}{10c} \right)}{6cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x]

[Out] $((x^2*(a + x*(b + c*x)))^{3/2}*((-7*b*(a + x*(b + c*x)))/(10*c) + x*(a + x*(b + c*x)) + ((7*b^2 - 4*a*c)*(2*\sqrt{c}*(b + 2*c*x)*\sqrt{a + x*(b + c*x)}*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + x*(b + c*x)}])))/(512*c^{7/2}*(a + x*(b + c*x))^{3/2}))/6*c*x^3$

IntegrateAlgebraic [A] time = 3.42, size = 220, normalized size = 0.76

$$\frac{\sqrt{ax^2 + bx^3 + cx^4} (-1296a^2bc^2 + 480a^2c^3x + 760ab^3c - 432ab^2c^2x + 288abc^3x^2 + 2240ac^4x^3 - 105b^5 + 70b^4cx - 56b^3c^2x^2 + 48b^2c^3x^3 + 1664bc^4x^4 + 1280c^5x^5)}{7680c^4x} + \frac{(64a^3c^3 - 144a^2b^2c^2 + 60ab^4c - 7b^6) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a + x(b + cx^2)}}\right)}{512c^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]

[Out] $(\sqrt{ax^2 + bx^3 + cx^4}*(-105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2 + 70*b^4*c*x - 432*a*b^2*c^2*x + 480*a^2*c^3*x - 56*b^3*c^2*x^2 + 288*a*b*c^3*x^2 + 48*b^2*c^3*x^3 + 2240*a*c^4*x^3 + 1664*b*c^4*x^4 + 1280*c^5*x^5))/(7680*c^4*x) + ((-7*b^6 + 60*a*b^4*c - 144*a^2*b^2*c^2 + 64*a^3*c^3)*\text{ArcTanh}[(\sqrt{c}*x^2)/(\sqrt{a}*x - \sqrt{a*x^2 + b*x^3 + c*x^4})])/(512*c^{9/2})$

fricas [A] time = 1.17, size = 474, normalized size = 1.65

$$\frac{\sqrt{ax^2 + bx^3 + cx^4} (-1296a^2bc^2 + 480a^2c^3x + 760ab^3c - 432ab^2c^2x + 288abc^3x^2 + 2240ac^4x^3 - 105b^5 + 70b^4cx - 56b^3c^2x^2 + 48b^2c^3x^3 + 1664bc^4x^4 + 1280c^5x^5)}{7680c^4x} + \frac{(64a^3c^3 - 144a^2b^2c^2 + 60ab^4c - 7b^6) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a + x(b + cx^2)}}\right)}{512c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")

[Out] $[-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*\sqrt{c}*x*\log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*\sqrt{c*x^4 + b*x^3 + a*x^2})/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*\sqrt{-c}*x*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*\sqrt{c*x^4 + b*x^3 + a*x^2})/(c^5*x)]$

giac [A] time = 0.99, size = 365, normalized size = 1.27

$$\frac{\sqrt{ax^2 + bx^3 + cx^4} (-1296a^2bc^2 + 480a^2c^3x + 760ab^3c - 432ab^2c^2x + 288abc^3x^2 + 2240ac^4x^3 - 105b^5 + 70b^4cx - 56b^3c^2x^2 + 48b^2c^3x^3 + 1664bc^4x^4 + 1280c^5x^5)}{7680c^4x} + \frac{(64a^3c^3 - 144a^2b^2c^2 + 60ab^4c - 7b^6) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a + x(b + cx^2)}}\right)}{512c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
(3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b*c
^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*c^4
*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b*c^3
*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2*c^2*
sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
sqrt(c) - b))/c^(9/2) + 1/15360*(105*b^6*log(abs(-b + 2*sqrt(a)*sqrt(c))) -
900*a*b^4*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(abs(-b
+ 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210
*sqrt(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/2))*s
gn(x)/c^(9/2)
```

maple [A] time = 0.01, size = 431, normalized size = 1.50

$(c^2 + b^2 + a^2) \sqrt{c^2 + b^2 + a^2} \left(\frac{105 b^6 \sqrt{c^2 + b^2 + a^2}}{15360 (c^2 + b^2 + a^2)^{9/2}} + \frac{2160 a^2 b^2 c^2 \log(\sqrt{c^2 + b^2 + a^2} \sqrt{c})}{15360 (c^2 + b^2 + a^2)^{9/2}} - \frac{900 a b^4 c \log(\sqrt{c^2 + b^2 + a^2} \sqrt{c})}{15360 (c^2 + b^2 + a^2)^{9/2}} + \frac{210 \sqrt{a} b^5 \sqrt{c}}{15360 (c^2 + b^2 + a^2)^{9/2}} - \frac{1520 a^{3/2} b^3 c^{3/2}}{15360 (c^2 + b^2 + a^2)^{9/2}} + \frac{2592 a^{5/2} b c^{5/2}}{15360 (c^2 + b^2 + a^2)^{9/2}} \right) \operatorname{sgn}(x) / c^{9/2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 1/15360*(c*x^4+b*x^3+a*x^2)^(3/2)*(2560*x*(c*x^2+b*x+a)^(5/2)*c^(9/2)-1792*
c^(7/2)*(c*x^2+b*x+a)^(5/2)*b-640*c^(9/2)*(c*x^2+b*x+a)^(3/2)*x*a+1120*c^(7
/2)*(c*x^2+b*x+a)^(3/2)*x*b^2-320*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b+560*c^(5/
2)*(c*x^2+b*x+a)^(3/2)*b^3-960*c^(9/2)*(c*x^2+b*x+a)^(1/2)*x*a^2+1920*c^(7/
2)*(c*x^2+b*x+a)^(1/2)*x*a*b^2-420*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b^4-480*c^(
7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b+960*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^3-210*c
^(3/2)*(c*x^2+b*x+a)^(1/2)*b^5-960*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(
1/2))/c^(1/2))*a^3*c^4+2160*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))
/c^(1/2))*a^2*b^2*c^3-900*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(
1/2))*a*b^4*c^2+105*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2)
)*b^6*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)`

[Out] `Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)`

$$3.41 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx$$

Optimal. Leaf size=198

$$\frac{3bx(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)}{128c^3x}$$

Rubi [A] time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1917, 1918, 1914, 621, 206}

$$\frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{3bx(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]

[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_) + (a_.)*(x_)^(q_) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||

EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1917

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1)))^(p + 1)
)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^2} dx &= \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} - \frac{b \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^3} dx}{2c} \\
 &= -\frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} + \frac{(3b(b^2 - 4ac)) \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx}{32c^2} \\
 &= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} \\
 &= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} \\
 &= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} \\
 &= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5}
 \end{aligned}$$

Mathematica [A] time = 0.17, size = 163, normalized size = 0.82

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(4b^2c(2cx^2-25a)+8bc^2x(7a+22cx^2)+128c^2(a+cx^2)^2+15b^4-10b^3cx\right)-15b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{1280c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a + 22*c*x^2)) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 2.64, size = 175, normalized size = 0.88

$$\frac{3(16a^2bc^2 - 8ab^3c + b^5)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{ax-\sqrt{ax^2+bx^3+cx^4}}}\right)}{128c^{7/2}} + \frac{\sqrt{ax^2+bx^3+cx^4}(128a^2c^2-100ab^2c+56abc^2x+256ac^3x^2+15b^4-10b^3cx+8b^2c^2x^2+176bc^3x^3+128c^4x^4)}{640c^3x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]

[Out] (Sqrt[a*x^2 + b*x^3 + c*x^4]*(15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 10*b^3*c*x + 56*a*b*c^2*x + 8*b^2*c^2*x^2 + 256*a*c^3*x^2 + 176*b*c^3*x^3 + 128*c^4*x^4))/(640*c^3*x) + (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*c^(7/2))

fricas [A] time = 1.21, size = 384, normalized size = 1.94

$$\frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{c}\log\left(\frac{-x^2\sqrt{ax^2+bx^3+cx^4}-\sqrt{c}\sqrt{ax-\sqrt{ax^2+bx^3+cx^4}}}{2560c^3}\right) + 4(128c^2a^4 + 176bc^2a^3 + 15b^5c - 100ab^2c^2 + 128a^2c^3 + 8(b^2c^2 + 32ac^2)x^2 - 2(b^2c^2 - 28abc^2)x)\sqrt{c^2+bx^3+cx^4}}{2560c^3} + \frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}{2\sqrt{c}\sqrt{ax-\sqrt{ax^2+bx^3+cx^4}}}\right) + 2(128c^2a^4 + 176bc^2a^3 + 15b^5c - 100ab^2c^2 + 128a^2c^3 + 8(b^2c^2 + 32ac^2)x^2 - 2(b^2c^2 - 28abc^2)x)\sqrt{c^2+bx^3+cx^4}}{1280c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]

giac [A] time = 0.93, size = 284, normalized size = 1.43

$$\frac{1}{640}\sqrt{c^2+bx^3+cx^4}\left(c\left(\frac{1}{c}\left(15b^5c^2+11b^4c^2\log(c)+\frac{b^2c^2\log(c)+32ac^2\log(c)}{c^2}-\frac{5b^2c^2\log(c)-28abc^2\log(c)}{c^2}\right)\right)+\frac{15b^5\log(c)-100ab^2c^2\log(c)+128a^2c^3\log(c)}{c^2}\right)+\frac{3(b^5\log(c)-8ab^3\log(c)+16a^2bc^2\log(c))\log\left(\frac{-2(\sqrt{c}+\sqrt{c^2+bx^3+cx^4})\sqrt{c}-b}{256c^3}\right)}{256c^3}+\frac{(15b^5\log(-b+2\sqrt{c}\sqrt{c^2+bx^3+cx^4})-120ab^2c\log(-b+2\sqrt{c}\sqrt{c^2+bx^3+cx^4})+240b^2c^2\log(-b+2\sqrt{c}\sqrt{c^2+bx^3+cx^4})+30\sqrt{c}b^4\sqrt{c}-200b^2b^2\sqrt{c}+256a^2b^2\sqrt{c})\log(c)}{1280c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")

[Out] $\frac{1}{640}\sqrt{c^2x^2 + bx + a}(2*(4*(2*(8*c*x*\text{sgn}(x) + 11*b*\text{sgn}(x))*x + (b^2*c^3*\text{sgn}(x) + 32*a*c^4*\text{sgn}(x))/c^4)*x - (5*b^3*c^2*\text{sgn}(x) - 28*a*b*c^3*\text{sgn}(x)))/c^4)*x + (15*b^4*c*\text{sgn}(x) - 100*a*b^2*c^2*\text{sgn}(x) + 128*a^2*c^3*\text{sgn}(x))/c^4) + \frac{3}{256}(b^5*\text{sgn}(x) - 8*a*b^3*c*\text{sgn}(x) + 16*a^2*b*c^2*\text{sgn}(x))*\log(\text{abs}(-2*(\sqrt{c})x - \sqrt{c^2x^2 + bx + a})*\sqrt{c} - b))/c^{7/2} - \frac{1}{1280}(15*b^5*\log(\text{abs}(-b + 2*\sqrt{a}*\sqrt{c})) - 120*a*b^3*c*\log(\text{abs}(-b + 2*\sqrt{a}*\sqrt{c})) + 240*a^2*b*c^2*\log(\text{abs}(-b + 2*\sqrt{a}*\sqrt{c})) + 30*\sqrt{a}*b^4*\sqrt{c} - 200*a^{3/2}*b^2*c^{3/2} + 256*a^{5/2}*c^{5/2})*\text{sgn}(x)/c^{7/2}$

maple [A] time = 0.01, size = 289, normalized size = 1.46

$$\frac{(cx^4 + bx^3 + ax^2)^{3/2} \left(-240a^2b^2c^2 \ln\left(\frac{2a+b\sqrt{c^2+bx+a}}{2c}\right) + 120ab^2c^2 \ln\left(\frac{2a+b\sqrt{c^2+bx+a}}{2c}\right) - 15b^5c \ln\left(\frac{2a+b\sqrt{c^2+bx+a}}{2c}\right) - 240\sqrt{c^2+bx+a} ab^2c^2x + 60\sqrt{c^2+bx+a} b^3c^2x - 120\sqrt{c^2+bx+a} a^2c^3 + 30\sqrt{c^2+bx+a} b^4c^3 - 160(c^2+bx+a)^{3/2} b^2c^3 - 80(c^2+bx+a)^{3/2} b^2c^3 + 256(c^2+bx+a)^{3/2} c^3 \right)}{1280(c^2+bx+a)^{3/2} c^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x)

[Out] $\frac{1}{1280}(c^2x^2 + bx + a)^{5/2}c^{7/2} - 160(c^2x^2 + bx + a)^{3/2}b^2c^{7/2}x - 80(c^2x^2 + bx + a)^{3/2}b^2c^{5/2} - 240(c^2x^2 + bx + a)^{1/2}a^2b^2c^{7/2}x + 60(c^2x^2 + bx + a)^{1/2}b^3c^{5/2}x - 120(c^2x^2 + bx + a)^{1/2}a^2b^2c^{5/2} + 30(c^2x^2 + bx + a)^{1/2}b^4c^{3/2} - 240a^2b^2c^3 \ln(1/2*(2*c*x+b+2*(c^2x^2+bx+a)^{1/2})*c^{1/2})/c^{1/2} + 120a^2b^3c^2 \ln(1/2*(2*c*x+b+2*(c^2x^2+bx+a)^{1/2})*c^{1/2})/c^{1/2} - 15b^5c \ln(1/2*(2*c*x+b+2*(c^2x^2+bx+a)^{1/2})*c^{1/2})/c^{1/2}))/x^3/(c^2x^2+bx+a)^{3/2}/c^{9/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)`

[Out] `Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)`

$$3.42 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3} dx$$

Optimal. Leaf size=165

$$\frac{3x(b^2-4ac)^2 \sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1918, 1914, 621, 206}

$$-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{3x(b^2-4ac)^2 \sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]

[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||

EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1918

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, n - q]

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^3} dx &= \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} - \frac{(3(b^2 - 4ac)) \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx}{16c} \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{3(b^2 - 4ac)}{16c} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{3(b^2 - 4ac)}{16c} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{3(b^2 - 4ac)}{16c} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{3(b^2 - 4ac)}{16c} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx \end{aligned}$$

Mathematica [A] time = 0.06, size = 132, normalized size = 0.80

$$\frac{x\sqrt{a + x(b + cx)} \left(2\sqrt{c}(b + 2cx)\sqrt{a + x(b + cx)} (4c(5a + 2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right) \right)}{128c^{5/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)

$$\frac{1}{(2\sqrt{c}\sqrt{a+bx+cx^2})^{5/2}} \int \frac{1}{(128c^{5/2}\sqrt{x^2(a+bx+cx^2)})} dx$$

IntegrateAlgebraic [A] time = 1.58, size = 143, normalized size = 0.87

$$\frac{(20abc + 40ac^2x - 3b^3 + 2b^2cx + 24bc^2x^2 + 16c^3x^3) \sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} - \frac{3(16a^2c^2 - 8ab^2c + b^4) \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{64c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]

[Out] ((-3*b^3 + 20*a*b*c + 2*b^2*c*x + 40*a*c^2*x + 24*b*c^2*x^2 + 16*c^3*x^3)*sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) - (3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*ArcTanh[(sqrt[c]*x^2)/(sqrt[a]*x - sqrt[a*x^2 + b*x^3 + c*x^4])])/(64*c^(5/2))

fricas [A] time = 1.31, size = 320, normalized size = 1.94

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{c}x \log\left(\frac{8c^2x^4 + 4\sqrt{c^2x^4 + b^2x^3 + a^2x^2} + 4(16c^4x^3 + 24b^2c^2x^2 - 3b^3c + 20abc^2 + 2(b^2c^2 + 20ac^3))\sqrt{cx^4 + bx^3 + ax^2}}{256c^2x}\right) - 2(16c^4x^3 + 24b^2c^2x^2 - 3b^3c + 20abc^2 + 2(b^2c^2 + 20ac^3))\sqrt{cx^4 + bx^3 + ax^2}}{128c^2x} - \frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-c}x \arctan\left(\frac{\sqrt{c^2x^4 + b^2x^3 + a^2x^2}}{2(\sqrt{c^2x^4 + b^2x^3 + a^2x^2})}\right)}{128c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2)/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]

giac [A] time = 1.01, size = 232, normalized size = 1.41

$$\frac{1}{64} \sqrt{cx^2 + bx + a} \left(z \left(4(2c \operatorname{sgn}(x) + 3b \operatorname{sgn}(x))x + \frac{b^2c^2 \operatorname{sgn}(x) + 20abc^2 \operatorname{sgn}(x)}{c^3} \right) - \frac{3b^3 \operatorname{sgn}(x) - 20ab^2c \operatorname{sgn}(x)}{c^3} \right) \log\left(\frac{-z(\sqrt{cx^2 + bx + a})\sqrt{c} - 4}{128c^2} \right) - \frac{(3b^4 \log(|-b + 2\sqrt{a}\sqrt{c}|) - 24ab^2c \log(|-b + 2\sqrt{a}\sqrt{c}|) + 48a^2c^2 \log(|-b + 2\sqrt{a}\sqrt{c}|) + 6\sqrt{a}b^3\sqrt{c} - 40a^2b^3)\operatorname{sgn}(x)}{128c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")

[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*sgn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^3) - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/128*(3*b^4*log(a

$\text{bs}(-b + 2\sqrt{a}\sqrt{c})) - 24ab^2c \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 48a^2c^2 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 6\sqrt{a}b^3\sqrt{c} - 40a^{3/2}b^2c^{3/2}) \text{sgn}(x)/c^{5/2}$

maple [A] time = 0.00, size = 265, normalized size = 1.61

$$\frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(48a^2c^2 \ln\left(\frac{2ax + 2\sqrt{cx^2 + bx + a} \sqrt{c}}{2\sqrt{c}}\right) - 24ab^2c^2 \ln\left(\frac{2ax + 2\sqrt{cx^2 + bx + a} \sqrt{c}}{2\sqrt{c}}\right) + 3b^4c \ln\left(\frac{2ax + 2\sqrt{cx^2 + bx + a} \sqrt{c}}{2\sqrt{c}}\right) + 48\sqrt{cx^2 + bx + a} ac^{\frac{7}{2}}x - 12\sqrt{cx^2 + bx + a} b^2c^{\frac{5}{2}}x + 24\sqrt{cx^2 + bx + a} abc^{\frac{5}{2}} - 6\sqrt{cx^2 + bx + a} b^3c^{\frac{3}{2}} + 32(c^2x + bx + a)^{\frac{3}{2}}c^{\frac{7}{2}}x + 16(c^2x + bx + a)^{\frac{3}{2}}bc^{\frac{5}{2}} \right)}{128(c^2x + bx + a)^{\frac{3}{2}}c^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x)`

[Out] $\frac{1}{128}(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left((32(cx^2 + bx + a)^{\frac{3}{2}}c^{\frac{7}{2}}x + 16(cx^2 + bx + a)^{\frac{3}{2}}b^2c^{\frac{5}{2}} + 48(cx^2 + bx + a)^{\frac{1}{2}}ac^{\frac{7}{2}}x - 12(cx^2 + bx + a)^{\frac{1}{2}}b^2c^{\frac{5}{2}}x + 24(cx^2 + bx + a)^{\frac{1}{2}}ab^2c^{\frac{5}{2}} - 6(cx^2 + bx + a)^{\frac{1}{2}}b^3c^{\frac{3}{2}} + 48a^2c^3 \ln\left(\frac{1}{2}(2cx + b + 2\sqrt{cx^2 + bx + a})\sqrt{c}\right) - 24a^2b^2c^2 \ln\left(\frac{1}{2}(2cx + b + 2\sqrt{cx^2 + bx + a})\sqrt{c}\right) + 3b^4c \ln\left(\frac{1}{2}(2cx + b + 2\sqrt{cx^2 + bx + a})\sqrt{c}\right) \right) / x^3 / (cx^2 + bx + a)^{\frac{3}{2}} / c^{\frac{7}{2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**3/2/x**3, x)
```


$$3.43 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx$$

Optimal. Leaf size=227

$$\frac{a^{3/2}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{bx(b^2-12ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(8ac+b^2)}{3x^3}$$

Rubi [A] time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1921, 1945, 1933, 843, 621, 206, 724}

$$\frac{a^{3/2}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{bx(b^2-12ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]

[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1921

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^p, x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]

Rule 1933

Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]

Rule 1945

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^p, x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p + A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n - q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n - q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} dx &= \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{1}{2} \int \frac{(2a + bx)\sqrt{ax^2 + bx^3 + cx^4}}{x^2} dx \\
&= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{\int \frac{8a^2c - \frac{1}{2}b(b^2 - 12ac)x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c} \\
&= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{(x\sqrt{a + bx + cx^2})}{8c\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{(a^2x\sqrt{a + bx + cx^2})}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} - \frac{(2a^2x\sqrt{a + bx + cx^2})}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a + bx + cx^2}}{\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 166, normalized size = 0.73

$$\frac{x\sqrt{a+x(b+cx)}\left(-48a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\sqrt{a+x(b+cx)}(8c(4a+cx^2)+3b^2+14bcx)-3b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{48c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*b*c*x + 8*c*(4*a + c*x^2)) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 2.19, size = 168, normalized size = 0.74

$$a^{3/2}\log\left(2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4} - 2ax - bx^2\right) - 2a^{3/2}\log(x) + \frac{(b^3 - 12abc)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{8c^{3/2}} + \frac{(32ac + 3b^2 + 14bcx + 8c^2x^2)\sqrt{ax^2 + bx^3 + cx^4}}{24cx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x]

```
[Out] ((3*b^2 + 32*a*c + 14*b*c*x + 8*c^2*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c*x) + ((b^3 - 12*a*b*c)*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*c^(3/2)) - 2*a^(3/2)*Log[x] + a^(3/2)*Log[-2*a*x - b*x^2 + 2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]]
```

fricas [A] time = 1.42, size = 791, normalized size = 3.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, replacing 0 by `u`, a substitution variable should perhaps be pur
ged.Warning, replacing 0 by `u`, a substitution variable should perhaps be
purged.Warning, replacing 0 by `u`, a substitution variable should perhaps
s be purged.index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 222, normalized size = 0.98

$$\frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(48a^{\frac{3}{2}} c^{\frac{5}{2}} \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - 36ab c^2 \ln\left(\frac{2cx+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) + 3b^3 c \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) - 12\sqrt{cx^2+bx+a} b c^{\frac{5}{2}} x - 48\sqrt{cx^2+bx+a} a c^{\frac{5}{2}} - 6\sqrt{cx^2+bx+a} b^2 c^{\frac{3}{2}} - 16(cx^2+bx+a)^{\frac{3}{2}} c^{\frac{5}{2}} \right)}{48(cx^2+bx+a)^{\frac{3}{2}} c^{\frac{5}{2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x)

[Out] $-1/48*(cx^4+bx^3+ax^2)^{(3/2)}*(48*c^{(5/2)}*a^{(3/2)}*\ln((b*x+2*a+2*(cx^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)-16*(cx^2+b*x+a)^{(3/2)}*c^{(5/2)}-12*(cx^2+b*x+a)^{(1/2)}*b*c^{(5/2)}*x-48*c^{(5/2)}*(cx^2+b*x+a)^{(1/2)}*a-6*(cx^2+b*x+a)^{(1/2)}*b^2*c^{(3/2)}-36*a*b*c^2*\ln(1/2*(2*c*x+b+2*(cx^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)})+3*b^3*c*\ln(1/2*(2*c*x+b+2*(cx^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)}))/x^3/(cx^2+b*x+a)^{(3/2)}/c^{(5/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)

[Out] Integral((x**2*(a + b*x + c*x**2))**3/2/x**4, x)

$$3.44 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5} dx$$

Optimal. Leaf size=219

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} + \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - 3\sqrt{a}}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.24, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1920, 1945, 1933, 843, 621, 206, 724}

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} + \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]

[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p + A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n - q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n - q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{3}{2} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^2} dx \\
&= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{3 \int \frac{4abc + c(b^2 + 4ac)x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c} \\
&= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{\left(3x\sqrt{a + bx + cx^2}\right) \int \frac{4abc + c(b^2 + 4ac)}{x\sqrt{a + bx + cx^2}} dx}{8c\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{\left(3abx\sqrt{a + bx + cx^2}\right) \int \frac{4abc + c(b^2 + 4ac)}{x\sqrt{a + bx + cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} - \frac{\left(3abx\sqrt{a + bx + cx^2}\right) \operatorname{Subst}\left(\int \frac{4abc + c(b^2 + 4ac)}{x\sqrt{a + bx + cx^2}} dx, x, \sqrt{ax^2 + bx^3 + cx^4}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} - \frac{3\sqrt{a} bx \sqrt{a + bx + cx^2} \operatorname{tanh}^{-1}\left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + bx + cx^2}}\right)}{2\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 158, normalized size = 0.72

$$\frac{\sqrt{a + x(b + cx)} \left(3x(4ac + b^2) \operatorname{tanh}^{-1}\left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + x(b + cx)}}\right) + 2\sqrt{c} \sqrt{a + x(b + cx)} (x(5b + 2cx) - 4a) - 12\sqrt{a} b \sqrt{c} x \operatorname{tanh}^{-1}\left(\frac{2a + bx}{2\sqrt{a} \sqrt{a + x(b + cx)}}\right) \right)}{8\sqrt{c} \sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]

[Out] (Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2*c*x)) - 12*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]) + 3*(b^2 + 4*a*c)*x*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 2.07, size = 160, normalized size = 0.73

$$-\frac{3(4ac + b^2) \operatorname{tanh}^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{ax - \sqrt{ax^2 + bx^3 + cx^4}}}\right)}{4\sqrt{c}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-4a + 5bx + 2cx^2)}{4x^2} + \frac{3}{2}\sqrt{a}b \log\left(2\sqrt{a} \sqrt{ax^2 + bx^3 + cx^4} - 2ax - bx^2\right) - 3\sqrt{a}b \log(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]


```
[Out] ((-4*a + 5*b*x + 2*c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (3*(b^2 + 4*a*c)*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(4*Sqrt[c]) - 3*Sqrt[a]*b*Log[x] + (3*Sqrt[a]*b*Log[-2*a*x - b*x^2 + 2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]])/2
```

fricas [A] time = 1.32, size = 757, normalized size = 3.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c)/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c)/(c*x^2), 1/16*(24*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c)/(c*x^2), 1/8*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c)/(c*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%{%%{[1,0]:[1,0,%%{-1,[1]
%%}%], [4,0,0,0]%%}%+%%{%%{[-2,0]:[1,0,%%{-1,[1]%%}%], [2,0,1,0]%%}%+
%%{%%{[1,0]:[1,0,%%{-1,[1]%%}%], [0,0,2,0]%%}% / %%{%%{1,[1]%%}, [4,0
```

,0,0]%%}+%%{%%{-2, [1]%%}, [2,0,1,0]%%}+%%{%%{1, [1]%%}, [0,0,2,0]%%}
 Error: Bad Argument Value

maple [A] time = 0.01, size = 254, normalized size = 1.16

$$\frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(12a^2c^2x \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) - 12a^{\frac{3}{2}}bc^{\frac{3}{2}}x \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{c}}{x}\right) + 3ab^2cx \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) + 12\sqrt{cx^2+bx+a}ac^{\frac{3}{2}}x^2 + 18\sqrt{cx^2+bx+a}abc^{\frac{3}{2}}x + 8(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{3}{2}}x^2 + 8(cx^2+bx+a)^{\frac{3}{2}}bc^{\frac{3}{2}}x - 8(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{3}{2}} \right)}{8(cx^2+bx+a)^{\frac{3}{2}}ac^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x)

[Out] $\frac{1}{8} * (c*x^4 + b*x^3 + a*x^2)^{\frac{3}{2}} * (8*c^{\frac{5}{2}} * (c*x^2 + b*x + a)^{\frac{3}{2}} * x^2 + 12*c^{\frac{5}{2}} * (c*x^2 + b*x + a)^{\frac{1}{2}} * x^2 * a - 12*c^{\frac{3}{2}} * a^{\frac{3}{2}} * \ln((b*x + 2*a + 2*(c*x^2 + b*x + a)^{\frac{1}{2}} * a^{\frac{1}{2}}) / x) * x * b - 8*(c*x^2 + b*x + a)^{\frac{5}{2}} * c^{\frac{3}{2}} + 8*c^{\frac{3}{2}} * (c*x^2 + b*x + a)^{\frac{3}{2}} * x * b + 18*c^{\frac{3}{2}} * (c*x^2 + b*x + a)^{\frac{1}{2}} * x * a * b + 12 * \ln(1/2 * (2*c*x + b + 2*(c*x^2 + b*x + a)^{\frac{1}{2}} * c^{\frac{1}{2}}) / c^{\frac{1}{2}}) * x * a^2 * c^2 + 3*c * \ln(1/2 * (2*c*x + b + 2*(c*x^2 + b*x + a)^{\frac{1}{2}} * c^{\frac{1}{2}}) / c^{\frac{1}{2}}) * x * a * b^2) / x^4 / (c*x^2 + b*x + a)^{\frac{3}{2}} / a / c^{\frac{3}{2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

$$3.45 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6} dx$$

Optimal. Leaf size=219

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} - \frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.24, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1920, 1941, 1933, 843, 621, 206, 724}

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]

[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Simp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m + p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} + \frac{3}{4} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
&= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} - \frac{3}{8} \int \frac{-b^2 - 4ac - 4bcx}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} - \frac{(3x\sqrt{a + bx + cx^2}) \int \frac{-b^2 - 4ac - 4bcx}{x\sqrt{a + bx + cx^2}} dx}{8\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} + \frac{(3bcx\sqrt{a + bx + cx^2}) \int \frac{-b^2 - 4ac - 4bcx}{\sqrt{a + bx + cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} + \frac{(3bcx\sqrt{a + bx + cx^2}) \operatorname{Subst}\left(\int \frac{-b^2 - 4ac - 4bcx}{\sqrt{ax^2 + bx^3 + cx^4}} dx, x, \sqrt{a + bx + cx^2}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} - \frac{3(b^2 + 4ac)x\sqrt{a + bx + cx^2}}{8\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 162, normalized size = 0.74

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(3x^2(4ac + b^2) \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right) + 2\sqrt{a} \left((2a + x(5b - 4cx))\sqrt{a + x(b + cx)} - 6b\sqrt{c}x^2 \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right) \right) \right)}{8\sqrt{a}x^3\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x]

[Out] -1/8*(Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a + x*(b + c*x)] - 6*b*Sqrt[c]*x^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 2.05, size = 167, normalized size = 0.76

$$\frac{3(4ac + b^2) \log\left(2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4} - 2ax - bx^2\right)}{8\sqrt{a}} - \frac{3 \log(x)(4ac + b^2)}{4\sqrt{a}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-2a - 5bx + 4cx^2)}{4x^3} - 3b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x]

```
[Out] ((-2*a - 5*b*x + 4*c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^3) - 3*b*Sqrt[c]
]*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])] - (3*(b^
2 + 4*a*c)*Log[x])/(4*Sqrt[a]) + (3*(b^2 + 4*a*c)*Log[-2*a*x - b*x^2 + 2*Sq
rt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(8*Sqrt[a])
```

fricas [A] time = 1.59, size = 757, normalized size = 3.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt
(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c
*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*sq
rt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))
- 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
- 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 +
b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b*sqrt(c)*
x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sq
rt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))
+ 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1
/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan
(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 +
a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x
^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 338, normalized size = 1.54

$$\frac{(c^2 + b^2 + a^2)^2 \left(12a^2 c^2 x^2 \ln\left(\frac{(2a^2 + 2\sqrt{a^2 + b^2 + c^2})x + a^2}{4}\right) - 12a^2 b^2 c^2 x \ln\left(\frac{(2a^2 + 2\sqrt{a^2 + b^2 + c^2})x + a^2}{4}\right) + 3a^2 b^2 c^2 \ln\left(\frac{(2a^2 + 2\sqrt{a^2 + b^2 + c^2})x + a^2}{4}\right) - 6\sqrt{c^2 + b^2 + a^2} ab^2 c^2 x^3 - 12\sqrt{c^2 + b^2 + a^2} a^2 c^2 x^2 - 6\sqrt{c^2 + b^2 + a^2} a^2 b^2 c^2 x - 2(c^2 + b^2 + a^2)^2 bc^2 x^3 - 4(c^2 + b^2 + a^2)^2 a^2 c^2 x^2 - 2(c^2 + b^2 + a^2)^2 b^2 c^2 x + 4(c^2 + b^2 + a^2)^2 a^2 c^2 \right)}{8(c^2 + b^2 + a^2)^2 a^2 c^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x)`

[Out]
$$-1/8*(c*x^4+b*x^3+a*x^2)^{(3/2)}*(12*c^{(5/2)}*a^{(5/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*x^2-2*c^{(5/2)}*(c*x^2+b*x+a)^{(3/2)}*x^3*b-4*c^{(5/2)}*(c*x^2+b*x+a)^{(3/2)}*x^2*a-6*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*x^3*a*b+3*c^{(3/2)}*a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*x^2*b^2-12*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*x^2*a^2+2*c^{(3/2)}*(c*x^2+b*x+a)^{(5/2)}*x*b-2*c^{(3/2)}*(c*x^2+b*x+a)^{(3/2)}*x^2*b^2+4*(c*x^2+b*x+a)^{(5/2)}*a*c^{(3/2)}-6*c^{(3/2)}*(c*x^2+b*x+a)^{(1/2)}*x^2*a*b^2-12*c^2*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)})*x^2*a^2*b)/x^5/(c*x^2+b*x+a)^{(3/2)}/a^2/c^{(3/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x)`

[Out] `int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)`

[Out] `Integral((x**2*(a + b*x + c*x**2))** (3/2)/x**6, x)`

$$3.46 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7} dx$$

Optimal. Leaf size=257

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}}{\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.35, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1920, 1951, 1941, 1933, 843, 621, 206, 724}

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x]

[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(4*a*x^5) + (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 1920

Int[(x_)^(m_)*((b_.)*(x_)^(n_) + (a_.)*(x_)^(q_) + (c_.)*(x_)^(r_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1933

Int[((A_) + (B_.)*(x_)^(j_))/Sqrt[(b_.)*(x_)^(n_) + (a_.)*(x_)^(q_) + (c_.)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]

Rule 1941

Int[(x_)^(m_)*((c_.)*(x_)^(j_) + (b_.)*(x_)^(n_) + (a_.)*(x_)^(q_))^(p_)*(A_) + (B_.)*(x_)^(r_), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Simp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m + p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 1951

Int[(x_)^(m_)*((c_.)*(x_)^(j_) + (b_.)*(x_)^(n_) + (a_.)*(x_)^(q_))^(p_)*(A_) + (B_.)*(x_)^(r_), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b

$x^n + c x^{(2n - q)} \cdot (p + 1) / (a \cdot (m + p \cdot q + 1))$, $x] + \text{Dist}[1 / (a \cdot (m + p \cdot q + 1))$, $\text{Int}[x^{(m + n - q)} \cdot \text{Simp}[a \cdot B \cdot (m + p \cdot q + 1) - A \cdot b \cdot (m + p \cdot q + (n - q) \cdot (p + 1) + 1) - A \cdot c \cdot (m + p \cdot q + 2 \cdot (n - q) \cdot (p + 1) + 1) \cdot x^{(n - q)}$, $x] \cdot (a \cdot x^q + b \cdot x^n + c \cdot x^{(2n - q)})^p$, $x]$, $x] /;$ $\text{FreeQ}[\{a, b, c, A, B\}, x]$ && $\text{EqQ}[r, n - q]$ && $\text{EqQ}[j, 2 \cdot n - q]$ && $\text{IntegerQ}[p]$ && $\text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$ && $\text{IGtQ}[n, 0]$ && $\text{RationalQ}[m, p, q]$ && $((\text{GeQ}[p, -1]$ && $\text{LtQ}[p, 0]) \mid \mid \text{EqQ}[m + p \cdot q + (n - q) \cdot (2 \cdot p + 1) + 1, 0])$ && $\text{LeQ}[m + p \cdot q, -(n - q)]$ && $\text{NeQ}[m + p \cdot q + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} + \frac{1}{2} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx \\ &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} - \int \frac{\left(\frac{1}{2}(b^2 - 8ac) - bcx\right)\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\ &= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)}{4ax^5} \\ &= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)}{4ax^5} \\ &= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)}{4ax^5} \\ &= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)}{4ax^5} \\ &= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)}{4ax^5} \end{aligned}$$

Mathematica [A] time = 0.29, size = 175, normalized size = 0.68

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(3bx^3(b^2 - 12ac) \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right) - 2\sqrt{a} \left(\sqrt{a + x(b + cx)} (8a^2 + 2ax(7b + 16cx) + 3b^2x^2) - 24ac^{3/2}x^3 \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right) \right) \right)}{48a^{3/2}x^4\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x]

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(2*a + b*x)/(2*
*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^2
+ 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) - 24*a*c^(3/2)*x^3*ArcTanh[(b + 2*c*x)/(
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]))])))/(48*a^(3/2)*x^4*Sqrt[a + x*(b + c*x)]
)
```

IntegrateAlgebraic [A] time = 2.20, size = 187, normalized size = 0.73

$$\frac{\log(x)(b^3 - 12abc)}{8a^{3/2}} + \frac{(-8a^2 - 14abx - 32acx^2 - 3b^2x^2)\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(12abc - b^3)\log\left(\frac{-2a^{3/2}\sqrt{ax^2 + bx^3 + cx^4} + 2a^2x + abx^2}{16a^{3/2}}\right)}{16a^{3/2}} - 2c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((-8*a^2 - 14*a*b*x - 3*b^2*x^2 - 32*a*c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(24*a*x^4) - 2*c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^
3 + c*x^4])] + ((b^3 - 12*a*b*c)*Log[x])/(8*a^(3/2)) + ((-b^3 + 12*a*b*c)*L
og[2*a^2*x + a*b*x^2 - 2*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(16*a^(3/2))
```

fricas [A] time = 1.55, size = 815, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*s
qrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14
*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2*sqrt
(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*s
qrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b
*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2))*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/4
8*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^
2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(x)]
 Warning, choosing root of [1,0,%%{-2,[1,0,0,2]%%}+%%{-2,[0,1,0,1]%%}+%%
 %{-4,[0,0,1,0]%%}],0,%%{1,[2,0,0,4]%%}+%%{2,[1,1,0,3]%%}+%%{1,[0,2,0,2
]%%}] at parameters values [-97,-82,63.4443001123,-27]Warning, choosing ro
 ot of [1,0,%%{-2,[1,0,0,2]%%}+%%{-2,[0,1,0,1]%%}+%%{-4,[0,0,1,0]%%}],0
 ,%%{1,[2,0,0,4]%%}+%%{2,[1,1,0,3]%%}+%%{1,[0,2,0,2]%%}] at parameters
 values [63,-49,35.2935628123,-64]Warning, choosing root of [1,0,%%{-2,[2,
 1,0,0]%%}+%%{-2,[1,0,1,0]%%}+%%{-4,[0,0,0,1]%%}],0,%%{1,[4,2,0,0]%%}+
 %%{2,[3,1,1,0]%%}+%%{1,[2,0,2,0]%%}] at parameters values [22,42,56,43.
 9628838282]Sign error (%%{b-2*sqrt(a)*sqrt(c),0%%}+%%{-(-2*a*c+b*sqrt(a)
 *sqrt(c))/a,1%%}+%%{-4*a*c*sqrt(a)*sqrt(c)-b^2*sqrt(a)*sqrt(c)/(4*a^2),
 2%%}+%%{undef,3%%})Evaluation time: 0.45Limit: Max order reached or unab
 le to make series expansion Error: Bad Argument Value

maple [A] time = 0.01, size = 435, normalized size = 1.69

(c^3*x^12 + 3*c^2*b*x^11 + 3*c*b^2*x^10 + b^3*x^9 + 3*c^2*a*x^11 + 6*c*b*a*x^10 + 3*b^2*a*x^9 + a^3*x^8 + 3*c^2*b*x^10 + 6*c*b^2*x^9 + 3*b^3*x^8 + 3*c^2*a*x^10 + 6*c*b*a*x^9 + 3*b^2*a*x^8 + a^3*x^7 + 3*c^2*b*x^9 + 6*c*b^2*x^8 + 3*b^3*x^7 + 3*c^2*a*x^9 + 6*c*b*a*x^8 + 3*b^2*a*x^7 + a^3*x^6 + 3*c^2*b*x^8 + 6*c*b^2*x^7 + 3*b^3*x^6 + 3*c^2*a*x^8 + 6*c*b*a*x^7 + 3*b^2*a*x^6 + a^3*x^5 + 3*c^2*b*x^7 + 6*c*b^2*x^6 + 3*b^3*x^5 + 3*c^2*a*x^7 + 6*c*b*a*x^6 + 3*b^2*a*x^5 + a^3*x^4 + 3*c^2*b*x^6 + 6*c*b^2*x^5 + 3*b^3*x^4 + 3*c^2*a*x^6 + 6*c*b*a*x^5 + 3*b^2*a*x^4 + a^3*x^3 + 3*c^2*b*x^5 + 6*c*b^2*x^4 + 3*b^3*x^3 + 3*c^2*a*x^5 + 6*c*b*a*x^4 + 3*b^2*a*x^3 + a^3*x^2 + 3*c^2*b*x^4 + 6*c*b^2*x^3 + 3*b^3*x^2 + 3*c^2*a*x^4 + 6*c*b*a*x^3 + 3*b^2*a*x^2 + a^3*x + 3*c^2*b*x^3 + 6*c*b^2*x^2 + 3*b^3*x + 3*c^2*a*x^3 + 6*c*b*a*x^2 + 3*b^2*a*x + a^3)^(3/2)/x^7

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x)

[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*c^(7/2)*(c*x^2+b*x+a)^(3/2)*x^4*a-36*c^(
 5/2)*a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^3*b+48*c^(7/2)
 *(c*x^2+b*x+a)^(1/2)*x^4*a^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^4*b^2-32*c^(5/
 2)*(c*x^2+b*x+a)^(5/2)*x^2*a+28*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^3*a*b-6*c^(5/
 2)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^2+3*c^(3/2)*a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x
 +a)^(1/2)*a^(1/2))/x)*x^3*b^3+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b+2*c^(
 3/2)*(c*x^2+b*x+a)^(5/2)*x^2*b^2-2*c^(3/2)*(c*x^2+b*x+a)^(3/2)*x^3*b^3+4*c
 ^3/2*(c*x^2+b*x+a)^(5/2)*x*a*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x^3*a*b^3-16
 *(c*x^2+b*x+a)^(5/2)*a^2*c^(3/2)+48*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c
 ^1/2))/c^(1/2))*x^3*a^3*c^3)/x^6/(c*x^2+b*x+a)^(3/2)/a^3/c^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)

[Out] Integral((x**2*(a + b*x + c*x**2))**3/2/x**7, x)

$$3.47 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8} dx$$

Optimal. Leaf size=197

$$\frac{3(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}} + \frac{b(3b^2 - 20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b^2 - 12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

Rubi [A] time = 0.36, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$\frac{b(3b^2 - 20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{3(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}} - \frac{(b^2 - 12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]

[Out] -((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a*x^3) + (b*(3*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(4*x^7) - (3*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(5/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_)*(x_)^2 + (b_)*(x_)^(n_) + (c_)*(x_)^(r_)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1920

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
+ 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]

```

Rule 1941

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
- q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7} + \frac{3}{8} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^5} dx \\
&= -\frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7} + \frac{1}{16} \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7} \\
&= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} \\
&= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} \\
&= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} \\
&= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 141, normalized size = 0.72

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(2\sqrt{a} (2a + bx)\sqrt{a + x(b + cx)} (8a^2 + 4ax(2b + 5cx) - 3b^2x^2) + 3x^4 (b^2 - 4ac)^2 \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) \right)}{128a^{5/2}x^5\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x]

[Out] -1/128*(Sqrt[x^2*(a + x*(b + c*x))]*(2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x)) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))/(a^(5/2)*x^5*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 2.11, size = 175, normalized size = 0.89

$$\frac{3(16a^2c^2 - 8ab^2c + b^4) \log\left(2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4} - 2ax - bx^2\right)}{128a^{5/2}} - \frac{3 \log(x)(16a^2c^2 - 8ab^2c + b^4)}{64a^{5/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-16a^3 - 24a^2bx - 40a^2cx^2 - 2ab^2x^2 - 20abcx^3 + 3b^3x^3)}{64a^2x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]

[Out]
$$\frac{((-16a^3 - 24a^2bx - 2ab^2x^2 - 40a^2cx^2 + 3b^3x^3 - 20ab^2cx^3) \sqrt{ax^2 + bx^3 + cx^4}) / (64a^2x^5) - (3(b^4 - 8ab^2c + 16a^2c^2) \operatorname{Log}[x]) / (64a^{(5/2)}) + (3(b^4 - 8ab^2c + 16a^2c^2) \operatorname{Log}[-2ax - bx^2 + 2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}]) / (128a^{(5/2)})}$$

fricas [A] time = 1.63, size = 332, normalized size = 1.69

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{a}x^5 \log\left(\frac{8ab^2 + (b^2+4a)^2 + 8a^2c - 4\sqrt{c^2+2b^2+2c^2}(b+2a)\sqrt{c}}{a^3} - 4(24a^3bx + 16a^4 - (3ab^3 - 20a^2bc)x^2 + 2(a^2b^2 + 20a^2c^2)\sqrt{cx^2 + bx^3 + ax^2})\sqrt{-a}x^5 \arctan\left(\frac{\sqrt{cx^2 + bx^3 + ax^2}}{2(ax^2 + abx + a^2)}\right) - 2(24a^3bx + 16a^4 - (3ab^3 - 20a^2bc)x^2 + 2(a^2b^2 + 20a^2c^2)\sqrt{cx^2 + bx^3 + ax^2})\sqrt{-a}x^5}{256a^3x^5} - \frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-a}x^5 \arctan\left(\frac{\sqrt{cx^2 + bx^3 + ax^2}}{2(ax^2 + abx + a^2)}\right) - 2(24a^3bx + 16a^4 - (3ab^3 - 20a^2bc)x^2 + 2(a^2b^2 + 20a^2c^2)\sqrt{cx^2 + bx^3 + ax^2})\sqrt{-a}x^5}{128a^3x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{a}*x^5*\log(-(8*a*b*x^2 + (b^2 + 4*a*c))*x^3 + 8*a^2*x - 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{a})/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*\sqrt{c*x^4 + b*x^3 + a*x^2})/(a^3*x^5), 1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{-a}*x^5*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*\sqrt{c*x^4 + b*x^3 + a*x^2})/(a^3*x^5)] \end{aligned}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 501, normalized size = 2.54

$$\frac{(-1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^2*a^(7/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(3/2)*x^5*a*b-24*c*a^(5/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*b^2-16*c^2*(c*x^2+b*x+a)^(3/2)*x^4*a^2+24*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a^2*b-2*c*(c*x^2+b*x+a)^(3/2)*x^5*b^3-48*c^2*(c*x^2+b*x+a)^(1/2)*x^4*a^3-24*c*(c*x^2+b*x+a)^(5/2)*x^3*a*b+20*c^2*(c*x^2+b*x+a)^(3/2)*x^5*a^2*c^2)/x^8 + (3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*\operatorname{Log}[x]) / (64*a^{(5/2)}) + (3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*\operatorname{Log}[-2*a*x - b*x^2 + 2*\sqrt{a}*\sqrt{a*x^2 + b*x^3 + c*x^4}]) / (128*a^{(5/2)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x)

[Out]
$$\begin{aligned} & -1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^2*a^(7/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(3/2)*x^5*a*b-24*c*a^(5/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^4*b^2-16*c^2*(c*x^2+b*x+a)^(3/2)*x^4*a^2+24*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a^2*b-2*c*(c*x^2+b*x+a)^(3/2)*x^5*b^3-48*c^2*(c*x^2+b*x+a)^(1/2)*x^4*a^3-24*c*(c*x^2+b*x+a)^(5/2)*x^3*a*b+20 \end{aligned}$$

$*c*(c*x^2+b*x+a)^{(3/2)}*x^4*a*b^2-6*c*(c*x^2+b*x+a)^{(1/2)}*x^5*a*b^3+3*a^{(3/2)}$
 $)*ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*x^4*b^4+16*c*(c*x^2+b*x+a)^{(5/2)}$
 $*x^2*a^2+36*c*(c*x^2+b*x+a)^{(1/2)}*x^4*a^2*b^2+2*(c*x^2+b*x+a)^{(5/2)}*x^3*b^3-2*(c*x^2+b*x+a)^{(3/2)}$
 $*x^4*b^4+4*(c*x^2+b*x+a)^{(5/2)}*x^2*a*b^2-6*(c*x^2+b*x+a)^{(1/2)}*x^4*a*b^4-16*(c*x^2+b*x+a)^{(5/2)}$
 $*x*a^2*b+32*(c*x^2+b*x+a)^{(5/2)}*a^3)/x^7/(c*x^2+b*x+a)^{(3/2)}/a^4$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)

[Out] Integral((x**2*(a + b*x + c*x**2))** (3/2)/x**8, x)

$$3.48 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9} dx$$

Optimal. Leaf size=249

$$\frac{3b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}} + \frac{b(5b^2 - 28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}$$

Rubi [A] time = 0.50, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$\frac{(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2} + \frac{b(5b^2 - 28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} + \frac{3b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}} - \frac{(b^2 - 8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]

[Out] -((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(80*a*x^4) + (b*(5*b^2 - 28*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^4 - 100*a*b^2*c + 12*8*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*a^3*x^2) - (3*(b + 4*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*x^5) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(256*a^(7/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1920

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
+ 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]

```

Rule 1941

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
- q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} + \frac{3}{10} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^6} dx \\
&= -\frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} + \frac{3}{160} \int \frac{2(b^2 - 8ac) - 4bcx}{x^3\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} - \frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} \\
&= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} \\
&= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2 + 120b^2c - 15c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1280a^2x^6} \\
&= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2 + 120b^2c - 15c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1280a^2x^6} \\
&= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2 + 120b^2c - 15c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1280a^2x^6} \\
&= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2 + 120b^2c - 15c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1280a^2x^6}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 177, normalized size = 0.71

$$\frac{\sqrt{x^2(a+x(b+cx))} \left(15bx^5(b^2-4ac)^2 \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}\sqrt{a+x(b+cx)}(128a^4+16a^3x(11b+16cx)+8a^2x^2(b^2+7bcx+16c^2x^2)-10ab^2x^3(b+10cx)+15b^4x^4) \right)}{1280a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^4 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*a^2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))]/(1280*a^(7/2)*x^6*Sqrt[a + x*(b + c*x)])

IntegrateAlgebraic [A] time = 2.67, size = 211, normalized size = 0.85

$$\frac{3(16a^2bc^2 - 8ab^3c + b^5) \log\left(2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4} - 2ax - bx^2\right)}{256a^{7/2}} + \frac{3 \log(x)(16a^2bc^2 - 8ab^3c + b^5)}{128a^{7/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-128a^4 - 176a^2bx - 256a^2cx^2 - 8a^2b^2x^2 - 56a^2bcx^3 - 128a^2c^2x^4 + 10ab^2x^3 + 100ab^2cx^4 - 15b^4x^4)}{640a^3x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]

[Out] (Sqrt[a*x^2 + b*x^3 + c*x^4]*(-128*a^4 - 176*a^3*b*x - 8*a^2*b^2*x^2 - 256*a^3*c*x^2 + 10*a*b^3*x^3 - 56*a^2*b*c*x^3 - 15*b^4*x^4 + 100*a*b^2*c*x^4 - 128*a^2*c^2*x^4))/(640*a^3*x^6) + (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[x])/((128*a^(7/2))) - (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[-2*a*x - b*x^2 + 2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(256*a^(7/2))

fricas [A] time = 1.77, size = 394, normalized size = 1.58

$$\frac{15(b^5 - 8ab^3c + 16a^2b^2c^2)\sqrt{a}\log\left(\frac{3ab^2 + 4a^2b^3 + 4a^3c^2\sqrt{a}\sqrt{a^2b^2 + 4a^3c}}{256a^3}\right) - 4(176a^4b + 128a^5 + (15ab^4 - 100a^2b^2c + 128a^3c^2)x^4 - 2(5a^2b^3 - 28a^3b^2c)x^3 + 8(a^3b^2 + 32a^4c)x^2)\sqrt{a}\sqrt{a^2b^2 + 4a^3c}}{640a^3} - \frac{15(b^5 - 8ab^3c + 16a^2b^2c^2)\sqrt{a}\arctan\left(\frac{2(5a^2b^3 - 28a^3b^2c)x^3 + 8(a^3b^2 + 32a^4c)x^2}{21a^2b^2c^2}\right) + 2(176a^4b + 128a^5 + (15ab^4 - 100a^2b^2c + 128a^3c^2)x^4 - 2(5a^2b^3 - 28a^3b^2c)x^3 + 8(a^3b^2 + 32a^4c)x^2)\sqrt{a}\sqrt{a^2b^2 + 4a^3c}}{1280a^3}}{1280a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b^2*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b^2*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.01, size = 534, normalized size = 2.14

$$\frac{1}{1280} \left((c^2 a^7)^{1/2} \ln\left(\frac{(b x^2 + 2 a x + a^2) \sqrt{c x^2 + b x + a}}{x}\right) x^5 b + 120 c^2 (c x^2 + b x + a)^{3/2} x^6 a b^2 - 120 c a^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x)

[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(240*c^2*a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^5*b+120*c^2*(c*x^2+b*x+a)^(3/2)*x^6*a*b^2-120*c*a^(5

$$\begin{aligned} & /2) * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x) * x^5*b^3-80*c^2*(c*x^2+b*x \\ & +a)^{(3/2)}*x^5*a^2*b+120*c^2*(c*x^2+b*x+a)^{(1/2)}*x^6*a^2*b^2-10*c*(c*x^2+b*x \\ & +a)^{(3/2)}*x^6*b^4-240*c^2*(c*x^2+b*x+a)^{(1/2)}*x^5*a^3*b-120*c*(c*x^2+b*x+a) \\ & ^{(5/2)}*x^4*a*b^2+100*c*(c*x^2+b*x+a)^{(3/2)}*x^5*a*b^3-30*c*(c*x^2+b*x+a)^{(1/ \\ & 2)}*x^6*a*b^4+15*a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x) * x^5*b \\ & ^5+80*c*(c*x^2+b*x+a)^{(5/2)}*x^3*a^2*b+180*c*(c*x^2+b*x+a)^{(1/2)}*x^5*a^2*b^3 \\ & +10*(c*x^2+b*x+a)^{(5/2)}*x^4*b^4-10*(c*x^2+b*x+a)^{(3/2)}*x^5*b^5+20*(c*x^2+b* \\ & x+a)^{(5/2)}*x^3*a*b^3-30*(c*x^2+b*x+a)^{(1/2)}*x^5*a*b^5-80*(c*x^2+b*x+a)^{(5/2)} \\ &) * x^2*a^2*b^2+160*(c*x^2+b*x+a)^{(5/2)}*x*a^3*b-256*(c*x^2+b*x+a)^{(5/2)}*a^4) / \\ & x^8/(c*x^2+b*x+a)^{(3/2)}/a^5 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^4 + bx^3 + ax^2)^{3/2}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x)

[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)

[Out] Integral((x**2*(a + b*x + c*x**2))** (3/2)/x**9, x)

$$3.49 \quad \int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=143

$$\frac{x(3b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c}$$

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1928, 1949, 12, 1914, 621, 206}

$$\frac{x(3b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[

```
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1928

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^p_.), x_Symbol] := Simp[(x^(m - 2*n + q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + 2*(n - q)*p + 1)), x] - Dist[1/(c*(m + p*q + 2*(n - q)*p + 1)), Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q + (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^p_.), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] - Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{\int \frac{x\left(a + \frac{3bx}{2}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2c} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\int \frac{(3b^2 - 4ac)x}{4\sqrt{ax^2 + bx^3 + cx^4}} dx}{2c^2} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{(3b^2 - 4ac) \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c^2} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\left((3b^2 - 4ac)x\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{8c^2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\left((3b^2 - 4ac)x\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx\right)}{4c^2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{(3b^2 - 4ac)x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2cx - 3b}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.73

$$\frac{x \left((3b^2 - 4ac) \sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}} \right) + 2\sqrt{c} (2cx - 3b)(a + x(b + cx)) \right)}{8c^{5/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (3*b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.37, size = 122, normalized size = 0.85

$$\frac{\log(x)(3b^2 - 4ac)}{8c^{5/2}} + \frac{(4ac - 3b^2) \log\left(-2c^{5/2}\sqrt{ax^2 + bx^3 + cx^4} + bc^2x + 2c^3x^2\right)}{8c^{5/2}} + \frac{(2cx - 3b)\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] $((-3*b + 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4*c^2*x) + ((3*b^2 - 4*a*c)*\text{Log}[x])/(8*c^{(5/2)}) + ((-3*b^2 + 4*a*c)*\text{Log}[b*c^2*x + 2*c^3*x^2 - 2*c^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]])/(8*c^{(5/2)})$

fricas [A] time = 1.02, size = 226, normalized size = 1.58

$$\left[\frac{(3b^2 - 4ac)\sqrt{c}x \log\left(\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c} + (b^2 + 4ac)x}{x}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}(2c^2x - 3bc)}{16c^3x}, \frac{(3b^2 - 4ac)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{-c}}{2(c^2x^3 + bcx^2 + acx)}\right) - 2\sqrt{cx^4 + bx^3 + ax^2}(2c^2x - 3bc)}{8c^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] $[-1/16*((3*b^2 - 4*a*c)*\text{sqrt}(c)*x*\log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*\text{sqrt}(c) + (b^2 + 4*a*c)*x)/x) - 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*\text{sqrt}(-c)*x*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*\text{sqrt}(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)

maple [A] time = 0.01, size = 144, normalized size = 1.01

$$\frac{\sqrt{cx^2 + bx + a} \left(-4ac^2 \ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a}\sqrt{c}}{2\sqrt{c}}\right) + 3b^2c \ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a}\sqrt{c}}{2\sqrt{c}}\right) + 4\sqrt{cx^2 + bx + a}c^{\frac{5}{2}}x - 6\sqrt{cx^2 + bx + a}bc^{\frac{3}{2}} \right)}{8\sqrt{cx^4 + bx^3 + ax^2}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] $1/8*x*(c*x^2+b*x+a)^{(1/2)}*(4*(c*x^2+b*x+a)^{(1/2)}*c^{(5/2)}*x-6*(c*x^2+b*x+a)^{(1/2)}*b*c^{(3/2)}-4*a*c^2*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)}))+3*b^2*c*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)})/c^{(1/2)}))/c*(c*x^4+b*x^3+a*x^2)^{(1/2)}/c^{(7/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)

[Out] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)

$$3.50 \quad \int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=103

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1917, 1914, 621, 206}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_) + (a_.)*(x_)^(q_) + (c_.)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1917

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1))/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && EqQ[m + p*(n - 1) - 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{b \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2c} \\ &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{(bx\sqrt{a + bx + cx^2}) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{2c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{(bx\sqrt{a + bx + cx^2}) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{2c^{3/2}\sqrt{ax^2 + bx^3 + cx^4}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.86

$$\frac{x\left(2\sqrt{c}(a + x(b + cx)) - b\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)\right)}{2c^{3/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.24, size = 91, normalized size = 0.88

$$\frac{b \log\left(-2c^{3/2}\sqrt{ax^2 + bx^3 + cx^4} + bcx + 2c^2x^2\right)}{2c^{3/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{b \log(x)}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*Log[x])/(2*c^(3/2)) + (b*Log[b*c*x + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(2*c^(3/2))

fricas [A] time = 1.13, size = 188, normalized size = 1.83

$$\left[\frac{b\sqrt{c}x \log\left(-\frac{8c^2x^3+8bcx^2-4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right) + 4\sqrt{cx^4+bx^3+ax^2}c}{4c^2x}, \frac{b\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right) + 2\sqrt{cx^4+bx^3+ax^2}c}{2c^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c^2*x)]

giac [A] time = 0.91, size = 108, normalized size = 1.05

$$\frac{b \arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c} + \frac{b\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-2\sqrt{a}c}{\left(\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)^2-c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="giac")

[Out] b*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c) + (b*(sqrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/(((sqrt(c + b/x + a/x^2) - sqrt(a)/x)^2 - c)*c)

maple [A] time = 0.01, size = 88, normalized size = 0.85

$$\frac{\sqrt{cx^2+bx+a} \left(-bc \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) + 2\sqrt{cx^2+bx+a}c^{\frac{3}{2}}\right)x}{2\sqrt{cx^4+bx^3+ax^2}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)`

[Out] $\frac{1}{2}x(c^2x^2+bx+a)^{1/2}(2(c^2x^2+bx+a)^{1/2}c^{3/2}-b\ln(1/2(2cx+b+2(c^2x^2+bx+a)^{1/2}c^{1/2}))/c^{1/2}))/c^{5/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)`

[Out] `int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)`

[Out] `Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)`

$$3.51 \quad \int \frac{x}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=71

$$\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1914, 621, 206}

$$\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]))/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= \frac{\left(x\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\left(2x\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.93

$$\frac{x\sqrt{a + bx + cx^2} \log\left(2\sqrt{c}\sqrt{a + bx + cx^2} + b + 2cx\right)}{\sqrt{c}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.06, size = 54, normalized size = 0.76

$$\frac{\log(x)}{\sqrt{c}} - \frac{\log\left(-2\sqrt{c}\sqrt{ax^2 + bx^3 + cx^4} + bx + 2cx^2\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] Log[x]/Sqrt[c] - Log[b*x + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4]]/Sqrt[c]

fricas [A] time = 1.06, size = 129, normalized size = 1.82

$$\left[\frac{\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)}{2\sqrt{c}}, -\frac{\sqrt{-c} \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)}{c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]

giac [A] time = 0.91, size = 37, normalized size = 0.52

$$-\frac{2 \arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] -2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/sqrt(-c)

maple [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{\sqrt{cx^2 + bx + a} x \ln\left(\frac{2cx+b+2\sqrt{cx^2+bx+a} \sqrt{c}}{2\sqrt{c}}\right)}{\sqrt{cx^4 + bx^3 + ax^2} \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] 1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/c^(1/2))/c^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)`

[Out] `int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2), x)`

[Out] `Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)`

$$3.52 \quad \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x(2a + bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.56

$$\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))

IntegrateAlgebraic [A] time = 0.19, size = 49, normalized size = 1.09

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}x^2 - \sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (2*ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^2 - Sqrt[a*x^2 + b*x^3 + c*x^4])])/Sqrt[a]

fricas [A] time = 1.27, size = 130, normalized size = 2.89

$$\left[\frac{\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{-a}}{2(acx^3+abx^2+a^2x)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]

giac [A] time = 0.92, size = 59, normalized size = 1.31

$$-\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a} \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] $-2*\arctan(\sqrt{a}/\sqrt{-a})*\operatorname{sgn}(x)/\sqrt{-a} + 2*\arctan(-(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})/\sqrt{-a})/(\sqrt{-a})*\operatorname{sgn}(x)$

maple [A] time = 0.00, size = 66, normalized size = 1.47

$$-\frac{\sqrt{cx^2 + bx + a} x \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a} \sqrt{a}}{x}\right)}{\sqrt{cx^4 + bx^3 + ax^2} \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] $-1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)

[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

$$3.53 \quad \int \frac{1}{x \sqrt{ax^2 + bx^3 + cx^4}} dx$$

Optimal. Leaf size=77

$$\frac{b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}} - \frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1927, 1904, 206}

$$\frac{b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}} - \frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1927

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := -Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q)))^(p + 1)/(2*a*(n - q)*(p + 1)), x] - Dist[b/(2*a), Int[x^(m + n - q)*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p + 1)]

]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{ax^2 + bx^3 + cx^4}} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2} - \frac{b \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2} + \frac{b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{2a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 1.16

$$\frac{bx\sqrt{a+x(b+cx)} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}(a+x(b+cx))}{2a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]), x]

[Out] $(-2*\operatorname{Sqrt}[a]*(a + x*(b + c*x)) + b*x*\operatorname{Sqrt}[a + x*(b + c*x)]*\operatorname{ArcTanh}[(2*a + b*x)/(2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[a + x*(b + c*x)])])/(2*a^{(3/2)}*\operatorname{Sqrt}[x^2*(a + x*(b + c*x))])$

IntegrateAlgebraic [A] time = 0.29, size = 79, normalized size = 1.03

$$-\frac{b \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}x^2 - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{a^{3/2}} - \frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]), x]

[Out] $-(\operatorname{Sqrt}[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) - (b*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*x)/(\operatorname{Sqrt}[c]*x^2 - \operatorname{Sqrt}[a*x^2 + b*x^3 + c*x^4])])/a^{(3/2)}$

fricas [A] time = 1.28, size = 194, normalized size = 2.52

$$\left[\frac{\sqrt{a}bx^2 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}a}{4a^2x^2}, -\frac{\sqrt{-a}bx^2 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{-a}}{2(acx^3 + abx^2 + a^2x)}\right) + 2\sqrt{cx^4 + bx^3 + ax^2}a}{2a^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a^2*x^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-4,[1,0,0,0]%%}+%%{-2,[0,1,0,1]%%}+%%{-2,[0,0,1,2]%%},0,%%{1,[0,2,0,2]%%}+%%{2,[0,1,1,3]%%}+%%{1,[0,0,2,4]%%}] at parameters values [54.7579903365,-49,-33,-70]-1/a*sqrt(a*(1/x)^2+b/x+c)-2*b/4/a/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x+c)-sqrt(a)/x)-b))

maple [A] time = 0.01, size = 88, normalized size = 1.14

$$\frac{\sqrt{cx^2 + bx + a} \left(-abx \ln \left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a} \sqrt{a}}{x} \right) + 2\sqrt{cx^2 + bx + a} a^{\frac{3}{2}} \right)}{2\sqrt{cx^4 + bx^3 + ax^2} a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] -1/2*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*a^(3/2)-b*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*a*x)/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sqrt{c x^4 + b x^3 + a x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)

[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{x^2 (a + b x + c x^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2), x)

[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)

$$3.54 \quad \int \frac{1}{x^2 \sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=119

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Rubi [A] time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1929, 1951, 12, 1904, 206}

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(5/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1929

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q)))^(p + 1)

)/(a*(m + p*q + 1)), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]

Rule 1951

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_) * ((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{\int \frac{-\frac{3b}{2} - cx}{x \sqrt{ax^2 + bx^3 + cx^4}} dx}{2a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} - \frac{\int \frac{-\frac{3b^2}{4} + ac}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a^2} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} + \frac{(3b^2 - 4ac) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8a^2} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} - \frac{(3b^2 - 4ac) \text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x}{\sqrt{a}}\right)}{4a^2} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} - \frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a} \sqrt{ax^2 + bx^3 + cx^4}}\right)}{8a^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 0.94

$$\frac{-\left(x^2(3b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right)\right) - 2\sqrt{a}(2a - 3bx)(a + x(b + cx))}{8a^{5/2}x\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] (-2*Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x)) - (3*b^2 - 4*a*c)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(8*a^(5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 0.44, size = 100, normalized size = 0.84

$$\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}x^2 - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{4a^{5/2}} + \frac{(3bx - 2a)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] ((-2*a + 3*b*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*x^3) + (((3*b^2 - 4*a*c)*ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^2 - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(4*a^(5/2)))

fricas [A] time = 1.29, size = 232, normalized size = 1.95

$$\left[\frac{(3b^2 - 4ac)\sqrt{a}x^3 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}(3abx - 2a^2)}{16a^3x^3}, \frac{(3b^2 - 4ac)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{-a}}{2(ax^3 + abx^2 + a^2x)}\right) + 2\sqrt{cx^4 + bx^3 + ax^2}(3abx - 2a^2)}{8a^3x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%{-4,[1,0,0,0]%%}+%%{-2,[0,1,0,1]%%}+%%{-2,

[0,0,1,2]%%},0,%%{1,[0,2,0,2]%%}+%%{2,[0,1,1,3]%%}+%%{1,[0,0,2,4]%%}
] at parameters values [54.7579903365,-49,-33,-70]2*(-4*a/16/a^2/x+6*b/16/a
 ^2)*sqrt(a*(1/x)^2+b/x+c)+2*(-4*a*c+3*b^2)/16/a^2/sqrt(a)*ln(abs(2*sqrt(a)*
 (sqrt(a*(1/x)^2+b/x+c)-sqrt(a)/x)-b))

maple [A] time = 0.01, size = 152, normalized size = 1.28

$$\frac{\sqrt{cx^2 + bx + a} \left(-4a^2cx^2 \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) + 3ab^2x^2 \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - 6\sqrt{cx^2 + bx + a} a^{\frac{3}{2}}bx + 4\sqrt{cx^2 + bx + a} a^{\frac{5}{2}} \right)}{8\sqrt{cx^4 + bx^3 + ax^2} a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] -1/8*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*a^(5/2)-6*(c*x^2+b*x+a)^(1/2)*a^(3/2)*x*b-4*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^2*a^2+3*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*x^2*a*b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(7/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)

[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{x^2 (a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

$$3.55 \quad \int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=262

$$\frac{3x(5b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) - b(15b^2 - 52ac) \sqrt{ax^2 + bx^3 + cx^4}}{8c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{4c^3x(b^2 - 4ac)} + \frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)}$$

Rubi [A] time = 0.51, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac) \sqrt{ax^2 + bx^3 + cx^4}}{4c^3x(b^2 - 4ac)} + \frac{3x(5b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2bx\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/((2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

```

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2])) && EqQ[n, 3] && EqQ[q, 1])

```

Rule 1923

```

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
-1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]

```

Rule 1949

```

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
)*(A_) + (B_)*(x_)^(r_), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{x^3(6a+3bx)}{\sqrt{ax^2+bx^3+cx^4}} dx}{b^2 - 4ac} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2bx\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{2 \int \frac{x^2(6ab+\frac{3}{2}(5b^2-12ac)x)}{\sqrt{ax^2+bx^3+cx^4}} dx}{3c(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{2bx\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)}{4c^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 183, normalized size = 0.70

$$\frac{x \left(2\sqrt{c} (4a^2c(6cx - 13b) + a(15b^3 - 62b^2cx - 20bc^2x^2 + 8c^3x^3)) + b^2x(15b^2 + 5bcx - 2c^2x^2) \right) - 3(16a^2c^2 - 24ab^2c + 5b^4)\sqrt{a+x(b+cx)} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{8c^{7/2}(4ac - b^2)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (x*(2*sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x^2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)]))

rt[a + x*(b + c*x)])))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 2.80, size = 192, normalized size = 0.73

$$\frac{(-52a^2bc + 24a^2c^2x + 15ab^3 - 62ab^2cx - 20abc^2x^2 + 8ac^3x^3 + 15b^4x + 5b^3cx^2 - 2b^2c^2x^3)\sqrt{ax^2 + bx^3 + cx^4}}{4c^3x(4ac - b^2)(a + bx + cx^2)} - \frac{3(5b^2 - 4ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{4c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] ((15*a*b^3 - 52*a^2*b*c + 15*b^4*x - 62*a*b^2*c*x + 24*a^2*c^2*x + 5*b^3*c*x^2 - 20*a*b*c^2*x^2 - 2*b^2*c^2*x^3 + 8*a*c^3*x^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(-b^2 + 4*a*c)*x*(a + b*x + c*x^2)) - (3*(5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/(4*c^(7/2))

fricas [A] time = 1.55, size = 616, normalized size = 2.35

$$\frac{3(15a^2b^3c^2 + 16a^2b^2c^3 + (15a^2b^3c^2 - 24a^2b^2c^3 + 16a^2c^4)x^3 + (5b^5 - 24a^2b^3c + 16a^2b^2c^2)x^2 + (5a^2b^4 - 24a^2b^2c^2 + 16a^3c^2)x)\sqrt{c}\log(-8c^2x^3 + 8b^2cx^2 - 4\sqrt{c}(cx^4 + bx^3 + ax^2)(2cx + b)\sqrt{c} + (b^2 + 4ac)x)/x + 4(15a^2b^3c - 52a^2b^2c^2 - 2(b^2c^3 - 4a^2c^4)x^3 + 5(b^3c^2 - 4a^2b^2c^3)x^2 + (15b^4c - 62a^2b^2c^2 + 24a^2c^3)x)\sqrt{c^2x^4 + bx^3 + ax^2}}{4(15a^2b^3c^2 + 16a^2b^2c^3 + (15a^2b^3c^2 - 24a^2b^2c^3 + 16a^2c^4)x^3 + (5b^5 - 24a^2b^3c + 16a^2b^2c^2)x^2 + (5a^2b^4 - 24a^2b^2c^2 + 16a^3c^2)x)\sqrt{c}\operatorname{arctan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)} + \frac{3(15a^2b^3c^2 + 16a^2b^2c^3 + (15a^2b^3c^2 - 24a^2b^2c^3 + 16a^2c^4)x^3 + (5b^5 - 24a^2b^3c + 16a^2b^2c^2)x^2 + (5a^2b^4 - 24a^2b^2c^2 + 16a^3c^2)x)\sqrt{c}\log(-8c^2x^3 + 8b^2cx^2 - 4\sqrt{c}(cx^4 + bx^3 + ax^2)(2cx + b)\sqrt{c} + (b^2 + 4ac)x)/x + 4(15a^2b^3c - 52a^2b^2c^2 - 2(b^2c^3 - 4a^2c^4)x^3 + 5(b^3c^2 - 4a^2b^2c^3)x^2 + (15b^4c - 62a^2b^2c^2 + 24a^2c^3)x)\sqrt{c^2x^4 + bx^3 + ax^2}}{4(15a^2b^3c^2 + 16a^2b^2c^3 + (15a^2b^3c^2 - 24a^2b^2c^3 + 16a^2c^4)x^3 + (5b^5 - 24a^2b^3c + 16a^2b^2c^2)x^2 + (5a^2b^4 - 24a^2b^2c^2 + 16a^3c^2)x)\sqrt{c}\operatorname{arctan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="fricas")

[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*log(-8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2)]/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2)]/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

maple [A] time = 0.01, size = 283, normalized size = 1.08

$$\frac{(c^2 + bx + a) \left(16a^2 c^3 x^3 - 4b^2 c^2 x^3 - 40ab c^2 x^2 + 10b^2 c^2 x^2 + 48a^2 c^2 x - 124a b^2 c^2 x + 30b^4 c^2 x - 48\sqrt{c^2 + bx + a} a^2 c^3 \ln\left(\frac{2cx+b+2\sqrt{c^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) + 72\sqrt{c^2 + bx + a} a b^2 c^2 \ln\left(\frac{2cx+b+2\sqrt{c^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) - 15\sqrt{c^2 + bx + a} b^4 c \ln\left(\frac{2cx+b+2\sqrt{c^2+bx+a}\sqrt{c}}{2\sqrt{c}}\right) - 104a^2 b c^2 + 30a b^3 c^2 \right) x^3}{8(c^2 + bx + a)^2 (4ac - b^2) c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] $\frac{1}{8}x^3(c^2+bx+a)/c^{9/2} * (16c^{9/2}x^3a - 4c^{7/2}x^3b^2 - 40c^{7/2}x^2ab + 10c^{5/2}x^2b^3 + 48c^{7/2}xa^2 - 124c^{5/2}xab^2 + 30c^{3/2}x^2b^4 - 104c^{5/2}a^2b + 30c^{3/2}ab^3 - 48\ln(1/2 * (2cx+b+2\sqrt{c^2+bx+a})^{1/2} * c^{1/2})) / c^{1/2} * (c^2+bx+a)^{1/2} * a^2c^3 + 72\ln(1/2 * (2cx+b+2\sqrt{c^2+bx+a})^{1/2} * c^{1/2})) / c^{1/2} * (c^2+bx+a)^{1/2} * a^2c^3 + 72\ln(1/2 * (2cx+b+2\sqrt{c^2+bx+a})^{1/2} * c^{1/2})) / c^{1/2} * (c^2+bx+a)^{1/2} * ab^2c^2 - 15\ln(1/2 * (2cx+b+2\sqrt{c^2+bx+a})^{1/2} * c^{1/2})) / c^{1/2} * (c^2+bx+a)^{1/2} * b^4c) / (c^2+bx+a)^{3/2} / (4ac - b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{(cx^4 + bx^3 + ax^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)

[Out] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```


$$3.56 \quad \int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=201

$$\frac{(3b^2 - 8ac) \sqrt{ax^2 + bx^3 + cx^4}}{c^2x(b^2 - 4ac)} + \frac{2x^3(2a + bx)}{(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} - \frac{3bx\sqrt{a + bx + cx^2} \tanh^{-1}}{2c^{5/2}\sqrt{ax^2 + bx^3}}$$

Rubi [A] time = 0.30, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{(3b^2 - 8ac) \sqrt{ax^2 + bx^3 + cx^4}}{c^2x(b^2 - 4ac)} + \frac{2x^3(2a + bx)}{(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} - \frac{3bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2])) && EqQ[n, 3] && EqQ[q, 1])
```

Rule 1923

```
Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
-1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
)*(A_) + (B_)*(x_)^(r_), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p, x), x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{x^2(4a+2bx)}{\sqrt{ax^2+bx^3+cx^4}} dx}{b^2 - 4ac} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{\int \frac{x(2ab+(3b^2-8ac)x)}{\sqrt{ax^2+bx^3+cx^4}} dx}{c(b^2 - 4ac)} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)} x \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)} x \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)} x \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)} x
\end{aligned}$$

Mathematica [A] time = 0.15, size = 141, normalized size = 0.70

$$\frac{x \left(2\sqrt{c} (8a^2c + a(-3b^2 + 10bcx + 4c^2x^2)) - b^2x(3b + cx) \right) + 3b(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right)}{2c^{5/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*x^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 2.16, size = 145, normalized size = 0.72

$$\frac{\sqrt{ax^2 + bx^3 + cx^4} (8a^2c - 3ab^2 + 10abcx + 4ac^2x^2 - 3b^3x - b^2cx^2)}{c^2x(4ac - b^2)(a + bx + cx^2)} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}x - \sqrt{ax^2 + bx^3 + cx^4}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] ((-3*a*b^2 + 8*a^2*c - 3*b^3*x + 10*a*b*c*x - b^2*c*x^2 + 4*a*c^2*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*(-b^2 + 4*a*c)*x*(a + b*x + c*x^2)) + (3*b*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/c^(5/2)

fricas [A] time = 1.36, size = 486, normalized size = 2.42

$$\frac{3((b^3c - 4abc^2)x^3 + (b^4 - 4a^2bc^2)x^2 + (ab^3 - 4a^2b^2c)x) \sqrt{c} \log\left(\frac{b^3c^2 - 4abc^2 + \sqrt{4a^2c^2 - 4ab^2c + 4a^2c^2}}{2\sqrt{c^2 + bx^3 + ax^2}}\right) + 4\sqrt{c^2 + bx^3 + ax^2}(3ab^2c - 8a^2c^2 + (b^2c - 4ac^2)x^2 + (3b^3c - 10abc^2)x) + 3((b^3c - 4abc^2)x^3 + (b^4 - 4a^2bc^2)x^2 + (ab^3 - 4a^2b^2c)x) \sqrt{-c} \arctan\left(\frac{\sqrt{c^2 + bx^3 + ax^2}}{2\sqrt{c^2 + bx^3 + ax^2}}\right) + 2\sqrt{c^2 + bx^3 + ax^2}(3ab^2c - 8a^2c^2 + (b^2c - 4ac^2)x^2 + (3b^3c - 10abc^2)x)}{4((b^3c - 4abc^2)x^3 + (b^4 - 4a^2bc^2)x^2 + (ab^3 - 4a^2b^2c)x) \sqrt{c} + 2((b^3c - 4abc^2)x^3 + (b^4 - 4a^2bc^2)x^2 + (ab^3 - 4a^2b^2c)x) \sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a^2*b*c^2)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x)/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a^2*b*c^2)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x)/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^4)*x)]

giac [A] time = 0.89, size = 195, normalized size = 0.97

$$\frac{2\left(\frac{b^3c^2 - 3abc^3}{b^2c^4 - 4ac^5} + \frac{ab^2c^2 - 2a^2c^3}{(b^2c^4 - 4ac^5)x}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}} + \frac{3b \arctan\left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2} + \frac{b\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right) - 2\sqrt{a}c}{\left(\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right)^2 - c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="giac")

[Out] $2*((b^3*c^2 - 3*a*b*c^3)/(b^2*c^4 - 4*a*c^5) + (a*b^2*c^2 - 2*a^2*c^3)/((b^2*c^4 - 4*a*c^5)*x))/\sqrt{c + b/x + a/x^2} + 3*b*\arctan((\sqrt{c + b/x + a/x^2} - \sqrt{a}/x)/\sqrt{-c})/(\sqrt{-c}*c^2) + (b*(\sqrt{c + b/x + a/x^2} - \sqrt{a}/x) - 2*\sqrt{a}*c)/(((\sqrt{c + b/x + a/x^2} - \sqrt{a}/x)^2 - c)*c^2)$

maple [A] time = 0.01, size = 199, normalized size = 0.99

$$\frac{(cx^2 + bx + a) \left(8a^7 c^2 x^2 - 2b^2 c^5 x^2 + 20ab c^5 x - 6b^3 c^3 x - 12\sqrt{cx^2 + bx + a} ab c^2 \ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a} \sqrt{c}}{2\sqrt{c}}\right) + 3\sqrt{cx^2 + bx + a} b^3 c \ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a} \sqrt{c}}{2\sqrt{c}}\right) + 16a^2 c^5 - 6ab^2 c^3 \right) x^3}{2(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} (4ac - b^2) c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6/(c*x^4+b*x^3+a*x^2)^{(3/2)}, x)$

[Out] $1/2*x^3*(c*x^2+b*x+a)/c^{(7/2)}*(8*c^{(7/2)}*x^2*a-2*c^{(5/2)}*x^2*b^2+20*c^{(5/2)}*x*a*b-6*c^{(3/2)}*x*b^3+16*c^{(5/2)}*a^2-6*c^{(3/2)}*a*b^2-12*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}))/c^{(1/2)})*(c*x^2+b*x+a)^{(1/2)}*a*b*c^2+3*\ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}))/c^{(1/2)})*(c*x^2+b*x+a)^{(1/2)}*b^3*c)/(c*x^4+b*x^3+a*x^2)^{(3/2)}/(4*a*c-b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^6/(c*x^4+b*x^3+a*x^2)^{(3/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(x^6/(c*x^4 + b*x^3 + a*x^2)^{(3/2)}, x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(cx^4 + bx^3 + ax^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^6/(a*x^2 + b*x^3 + c*x^4)^{(3/2)}, x)$

[Out] $\text{int}(x^6/(a*x^2 + b*x^3 + c*x^4)^{(3/2)}, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

$$3.57 \quad \int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=153

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx(b^2-4ac)} + \frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx(b^2-4ac)} + \frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2])) && EqQ[n, 3] && EqQ[q, 1])
```

Rule 1923

```
Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
-1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
)*(A_) + (B_)*(x_)^(r_), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p, x), x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}} dx}{b^2 - 4ac} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{2 \int \frac{(b^2-4ac)x}{2\sqrt{ax^2+bx^3+cx^4}} dx}{c(b^2 - 4ac)} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}} dx}{c} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{(x\sqrt{a + bx + cx^2}) \int \frac{1}{\sqrt{a+bx}}}{c\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{(2x\sqrt{a + bx + cx^2}) \text{Subst}}{c\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{1}{2}\right)}{c^{3/2}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 112, normalized size = 0.73

$$\frac{x \left(2\sqrt{c} (-ab + 2acx + b^2(-x)) + (b^2 - 4ac) \sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c} \sqrt{a+x(b+cx)}} \right) \right)}{c^{3/2} (4ac - b^2) \sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] -((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)])*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 1.83, size = 116, normalized size = 0.76

$$\frac{2(ab - 2acx + b^2x)\sqrt{ax^2 + bx^3 + cx^4}}{cx(4ac - b^2)(a + bx + cx^2)} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{ax - \sqrt{ax^2 + bx^3 + cx^4}}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (2*(a*b + b^2*x - 2*a*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(-b^2 + 4*a*c)*x*(a + b*x + c*x^2)) - (2*ArcTanh[(Sqrt[c]*x^2)/(Sqrt[a]*x - Sqrt[a*x^2 + b*x^3 + c*x^4])])/c^(3/2)

fricas [A] time = 1.18, size = 414, normalized size = 2.71

$$\frac{\left((b^2c - 4ac^2)x^3 + (b^2 - 4abc)x^2 + (ab^2 - 4a^2c)x \right) \sqrt{c} \log\left(\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{c^2bx^2 + a^2}\sqrt{c} + (b^2 + 4ac)x}{2((b^2c - 4ac^2)x^3 + (b^2 - 4abc)x^2 + (ab^2 - 4a^2c)x)} \right) - 4\sqrt{cx^2 + bx + a} (abc + (b^2c - 2ac^2)x) - ((b^2c - 4ac^2)x^3 + (b^2 - 4abc)x^2 + (ab^2 - 4a^2c)x) \sqrt{-c} \arctan\left(\frac{\sqrt{cx^2 + bx + a} \sqrt{-c}}{2(b^2 - 4abc)x} \right) + 2\sqrt{cx^2 + bx + a} (abc + (b^2c - 2ac^2)x)}{(b^2c - 4ac^2)x^3 + (b^2 - 4abc)x^2 + (ab^2 - 4a^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]

giac [A] time = 0.98, size = 110, normalized size = 0.72

$$\frac{2 \left(\frac{abc}{(b^2c^2 - 4ac^3)x} + \frac{b^2c - 2ac^2}{b^2c^2 - 4ac^3} \right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}} - \frac{2 \arctan\left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}} \right)}{\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] -2*(a*b*c/((b^2*c^2 - 4*a*c^3)*x) + (b^2*c - 2*a*c^2)/(b^2*c^2 - 4*a*c^3))/sqrt(c + b/x + a/x^2) - 2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c)

maple [A] time = 0.01, size = 166, normalized size = 1.08

$$\frac{(cx^2 + bx + a) \left(-4ac^{\frac{5}{2}}x + 2b^2c^{\frac{3}{2}}x + 4\sqrt{cx^2 + bx + a}ac^2 \ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a}\sqrt{c}}{2\sqrt{c}} \right) - \sqrt{cx^2 + bx + a}b^2c \ln\left(\frac{2cx + b + 2\sqrt{cx^2 + bx + a}\sqrt{c}}{2\sqrt{c}} \right) + 2abc^{\frac{3}{2}} \right) x^3}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}(4ac - b^2)c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out] $x^3(c*x^2+b*x+a)/c^{5/2}*(-4*c^{5/2}*x*a+2*c^{3/2}*x*b^2+4*(c*x^2+b*x+a)^{1/2}*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{1/2}*c^{1/2}))/c^{1/2})*a*c^2-(c*x^2+b*x+a)^{1/2}*ln(1/2*(2*c*x+b+2*(c*x^2+b*x+a)^{1/2}*c^{1/2}))/c^{1/2})*b^2*c+2*c^{3/2}*a*b)/(c*x^4+b*x^3+a*x^2)^{3/2}/(4*a*c-b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(cx^4 + bx^3 + ax^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)`

[Out] `int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral(x**5/(x**2*(a + b*x + c*x**2))**3/2, x)`

$$3.58 \quad \int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=40

$$\frac{2x(2a + bx)}{(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}}$$

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1916}

$$\frac{2x(2a + bx)}{(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1916

Int[(x_)^(m_)/((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(3/2), x_Symbol] :> Simp[(x^((n - 1)/2)*(4*a + 2*b*x))/((b^2 - 4*a*c)*Sqrt[a*x^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{x^4}{(ax^2 + bx^3 + cx^4)^{3/2}} dx = \frac{2x(2a + bx)}{(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.92

$$\frac{2x(2a + bx)}{(b^2 - 4ac) \sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] $(2*x*(2*a + b*x))/((b^2 - 4*a*c)*\text{Sqrt}[x^2*(a + x*(b + c*x))])$

IntegrateAlgebraic [A] time = 1.35, size = 56, normalized size = 1.40

$$\frac{2(2a + bx)\sqrt{ax^2 + bx^3 + cx^4}}{x(4ac - b^2)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $(-2*(2*a + b*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/((-b^2 + 4*a*c)*x*(a + b*x + c*x^2))$

fricas [A] time = 1.35, size = 73, normalized size = 1.82

$$\frac{2\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)}{(b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="fricas")

[Out] $2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)$

giac [A] time = 0.92, size = 45, normalized size = 1.12

$$\frac{2\left(\frac{b}{b^2-4ac} + \frac{2a}{(b^2-4ac)x}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="giac")

[Out] $2*(b/(b^2 - 4*a*c) + 2*a/((b^2 - 4*a*c)*x))/\text{sqrt}(c + b/x + a/x^2)$

maple [A] time = 0.00, size = 53, normalized size = 1.32

$$\frac{2(c x^2 + b x + a)(b x + 2 a) x^3}{(4 a c - b^2)(c x^4 + b x^3 + a x^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out] `-2*(c*x^2+b*x+a)*(b*x+2*a)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

mupad [B] time = 2.12, size = 75, normalized size = 1.88

$$\frac{\left(\frac{4ac}{4ac^2-b^2c} + \frac{2bcx}{4ac^2-b^2c}\right) \sqrt{cx^4 + bx^3 + ax^2}}{x(c x^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)`

[Out] `-(((4*a*c)/(4*a*c^2 - b^2*c) + (2*b*c*x)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3 + c*x^4)^(1/2))/(x*(a + b*x + c*x^2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)`

$$3.59 \quad \int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=39

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1915}

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1915

Int[(x_)^(m_)/((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(3/2), x_Symbol] :> Simp[(-2*x^((n-1)/2)*(b+2*c*x))/((b^2-4*a*c)*Sqrt[a*x^(n-1)+b*x^n+c*x^(n+1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3*(n-1))/2] && EqQ[q, n-1] && EqQ[r, n+1] && NeQ[b^2-4*a*c, 0]

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.92

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $(-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*\text{Sqrt}[x^2*(a + x*(b + c*x))])$

IntegrateAlgebraic [A] time = 0.87, size = 53, normalized size = 1.36

$$\frac{2(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x(b^2 - 4ac)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $(-2*(b + 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/((b^2 - 4*a*c)*x*(a + b*x + c*x^2))$

fricas [A] time = 1.35, size = 72, normalized size = 1.85

$$\frac{2\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)}{(b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="fricas")

[Out] $-2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)$

giac [A] time = 0.76, size = 45, normalized size = 1.15

$$\frac{2\left(\frac{2c}{b^2-4ac} + \frac{b}{(b^2-4ac)x}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="giac")

[Out] $-2*(2*c/(b^2 - 4*a*c) + b/((b^2 - 4*a*c)*x))/\text{sqrt}(c + b/x + a/x^2)$

maple [A] time = 0.00, size = 52, normalized size = 1.33

$$\frac{2(c x^2 + b x + a)(2 c x + b) x^3}{(4 a c - b^2)(c x^4 + b x^3 + a x^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out] $2*(c*x^2+b*x+a)*(2*c*x+b)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

mupad [B] time = 2.03, size = 75, normalized size = 1.92

$$\frac{\left(\frac{4c^2x}{4ac^2-b^2c} + \frac{2bc}{4ac^2-b^2c}\right) \sqrt{cx^4 + bx^3 + ax^2}}{x(cx^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)`

[Out] $\left(\frac{4c^2x}{4ac^2-b^2c} + \frac{2bc}{4ac^2-b^2c}\right) \sqrt{cx^4 + bx^3 + ax^2} / (x(a + bx + cx^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)`

$$3.60 \quad \int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=94

$$\frac{2x(-2ac + b^2 + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1922, 1904, 206}

$$\frac{2x(-2ac + b^2 + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1922

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n

- q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -(n - q)*(2*p + 3)
)]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{a} \\ &= \frac{2x(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{a} \\ &= \frac{2x(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.13, size = 109, normalized size = 1.16

$$\frac{x(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}x(-2ac + b^2 + bcx)}{a^{3/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (-2*Sqrt[a]*x*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(a^(3/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 1.84, size = 124, normalized size = 1.32

$$-\frac{2 \log(x)}{a^{3/2}} + \frac{\log\left(-2a^{3/2}\sqrt{ax^2 + bx^3 + cx^4} + 2a^2x + abx^2\right)}{a^{3/2}} + \frac{2\sqrt{ax^2 + bx^3 + cx^4}(2ac - b^2 - bcx)}{ax(4ac - b^2)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $(2*(-b^2 + 2*a*c - b*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(a*(-b^2 + 4*a*c)*x*(a + b*x + c*x^2)) - (2*\text{Log}[x])/a^{(3/2)} + \text{Log}[2*a^2*x + a*b*x^2 - 2*a^{(3/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/a^{(3/2)}$

fricas [B] time = 1.41, size = 411, normalized size = 4.37

$$\frac{\left(\frac{((b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x)\sqrt{a} \log\left(\frac{-8ab^2c(b^2+ac)^2 + 8a^2c^2 - 4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3} + 4\sqrt{cx^4+bx^3+ax^2}(abcx+ab^2-2a^2c)\right)}{2((a^2b^2c-4a^2c^2)x^3 + (a^2b^3-4a^2bc)x^2 + (a^3b^2-4a^4c)x)} + \frac{((b^2c-4ac^2)x^3 + (b^3-4abc)x^2 + (ab^2-4a^2c)x)\sqrt{-a} \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{2(bc^2+ab^2+ac^2)}\right) + 2\sqrt{cx^4+bx^3+ax^2}(abcx+ab^2-2a^2c)}{(a^2b^2c-4a^2c^2)x^3 + (a^2b^3-4a^2bc)x^2 + (a^3b^2-4a^4c)x} \right)}{2((a^2b^2c-4a^2c^2)x^3 + (a^2b^3-4a^2bc)x^2 + (a^3b^2-4a^4c)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] $[1/2*((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*\text{sqrt}(a)*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*\text{sqrt}(a))/x^3) + 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x), ((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*\text{sqrt}(-a)*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*\text{sqrt}(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of $[1, 0, \{\{-4, [1, 0, 0, 0]\}\} + \{\{-2, [0, 1, 0, 1]\}\} + \{\{-2, [0, 0, 1, 2]\}\}, 0, \{\{1, [0, 2, 0, 2]\}\} + \{\{2, [0, 1, 1, 3]\}\} + \{\{1, [0, 0, 2, 4]\}\}]$ at parameters values $[54.7579903365, -49, -33, -70] 2*(-(-4*a*c+2*b^2)/(8*a^2*c-2*a*b^2)/x-2*b*c/(8*a^2*c-2*a*b^2))*\text{sqrt}(a*(1/x)^2+b/x+c)/(a*(1/x)^2+b/x+c)+1/a/\text{sqrt}(a)*\ln(\text{abs}(2*\text{sqrt}(a)*(\text{sqrt}(a*(1/x)^2+b/x+c)-\text{sqrt}(a)/x)-b))$

maple [A] time = 0.01, size = 166, normalized size = 1.77

$$\frac{(cx^2 + bx + a) \left(2a^{\frac{3}{2}}bcx + 4\sqrt{cx^2 + bx + a} a^2c \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - \sqrt{cx^2 + bx + a} a b^2 \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - 4a^{\frac{5}{2}}c + 2a^{\frac{3}{2}}b^2 \right) x^3}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} (4ac - b^2) a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] $-x^3*(c*x^2+b*x+a)*(2*a^{(3/2)}*x*b*c+4*(c*x^2+b*x+a)^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*a^2*c-(c*x^2+b*x+a)^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)*a*b^2-4*a^{(5/2)}*c+2*a^{(3/2)}*b^2)/(c*x^4+b*x^3+a*x^2)^{(3/2)}/a^{(5/2)}/(4*a*c-b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(cx^4 + bx^3 + ax^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)`

[Out] `int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral(x**2/(x**2*(a + b*x + c*x**2))** (3/2), x)`

$$3.61 \quad \int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=144

$$\frac{3b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(b^2-4ac)} + \frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Rubi [A] time = 0.16, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(b^2-4ac)} + \frac{3b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}} + \frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1924

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q] && LtQ[m + p*q + 1, n - q]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*(A_) + (B_.)*(x_)^(r_.), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-\frac{3b^2}{2} + 4ac - bcx}{x\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} + \frac{2 \int -\frac{3b(b^2 - 4ac)}{4\sqrt{ax^2 + bx^3 + cx^4}} dx}{a^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} - \frac{(3b) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} + \frac{(3b) \text{Subst} \left(\int \frac{1}{4a\sqrt{ax^2 + bx^3 + cx^4}} dx \right)}{2a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} + \frac{3b \tanh^{-1} \left(\frac{x}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}} \right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 138, normalized size = 0.96

$$\frac{2\sqrt{a}(-4a^2c + a(b^2 - 10bcx - 8c^2x^2) + 3b^2x(b + cx)) - 3bx(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right)}{2a^{5/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2)) - 3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 2.34, size = 157, normalized size = 1.09

$$\frac{3b \log(x)}{a^{5/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-4a^2c + ab^2 - 10abcx - 8ac^2x^2 + 3b^3x + 3b^2cx^2)}{a^2x^2(4ac - b^2)(a + bx + cx^2)} - \frac{3b \log(-2a^{5/2}\sqrt{ax^2 + bx^3 + cx^4} + 2a^3x + a^2bx^2)}{2a^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $((a*b^2 - 4*a^2*c + 3*b^3*x - 10*a*b*c*x + 3*b^2*c*x^2 - 8*a*c^2*x^2)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(a^2*(-b^2 + 4*a*c)*x^2*(a + b*x + c*x^2)) + (3*b*\text{Log}[x])/a^{(5/2)} - (3*b*\text{Log}[2*a^3*x + a^2*b*x^2 - 2*a^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(2*a^{(5/2)})$

fricas [A] time = 1.02, size = 496, normalized size = 3.44

$$\frac{3((b^2 - 4ac^2)x^4 + (b^3 - 4ab^2c)x^3 + (ab^3 - 4a^2b^2c^2)\sqrt{a} \log\left(\frac{4ab^2(b^2 - 4ac^2)x^2 + 4\sqrt{a^3 + b^3 + ac^2}(a^2b^2 - 4a^2c + (3ab^2 - 8a^2c^2)x^2 + (3ab^3 - 10a^2b^2c))}{4((a^2bc - 4a^2c^2)x^2 + (a^2b^3 - 4a^2b^2c^2) + (a^2b^3 - 4a^2c^2)x^2)}\right) - 3((b^2 - 4ac^2)x^4 + (b^3 - 4ab^2c)x^3 + (ab^3 - 4a^2b^2c^2)\sqrt{a} \arctan\left(\frac{\sqrt{a^3 + b^3 + ac^2}(bx + 2a)\sqrt{a}}{2(a^2bc - 4a^2c^2)x^2 + (a^2b^3 - 4a^2b^2c^2) + (a^2b^3 - 4a^2c^2)x^2}\right) + 2\sqrt{a^3 + b^3 + ac^2}(a^2b^2 - 4a^2c + (3ab^2 - 8a^2c^2)x^2 + (3ab^3 - 10a^2b^2c))}{2((a^2bc - 4a^2c^2)x^2 + (a^2b^3 - 4a^2b^2c^2) + (a^2b^3 - 4a^2c^2)x^2)}}{4((a^2bc - 4a^2c^2)x^2 + (a^2b^3 - 4a^2b^2c^2) + (a^2b^3 - 4a^2c^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] $[1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(a)*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*\text{sqrt}(a))/x^3) - 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(-a)*\arctan(1/2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*\text{sqrt}(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)*x^2)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of $[1, 0, \{\{-4, [1, 0, 0, 0]\}\} + \{\{-2, [0, 1, 0, 1]\}\} + \{\{-2, [0, 0, 1, 2]\}\}, 0, \{\{1, [0, 2, 0, 2]\}\} + \{\{2, [0, 1, 1, 3]\}\} + \{\{1, [0, 0, 2, 4]\}\}]$ at parameters values $[54.7579903365, -49, -33, -70]2*((-8*a^2*c - 2*a*b^2)/(16*a^3*c - 4*a^2*b^2)/x - (20*a*b*c - 6*b^3)/(16*a^3*c - 4*a^2*b^2))/x - (16*a*c^2 - 6*b^2*c)/(16*a^3*c - 4*a^2*b^2)*\text{sqrt}(a*(1/x)^2 + b/x + c)/(a*(1/x)^2 + b/x + c) - 6*b/4/a^2/\text{sqrt}(a)*\ln(\text{abs}(2*\text{sqrt}(a)*(\text{sqrt}(a*(1/x)^2 + b/x + c) - \text{sqrt}(a)/x) - b))$

maple [A] time = 0.01, size = 201, normalized size = 1.40

$$\frac{(c^2 + bx + a)\left(-16a^5c^2x^2 + 6a^3b^2cx^2 + 12\sqrt{cx^2 + bx + a}a^2bcx \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a}\sqrt{a}}{x}\right) - 3\sqrt{cx^2 + bx + a}a^3bx \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a}\sqrt{a}}{x}\right) - 20a^5bcx + 6a^3b^3x - 8a^2c + 2a^5b^2\right)x^2}{2(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}(4ac - b^2)a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out] $\frac{1}{2}x^2(c x^2 + b x + a)(-16 a^{5/2} x^2 c^2 + 6 a^{3/2} x^2 b^2 c + 12(c x^2 + b x + a)^{1/2} \ln((b x + 2 a + 2(c x^2 + b x + a)^{1/2} a^{1/2})/x) x a^2 b c - 3(c x^2 + b x + a)^{1/2} \ln((b x + 2 a + 2(c x^2 + b x + a)^{1/2} a^{1/2})/x) x a b^3 - 20 a^{5/2} x b c + 6 a^{3/2} x b^3 - 8 a^{7/2} c + 2 a^{5/2} b^2)/(c x^4 + b x^3 + a x^2)^{3/2} / a^{7/2} / (4 a c - b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(c x^4 + b x^3 + a x^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(c x^4 + b x^3 + a x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)`

[Out] `int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^2(a + b x + c x^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)`

$$3.62 \quad \int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=209

$$\frac{3(5b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}} + \frac{b(15b^2 - 52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2(b^2 - 4ac)} - \frac{(5b^2 - 12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3(b^2 - 4ac)}$$

Rubi [A] time = 0.29, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1907, 1951, 12, 1904, 206}

$$\frac{b(15b^2 - 52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2(b^2 - 4ac)} - \frac{(5b^2 - 12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3(b^2 - 4ac)} - \frac{3(5b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}} + \frac{2(-2ac + b^2 + bcx)}{ax(b^2 - 4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2), x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/((2*a^2*(b^2 - 4*a*c)*x^3) + (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2)) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(7/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1907

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol]
:> -Simp[(x^(-q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1)*(b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol]
:> Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-2(b^2 - 2ac) + \frac{1}{2}(-b^2 + 4ac) - 2bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{\int \frac{-\frac{1}{4}b(15b^2 - 52)}{x\sqrt{ax^2 + bx^3 + cx^4}} dx}{a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52)}{4a^3} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52)}{4a^3} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52)}{4a^3} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52)}{4a^3}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 181, normalized size = 0.87

$$\frac{3x^2(16a^2c^2 - 24ab^2c + 5b^4)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) + 2\sqrt{a}(-8a^3c + 2a^2(b^2 + 10bcx - 12c^2x^2) + abx(-5b^2 + 62bcx + 52c^2x^2) - 15b^3x^2(b+cx))}{8a^{7/2}x(b^2 - 4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2), x]

[Out] -1/8*(2*sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 12*c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^2*sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])])/(a^(7/2)*(b^2 - 4*a*c)*x*sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 3.25, size = 216, normalized size = 1.03

$$\frac{3 \log(x) (4ac - 5b^2)}{4a^{7/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4} (-8a^3c + 2a^2b^2 + 20a^2bcx - 24a^2c^2x^2 - 5ab^3x + 62ab^2cx^2 + 52abc^2x^3 - 15b^4x^2 - 15b^3cx^3)}{4a^3x^3(4ac - b^2)(a + bx + cx^2)} - \frac{3(4ac - 5b^2) \log(-2a^{7/2}\sqrt{ax^2 + bx^3 + cx^4} + 2a^4x + a^3bx^2)}{8a^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]

[Out]
$$\frac{((2a^2b^2 - 8a^3c - 5ab^3x + 20a^2bcx - 15b^4x^2 + 62ab^2cx^2 - 24a^2c^2x^2 - 15b^3cx^3 + 52ab^2c^2x^3)\sqrt{ax^2 + bx^3 + cx^4})}{(4a^3(-b^2 + 4ac)x^3(a + bx + cx^2))} + \frac{(3(-5b^2 + 4ac)\text{Log}[x])}{(4a^{7/2})} - \frac{(3(-5b^2 + 4ac)\text{Log}[2a^4x + a^3bx^2 - 2a^{7/2}]\sqrt{ax^2 + bx^3 + cx^4})}{(8a^{7/2})}$$

fricas [A] time = 1.56, size = 630, normalized size = 3.01

$$\frac{3(5b^2 - 2ab^2 + 3a^2b^2 + (b^2 - 2ac)^2 + 16c^2b^2) + (5a^2 - 2ab^2 + 3a^2b^2 + 16c^2b^2)\sqrt{ax^2 + bx^3 + cx^4}}{8((5b^2 - 2ab^2 + 3a^2b^2 + (b^2 - 2ac)^2 + 16c^2b^2)\sqrt{ax^2 + bx^3 + cx^4})} + \frac{3(5b^2 - 2ab^2 + 3a^2b^2 + (b^2 - 2ac)^2 + 16c^2b^2)\sqrt{ax^2 + bx^3 + cx^4}}{8((5b^2 - 2ab^2 + 3a^2b^2 + (b^2 - 2ac)^2 + 16c^2b^2)\sqrt{ax^2 + bx^3 + cx^4})} - \frac{3(5b^2 - 2ab^2 + 3a^2b^2 + (b^2 - 2ac)^2 + 16c^2b^2)\sqrt{ax^2 + bx^3 + cx^4}}{8((5b^2 - 2ab^2 + 3a^2b^2 + (b^2 - 2ac)^2 + 16c^2b^2)\sqrt{ax^2 + bx^3 + cx^4})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(3*((5b^4c - 24a*b^2*c^2 + 16a^2*c^3)*x^5 + (5b^5 - 24a*b^3*c + 16a^2*b*c^2)*x^4 + (5a*b^4 - 24a^2*b^2*c + 16a^3*c^2)*x^3)*\text{sqrt}(a)*\log(- (8a*b*x^2 + (b^2 + 4a*c)*x^3 + 8a^2*x + 4*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)) * (b*x + 2a)*\text{sqrt}(a))/x^3) + 4*(2a^3*b^2 - 8a^4*c - (15a*b^3*c - 52a^2*b*c^2)*x^3 - (15a*b^4 - 62a^2*b^2*c + 24a^3*c^2)*x^2 - 5*(a^2*b^3 - 4a^3*b*c)*x)*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)] / ((a^4*b^2*c - 4a^5*c^2)*x^5 + (a^4*b^3 - 4a^5*b*c)*x^4 + (a^5*b^2 - 4a^6*c)*x^3), \\ & \frac{1}{8}*(3*((5b^4c - 24a*b^2*c^2 + 16a^2*c^3)*x^5 + (5b^5 - 24a*b^3*c + 16a^2*b*c^2)*x^4 + (5a*b^4 - 24a^2*b^2*c + 16a^3*c^2)*x^3)*\text{sqrt}(-a)*\arctan(\frac{1}{2}\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2a)*\text{sqrt}(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2a^3*b^2 - 8a^4*c - (15a*b^3*c - 52a^2*b*c^2)*x^3 - (15a*b^4 - 62a^2*b^2*c + 24a^3*c^2)*x^2 - 5*(a^2*b^3 - 4a^3*b*c)*x)*\text{sqrt}(c*x^4 + b*x^3 + a*x^2)] / ((a^4*b^2*c - 4a^5*c^2)*x^5 + (a^4*b^3 - 4a^5*b*c)*x^4 + (a^5*b^2 - 4a^6*c)*x^3) \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.01, size = 292, normalized size = 1.40

$$\frac{(c^2 + bx + a) \left(-104b^2c^2x^3 + 30b^2b^2cx^3 - 48\sqrt{cx^2 + bx + a}a^2c^2x^2 \ln\left(\frac{bx+2a+\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) + 72\sqrt{cx^2 + bx + a}a^2b^2cx^2 \ln\left(\frac{bx+2a+\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) - 15\sqrt{cx^2 + bx + a}a^2b^2cx^2 \ln\left(\frac{bx+2a+\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right) + 48b^2c^2x^2 - 124a^2b^2cx^2 + 30a^2b^2x^2 - 40a^2b^2cx + 10a^2b^2x + 16a^2c - 4a^2b^2 \right)}{8(c^2 + bx + a)^{\frac{3}{2}}(4ac - b^2)a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out]
$$-1/8*x*(c*x^2+b*x+a)*(48*a^{7/2}*x^2*c^2-104*a^{5/2}*x^3*b*c^2+16*a^{9/2}*c-40*a^{7/2}*x*b*c-124*a^{5/2}*x^2*b^2*c+30*a^{3/2}*x^3*b^3*c-4*a^{7/2}*b^2+10*a^{5/2}*x*b^3+30*a^{3/2}*x^2*b^4-48*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{1/2})*a^{1/2})/x)*(c*x^2+b*x+a)^{1/2}*x^2*a^3*c^2+72*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{1/2})*a^{1/2})/x)*(c*x^2+b*x+a)^{1/2}*x^2*a^2*b^2*c-15*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{1/2})*a^{1/2})/x)*(c*x^2+b*x+a)^{1/2}*x^2*a*b^4)/(c*x^4+b*x^3+a*x^2)^{3/2}/a^{9/2}/(4*a*c-b^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)`

[Out] `int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)`

[Out] `Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)`

$$3.63 \quad \int \frac{1}{x(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=271

$$\frac{5b(7b^2-12ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3(b^2-4ac)} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4(b^2-4ac)}$$

Rubi [A] time = 0.45, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2(b^2-4ac)} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3(b^2-4ac)} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4(b^2-4ac)} + \frac{5b(7b^2-12ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}} + \frac{2(-2ac+b^2+bcx)}{ax^2(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*sqrt[a*x^2 + b*x^3 + c*x^4]) - ((7*b^2 - 16*a*c)*sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4) + (b*(35*b^2 - 116*a*c)*sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))/(2*sqrt[a]*sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1924

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q] && LtQ[m + p*q + 1, n - q]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*(A_.) + (B_.)*(x_)^(r_.), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-\frac{7b^2}{2} + 8ac - 3bcx}{x^3\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{2 \int \frac{-\frac{1}{4}b(35b^2 - 11cx)}{x^2} dx}{3a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 11cx)}{12a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 11cx)}{12a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 11cx)}{12a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 11cx)}{12a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 11cx)}{12a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 11cx)}{12a}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 225, normalized size = 0.83

$$\frac{2\sqrt{a}(-32a^4c + 8a^3(b^2 + 7bcx + 16c^2x^2) + 2a^2x(-7b^3 - 86b^2cx + 244bc^2x^2 + 128c^3x^3) + 5ab^2x^2(7b^2 - 106bcx - 92c^2x^2) + 105b^3x^3(b + cx) - 15bx^3(48a^2c^2 - 40ab^2c + 7b^4)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right)}{48a^{9/2}x^2(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x]

[Out] (2*sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*c*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 - 86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*x^3*sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])])/(48*a^(9/2)*(-b^2 + 4*a*c)*x^2*sqrt[x^2*(a + x*(b + c*x))])

IntegrateAlgebraic [A] time = 3.61, size = 266, normalized size = 0.98

$$\frac{5 \log(x) (12abc - 7b^3)}{8a^{9/2}} + \frac{5(12abc - 7b^3) \log\left(\frac{-2a^{9/2}\sqrt{ax^2 + bx^3 + cx^4} + 2a^5x + a^4bx^2}{16a^{9/2}}\right)}{16a^{9/2}} + \frac{\sqrt{ax^2 + bx^3 + cx^4}(-32a^4c + 8a^3b^2 + 56a^2bcx + 128a^2c^2x^2 - 14a^2b^3x - 172a^2b^2cx^2 + 488a^2bc^2x^3 + 256a^2c^3x^4 + 35ab^4x^2 - 530ab^3cx^3 - 460ab^2c^2x^4 + 105b^5x^3 + 105b^4cx^4)}{24a^4x^4(4ac - b^2)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a*x^2 + b*x^3 + c*x^4]*(8*a^3*b^2 - 32*a^4*c - 14*a^2*b^3*x + 56*a^3*
b*c*x + 35*a*b^4*x^2 - 172*a^2*b^2*c*x^2 + 128*a^3*c^2*x^2 + 105*b^5*x^3 -
530*a*b^3*c*x^3 + 488*a^2*b*c^2*x^3 + 105*b^4*c*x^4 - 460*a*b^2*c^2*x^4 + 2
56*a^2*c^3*x^4))/(24*a^4*(-b^2 + 4*a*c)*x^4*(a + b*x + c*x^2)) - (5*(-7*b^3
+ 12*a*b*c)*Log[x])/(8*a^(9/2)) + (5*(-7*b^3 + 12*a*b*c)*Log[2*a^5*x + a^4
*b*x^2 - 2*a^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]])/(16*a^(9/2))
```

fricas [A] time = 2.08, size = 716, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4
*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt
(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c
- 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3
*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3
- 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 +
(a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((7*b^5*c
- 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*
x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^
4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3
*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3
+ a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b
^2 - 4*a^7*c)*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

maple [A] time = 0.01, size = 340, normalized size = 1.25

$$\frac{(c^2 + bx + a) \left(-512a^2c^2x^4 + 920a^2b^2c^2x^4 - 210a^2b^4cx^4 + 720a^2bc^2x^4 + 210a^2b^2c^2x^4 + 210a^2b^2c^2x^4 \ln\left(\frac{2a^2bx + 2\sqrt{a^2bx + c^2}}{a}\right) - 600a^2c^2x^2 + bx + a \right) b^2c^2 \ln\left(\frac{2a^2bx + 2\sqrt{a^2bx + c^2}}{a}\right) + 105a^2c^2x^2 + bx + a \right) b^2c^2 \ln\left(\frac{2a^2bx + 2\sqrt{a^2bx + c^2}}{a}\right) - 976a^2b^2c^2x^3 + 1060a^2b^2c^2x^3 - 210a^2b^2c^2x^3 - 256a^2c^2x^2 + 344a^2b^2c^2x^2 - 70a^2b^2c^2x^2 - 112a^2bcx + 28a^2b^2c + 64a^2c - 16a^2b^2}{48(c^2 + bx + a)^2(4ac - b^2)a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] $-1/48*(c*x^2+b*x+a)*(-512*a^(7/2)*x^4*c^3-256*a^(9/2)*x^2*c^2-976*a^(7/2)*x^3*b*c^2+920*a^(5/2)*x^4*b^2*c^2+64*a^(11/2)*c-112*a^(9/2)*x*b*c+344*a^(7/2)*x^2*b^2*c+1060*a^(5/2)*x^3*b^3*c-210*a^(3/2)*x^4*b^4*c-16*a^(9/2)*b^2+28*a^(7/2)*x*b^3-70*a^(5/2)*x^2*b^4-210*a^(3/2)*x^3*b^5+720*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a^3*b*c^2-600*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b^3*c+105*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a*b^5)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(11/2)/(4*a*c-b^2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x(c x^4 + b x^3 + a x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)

[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

$$3.64 \quad \int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=343

$$\frac{b(21b^2 - 68ac) \sqrt{ax^2 + bx^3 + cx^4}}{8a^3x^4(b^2 - 4ac)} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^5(b^2 - 4ac)} - \frac{15(16a^2c^2 - 56ab^2c + 21b^4) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}$$

Rubi [A] time = 0.62, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1951, 12, 1904, 206}

$$\frac{b(1808a^2c^2 - 1680ab^2c + 315b^4) \sqrt{ax^2 + bx^3 + cx^4}}{64a^5x^2(b^2 - 4ac)} - \frac{(240a^2c^2 - 448ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{32a^4x^3(b^2 - 4ac)} - \frac{15(16a^2c^2 - 56ab^2c + 21b^4) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}} + \frac{b(21b^2 - 68ac) \sqrt{ax^2 + bx^3 + cx^4}}{8a^3x^4(b^2 - 4ac)} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^5(b^2 - 4ac)} + \frac{2(-2ac + b^2 + bcx)}{ax^3(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4])
- ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5)
+ (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^4)
- ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a^4*(b^2 - 4*a*c)*x^3)
+ (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2)
- (15*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n]
```

- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q] && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_.) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-\frac{9b^2}{2} + 10ac - 4bcx}{x^4 \sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{\int \frac{-\frac{3}{4}b(21b^2 - 6bcx + 10ac)}{x^3 \sqrt{ax^2 + bx^3 + cx^4}} dx}{2a} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3 \sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac) \sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 6bcx + 10ac)}{8a^2}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 272, normalized size = 0.79

$$\frac{15x^4 (-64a^3c^3 + 240a^2b^2c^2 - 140ab^4c + 21b^6) \sqrt{a+x(b+cx)} \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a} (64a^3c - 16a^4(b^2 + 6bcx + 10c^2x^2) + 8a^3x(3b^3 + 26b^2cx + 98b^2x^2 - 60c^3x^3) + 2a^2bx^2(-21b^3 - 308b^2cx + 1352b^2x^2 + 904c^3x^3) + 105ab^3x^3(b^2 - 18bcx - 16c^2x^2) + 315b^5x^4(b+cx))}{128a^{11/2}x^3(4ac - b^2)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] (-2*Sqrt[a]*(64*a^5*c + 315*b^5*x^4*(b + c*x) + 105*a*b^3*x^3*(b^2 - 18*b*c*x - 16*c^2*x^2) - 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 8*a^3*x*(3*b^3 + 26*b^2*c*x + 98*b*c^2*x^2 - 60*c^3*x^3) + 2*a^2*b*x^2*(-21*b^3 - 308*b^2*c*x + 1352*b*c^2*x^2 + 904*c^3*x^3)) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c

$$c^2 - 64a^3c^3)x^4\sqrt{a + x(b + cx)}\operatorname{ArcTanh}\left[\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right]/(128a^{11/2}(-b^2 + 4ac)x^3\sqrt{x^2(a + x(b + cx))})$$

IntegrateAlgebraic [A] time = 5.24, size = 331, normalized size = 0.97

$$\frac{15(16a^2 - 56ab^2c + 21b^4)\log\left(\frac{2\sqrt{a}\sqrt{a^2 + bx^2 + cx^3 - 2ax - b^2}}{128a^{11/2}}\right) + 15\log(1)(16a^2c^2 - 56ab^2c + 21b^4)}{64a^{11/2}} + \frac{\sqrt{a^2 + bx^2 + cx^3}(-64a^2c + 16a^4b^2 + 96a^4bc + 160a^4c^2x^2 - 24a^3b^2x - 208a^3b^2cx^2 - 784a^3b^2c^2x^3 + 480a^3c^3x^4 + 42a^2b^3x^2 + 616a^2b^3cx^3 - 2704a^2b^2c^2x^4 - 1808a^2b^2c^2x^5 - 105ab^3x^3 + 1890ab^3cx^4 + 1680ab^3c^2x^5 - 315b^4x^4 - 315b^4cx^5)}{64a^{11/2}(4ac - b^2)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] $(\sqrt{a*x^2 + b*x^3 + c*x^4}*(16*a^4*b^2 - 64*a^5*c - 24*a^3*b^3*x + 96*a^4*b*c*x + 42*a^2*b^4*x^2 - 208*a^3*b^2*c*x^2 + 160*a^4*c^2*x^2 - 105*a*b^5*x^3 + 616*a^2*b^3*c*x^3 - 784*a^3*b*c^2*x^3 - 315*b^6*x^4 + 1890*a*b^4*c*x^4 - 2704*a^2*b^2*c^2*x^4 + 480*a^3*c^3*x^4 - 315*b^5*c*x^5 + 1680*a*b^3*c^2*x^5 - 1808*a^2*b*c^3*x^5))/(64*a^5*(-b^2 + 4*a*c)*x^5*(a + b*x + c*x^2)) - (15*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*\operatorname{Log}[x])/(64*a^{11/2}) + (15*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*\operatorname{Log}[-2*a*x - b*x^2 + 2*\sqrt{a}*\sqrt{a*x^2 + b*x^3 + c*x^4}])/(128*a^{11/2})$

fricas [A] time = 2.35, size = 866, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] $[1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*\sqrt{a}*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*\sqrt{(c*x^4 + b*x^3 + a*x^2)}]/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*\sqrt{-a}*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{-a})/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (c x^4 + b x^3 + a x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)

[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (x^2 (a + b x + c x^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2), x)

[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)

3.65 $\int x^m (ax + bx^3 + cx^5) dx$

Optimal. Leaf size=37

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a*x + b*x^3 + c*x^5), x]

[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x^m (ax + bx^3 + cx^5) dx &= \int (ax^{1+m} + bx^{3+m} + cx^{5+m}) dx \\ &= \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m} \end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.92

$$x^{m+2} \left(\frac{a}{m+2} + \frac{bx^2}{m+4} + \frac{cx^4}{m+6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a*x + b*x^3 + c*x^5), x]

[Out] x^(2 + m)*(a/(2 + m) + (b*x^2)/(4 + m) + (c*x^4)/(6 + m))

IntegrateAlgebraic [F] time = 0.31, size = 0, normalized size = 0.00

$$\int x^m (ax + bx^3 + cx^5) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(a*x + b*x^3 + c*x^5), x]

[Out] Defer[IntegrateAlgebraic][x^m*(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 1.26, size = 71, normalized size = 1.92

$$\frac{((cm^2 + 6cm + 8c)x^6 + (bm^2 + 8bm + 12b)x^4 + (am^2 + 10am + 24a)x^2)x^m}{m^3 + 12m^2 + 44m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x), x, algorithm="fricas")

[Out] ((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m + 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)

giac [B] time = 0.48, size = 107, normalized size = 2.89

$$\frac{cm^2x^6x^m + 6cmx^6x^m + bm^2x^4x^m + 8cx^6x^m + 8bmx^4x^m + am^2x^2x^m + 12bx^4x^m + 10amx^2x^m + 24ax^2x^m}{m^3 + 12m^2 + 44m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x), x, algorithm="giac")

[Out] (c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x^m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 + 12*m^2 + 44*m + 48)

maple [B] time = 0.00, size = 77, normalized size = 2.08

$$\frac{(cm^2x^4 + 6cmx^4 + bm^2x^2 + 8cx^4 + 8bmx^2 + am^2 + 12bx^2 + 10am + 24a)x^{m+2}}{(m+6)(m+4)(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(c*x^5+b*x^3+a*x), x)

[Out] x^(m+2)*(c*m^2*x^4+6*c*m*x^4+b*m^2*x^2+8*c*x^4+8*b*m*x^2+a*m^2+12*b*x^2+10*a*m+24*a)/(m+6)/(m+4)/(m+2)

maxima [A] time = 0.45, size = 37, normalized size = 1.00

$$\frac{cx^{m+6}}{m+6} + \frac{bx^{m+4}}{m+4} + \frac{ax^{m+2}}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] c*x^(m + 6)/(m + 6) + b*x^(m + 4)/(m + 4) + a*x^(m + 2)/(m + 2)

mupad [B] time = 2.08, size = 89, normalized size = 2.41

$$x^m \left(\frac{a x^2 (m^2 + 10 m + 24)}{m^3 + 12 m^2 + 44 m + 48} + \frac{b x^4 (m^2 + 8 m + 12)}{m^3 + 12 m^2 + 44 m + 48} + \frac{c x^6 (m^2 + 6 m + 8)}{m^3 + 12 m^2 + 44 m + 48} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(a*x + b*x^3 + c*x^5),x)

[Out] x^m*((a*x^2*(10*m + m^2 + 24))/(44*m + 12*m^2 + m^3 + 48) + (b*x^4*(8*m + m^2 + 12))/(44*m + 12*m^2 + m^3 + 48) + (c*x^6*(6*m + m^2 + 8))/(44*m + 12*m^2 + m^3 + 48))

sympy [A] time = 1.18, size = 280, normalized size = 7.57

$$\begin{cases} -\frac{a}{4x^4} - \frac{b}{2x^2} + c \log(x) & \text{for } m = -6 \\ -\frac{a}{2x^2} + b \log(x) + \frac{cx^2}{2} & \text{for } m = -4 \\ a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4} & \text{for } m = -2 \\ \frac{am^2x^2}{m^3+12m^2+44m+48} + \frac{10amx^2}{m^3+12m^2+44m+48} + \frac{24ax^2}{m^3+12m^2+44m+48} + \frac{bm^2x^4}{m^3+12m^2+44m+48} + \frac{8bm^2x^4}{m^3+12m^2+44m+48} + \frac{12bx^4}{m^3+12m^2+44m+48} + \frac{cm^2x^6}{m^3+12m^2+44m+48} + \frac{6cmx^6}{m^3+12m^2+44m+48} + \frac{8cx^6}{m^3+12m^2+44m+48} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(c*x**5+b*x**3+a*x),x)

[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) + b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -2)), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m**2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))

$$3.66 \quad \int x^2 (ax + bx^3 + cx^5) dx$$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8

Rule 14

Int[(u_)*((c_.)*(x_)^(m_.)), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (ax + bx^3 + cx^5) dx &= \int (ax^3 + bx^5 + cx^7) dx \\ &= \frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (ax + bx^3 + cx^5) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x^2*(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{1}{8}x^8c + \frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x), x, algorithm="fricas")

[Out] 1/8*x^8*c + 1/6*x^6*b + 1/4*x^4*a

giac [A] time = 0.38, size = 19, normalized size = 0.76

$$\frac{1}{8}cx^8 + \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x), x, algorithm="giac")

[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{8}cx^8 + \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^5+b*x^3+a*x), x)

[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{8}cx^8 + \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{cx^8}{8} + \frac{bx^6}{6} + \frac{ax^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a*x + b*x^3 + c*x^5),x)

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**5+b*x**3+a*x),x)

[Out] a*x**4/4 + b*x**6/6 + c*x**8/8

$$3.67 \quad \int x(ax + bx^3 + cx^5) dx$$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x(ax + bx^3 + cx^5) dx &= \int (ax^2 + bx^4 + cx^6) dx \\ &= \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(ax + bx^3 + cx^5) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x*(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 1.23, size = 19, normalized size = 0.76

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x), x, algorithm="fricas")

[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a

giac [A] time = 0.39, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x), x, algorithm="giac")

[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^5+b*x^3+a*x), x)

[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7

maxima [A] time = 0.43, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{cx^7}{7} + \frac{bx^5}{5} + \frac{ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x + b*x^3 + c*x^5),x)

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**5+b*x**3+a*x),x)

[Out] a*x**3/3 + b*x**5/5 + c*x**7/7

$$3.68 \quad \int (ax + bx^3 + cx^5) dx$$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[a*x + b*x^3 + c*x^5,x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6

Rubi steps

$$\int (ax + bx^3 + cx^5) dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[a*x + b*x^3 + c*x^5,x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax + bx^3 + cx^5) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a*x + b*x^3 + c*x^5,x]

[Out] IntegrateAlgebraic[a*x + b*x^3 + c*x^5, x]

fricas [A] time = 1.02, size = 19, normalized size = 0.76

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")

[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a

giac [A] time = 0.38, size = 19, normalized size = 0.76

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")

[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c*x^5+b*x^3+a*x,x)

[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6

maxima [A] time = 0.42, size = 19, normalized size = 0.76

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")

[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$\frac{cx^6}{6} + \frac{bx^4}{4} + \frac{ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x + b*x^3 + c*x^5,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

sympy [A] time = 0.06, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**5+b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

$$3.69 \quad \int \frac{ax+bx^3+cx^5}{x} dx$$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax + bx^3 + cx^5}{x} dx &= \int (a + bx^2 + cx^4) dx \\ &= ax + \frac{bx^3}{3} + \frac{cx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

fricas [A] time = 1.35, size = 16, normalized size = 0.80

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x

giac [A] time = 0.36, size = 16, normalized size = 0.80

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)/x,x)

[Out] a*x+1/3*b*x^3+1/5*c*x^5

maxima [A] time = 0.42, size = 16, normalized size = 0.80

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x

mupad [B] time = 0.03, size = 16, normalized size = 0.80

$$\frac{cx^5}{5} + \frac{bx^3}{3} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + b*x^3 + c*x^5)/x,x)

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

sympy [A] time = 0.07, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)/x,x)

[Out] a*x + b*x**3/3 + c*x**5/5

$$3.70 \quad \int \frac{ax+bx^3+cx^5}{x^2} dx$$

Optimal. Leaf size=21

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax + bx^3 + cx^5}{x^2} dx &= \int \left(\frac{a}{x} + bx + cx^3 \right) dx \\ &= \frac{bx^2}{2} + \frac{cx^4}{4} + a \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax + bx^3 + cx^5}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)/x^2, x]

fricas [A] time = 1.24, size = 17, normalized size = 0.81

$$\frac{1}{4} cx^4 + \frac{1}{2} bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)

giac [A] time = 0.41, size = 20, normalized size = 0.95

$$\frac{1}{4} cx^4 + \frac{1}{2} bx^2 + \frac{1}{2} a \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{cx^4}{4} + \frac{bx^2}{2} + a \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)/x^2,x)

[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)

maxima [A] time = 0.43, size = 17, normalized size = 0.81

$$\frac{1}{4} cx^4 + \frac{1}{2} bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{bx^2}{2} + \frac{cx^4}{4} + a \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + b*x^3 + c*x^5)/x^2,x)

[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)

sympy [A] time = 0.10, size = 17, normalized size = 0.81

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)/x**2,x)

[Out] a*log(x) + b*x**2/2 + c*x**4/4

$$3.71 \quad \int \frac{ax+bx^3+cx^5}{x^3} dx$$

Optimal. Leaf size=18

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3+cx^5}{x^3} dx &= \int \left(b + \frac{a}{x^2} + cx^2 \right) dx \\ &= -\frac{a}{x} + bx + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax + bx^3 + cx^5}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)/x^3, x]

fricas [A] time = 1.02, size = 20, normalized size = 1.11

$$\frac{cx^4 + 3bx^2 - 3a}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")

[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x

giac [A] time = 0.59, size = 16, normalized size = 0.89

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")

[Out] 1/3*c*x^3 + b*x - a/x

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{cx^3}{3} + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)/x^3,x)

[Out] -a/x+b*x+1/3*c*x^3

maxima [A] time = 0.43, size = 16, normalized size = 0.89

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")

[Out] 1/3*c*x^3 + b*x - a/x

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$bx - \frac{a}{x} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + b*x^3 + c*x^5)/x^3,x)

[Out] b*x - a/x + (c*x^3)/3

sympy [A] time = 0.10, size = 12, normalized size = 0.67

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)/x**3,x)

[Out] -a/x + b*x + c*x**3/3

$$3.72 \quad \int x^m (ax + bx^3 + cx^5)^2 dx$$

Optimal. Leaf size=76

$$\frac{a^2 x^{m+3}}{m+3} + \frac{x^{m+7} (2ac + b^2)}{m+7} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1585, 1108}

$$\frac{a^2 x^{m+3}}{m+3} + \frac{x^{m+7} (2ac + b^2)}{m+7} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7 + m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)

Rule 1108

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int x^m (ax + bx^3 + cx^5)^2 dx &= \int x^{2+m} (a + bx^2 + cx^4)^2 dx \\ &= \int (a^2 x^{2+m} + 2abx^{4+m} + (b^2 + 2ac)x^{6+m} + 2bcx^{8+m} + c^2 x^{10+m}) dx \\ &= \frac{a^2 x^{3+m}}{3+m} + \frac{2abx^{5+m}}{5+m} + \frac{(b^2 + 2ac)x^{7+m}}{7+m} + \frac{2bcx^{9+m}}{9+m} + \frac{c^2 x^{11+m}}{11+m} \end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.91

$$x^{m+3} \left(\frac{a^2}{m+3} + \frac{x^4(2ac+b^2)}{m+7} + \frac{2abx^2}{m+5} + \frac{2bcx^6}{m+9} + \frac{c^2x^8}{m+11} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]

[Out] x^(3 + m)*(a^2/(3 + m) + (2*a*b*x^2)/(5 + m) + ((b^2 + 2*a*c)*x^4)/(7 + m) + (2*b*c*x^6)/(9 + m) + (c^2*x^8)/(11 + m))

IntegrateAlgebraic [F] time = 0.41, size = 0, normalized size = 0.00

$$\int x^m (ax + bx^3 + cx^5)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(a*x + b*x^3 + c*x^5)^2,x]

[Out] Defer[IntegrateAlgebraic][x^m*(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.45, size = 241, normalized size = 3.17

$$\frac{((c^2m^4 + 24c^2m^3 + 206c^2m^2 + 744c^2m + 945c^2)x^{11} + 2(bcm^4 + 26bcm^3 + 236bcm^2 + 886bcm + 1155bc)x^9 + ((b^2 + 2ac)m^4 + 28(b^2 + 2ac)m^3 + 274(b^2 + 2ac)m^2 + 1485b^2 + 2970ac + 1092(b^2 + 2ac)m)x^7 + 2(abm^4 + 30abm^3 + 320abm^2 + 1410abm + 2079ab)x^5 + (a^2m^4 + 32a^2m^3 + 374a^2m^2 + 1888a^2m + 3465a^2)x^3)^m}{m^5 + 35m^4 + 470m^3 + 3010m^2 + 9129m + 10395}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] ((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11 + 2*(b*c*m^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c + 1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 1410*a*b*m + 2079*a*b)*x^5 + (a^2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m + 3465*a^2)*x^3)*x^m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)

giac [B] time = 0.52, size = 399, normalized size = 5.25

$$\frac{c^2m^4x^{11} + 24c^2m^3x^{11} + 206c^2m^2x^{11} + 744c^2mx^{11} + 945c^2x^{11} + 2bcm^4x^9 + 26bcm^3x^9 + 236bcm^2x^9 + 886bcmx^9 + 1155bcx^9 + ((b^2 + 2ac)m^4 + 28(b^2 + 2ac)m^3 + 274(b^2 + 2ac)m^2 + 1485b^2 + 2970ac + 1092(b^2 + 2ac)m)x^7 + 2(abm^4 + 30abm^3 + 320abm^2 + 1410abm + 2079ab)x^5 + (a^2m^4 + 32a^2m^3 + 374a^2m^2 + 1888a^2m + 3465a^2)x^3}{m^5 + 35m^4 + 470m^3 + 3010m^2 + 9129m + 10395} x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] (c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*x^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a*c

$m^4 x^7 x^m + 472 b c m^2 x^9 x^m + 945 c^2 x^{11} x^m + 28 b^2 m^3 x^7 x^m + 56 a c m^3 x^7 x^m + 1772 b c m x^9 x^m + 2 a b m^4 x^5 x^m + 274 b^2 m^2 x^7 x^m + 548 a c m^2 x^7 x^m + 2310 b c m x^9 x^m + 60 a b m^3 x^5 x^m + 1092 b^2 m x^7 x^m + 2184 a c m x^7 x^m + a^2 m^4 x^3 x^m + 640 a b m^2 x^5 x^m + 1485 b^2 x^7 x^m + 2970 a c x^7 x^m + 32 a^2 m^3 x^3 x^m + 2820 a b m x^5 x^m + 374 a^2 m^2 x^3 x^m + 4158 a b x^5 x^m + 1888 a^2 m x^3 x^m + 3465 a^2 x^3 x^m) / (m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395)$

maple [B] time = 0.00, size = 300, normalized size = 3.95

$(c^2 m^4 + 24 c^2 m^3 + 20 c m^4 + 206 c^2 m^2 + 520 c m^3 + 744 c m^4 + 2 a c m^4 + 472 b c m^2 + 945 c^2 + 56 a c m^3 + 282 m^2 + 1772 c m^2 + 2 a b m^4 + 548 a c m^2 + 274 m^2 + 2310 c^2 + 60 a b m^3 + 2184 a c m^2 + 1092 m^2 + a^2 m^4 + 640 a b m^2 + 2970 a c + 1485 b^2 + 32 m^3 + 2820 a b m + 374 m^2 + 4158 a b + 1888 m + 3465) x^{m+3} / (m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m(c x^5 + b x^3 + a x)^2, x)$

[Out] $x^{(m+3)} * (c^2 m^4 x^8 + 24 c^2 m^3 x^8 + 2 b c m^4 x^6 + 206 c^2 m^2 x^8 + 52 b c m^3 x^6 + 744 c^2 m x^8 + 2 a c m^4 x^4 + b^2 m^4 x^4 + 472 b c m^2 x^6 + 945 c^2 x^8 + 56 a c m^3 x^4 + 28 b^2 m^3 x^4 + 1772 b c m x^6 + 2 a b m^4 x^2 + 548 a c m^2 x^4 + 274 b^2 m^2 x^4 + 2310 b c m x^6 + 60 a b m^3 x^2 + 2184 a c m x^4 + 1092 b^2 m x^4 + a^2 m^4 + 640 a b m^2 x^2 + 2970 a c x^4 + 1485 b^2 x^4 + 32 a^2 m^3 + 2820 a b m x^2 + 374 a^2 m^2 + 4158 a b x^2 + 1888 a^2 m + 3465 a^2) / (m+11) / (m+9) / (m+7) / (m+5) / (m+3)$

maxima [A] time = 0.44, size = 85, normalized size = 1.12

$$\frac{c^2 x^{m+11}}{m+11} + \frac{2 b c x^{m+9}}{m+9} + \frac{b^2 x^{m+7}}{m+7} + \frac{2 a c x^{m+7}}{m+7} + \frac{2 a b x^{m+5}}{m+5} + \frac{a^2 x^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m(c x^5 + b x^3 + a x)^2, x, \text{algorithm}="maxima")$

[Out] $c^2 x^{(m+11)} / (m+11) + 2 b c x^{(m+9)} / (m+9) + b^2 x^{(m+7)} / (m+7) + 2 a c x^{(m+7)} / (m+7) + 2 a b x^{(m+5)} / (m+5) + a^2 x^{(m+3)} / (m+3)$

mupad [B] time = 2.19, size = 271, normalized size = 3.57

$\frac{a^2 x^m x^3 (m^4 + 32 m^3 + 374 m^2 + 1888 m + 3465)}{m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395} + \frac{c^2 x^m x^{11} (m^4 + 24 m^3 + 206 m^2 + 744 m + 945)}{m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395} + \frac{x^m x^7 (b^2 + 2 a c) (m^4 + 28 m^3 + 274 m^2 + 1092 m + 1485)}{m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395} + \frac{2 a b x^m x^5 (m^4 + 30 m^3 + 320 m^2 + 1410 m + 2079)}{m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395} + \frac{2 b c x^m x^9 (m^4 + 26 m^3 + 236 m^2 + 886 m + 1155)}{m^5 + 35 m^4 + 470 m^3 + 3010 m^2 + 9129 m + 10395}$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^m(a x + b x^3 + c x^5)^2, x)$

[Out] $(a^2 x^m x^3 (1888 m + 374 m^2 + 32 m^3 + m^4 + 3465)) / (9129 m + 3010 m^2 + 470 m^3 + 35 m^4 + m^5 + 10395) + (c^2 x^m x^{11} (744 m + 206 m^2 + 24 m^3 + m^4 + 945)) / (9129 m + 3010 m^2 + 470 m^3 + 35 m^4 + m^5 + 10395) + (x^m x^7 (2 a c + b^2) (1092 m + 274 m^2 + 28 m^3 + m^4 + 1485)) / (9129 m + 3010 m^2 + 470 m^3 + 35 m^4 + m^5 + 10395)$

$$\begin{aligned} &^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (2*a*b*x^m*x^5*(1410*m + 320*m^2 + 3 \\ &0*m^3 + m^4 + 2079))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + \\ &(2*b*c*x^m*x^9*(886*m + 236*m^2 + 26*m^3 + m^4 + 1155))/(9129*m + 3010*m^2 \\ &+ 470*m^3 + 35*m^4 + m^5 + 10395) \end{aligned}$$

sympy [A] time = 4.29, size = 1377, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)

[Out] Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) - b*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a*b/(2*x**4) - a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2, Eq(m, -9)), (-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4, Eq(m, -7)), (-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6, Eq(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 374*a**2*m**2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1888*a**2*m*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a**2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970*a*c*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + b**2*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b**2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 52*b*c*m**3*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 472*b*c*m**2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3

```

010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**11*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 206*c**2*m**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m
**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))

```

$$3.73 \quad \int x^2 (ax + bx^3 + cx^5)^2 dx$$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1585, 1108}

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^2*x^13)/13

Rule 1108

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int x^2 (ax + bx^3 + cx^5)^2 dx &= \int x^4 (a + bx^2 + cx^4)^2 dx \\ &= \int (a^2x^4 + 2abx^6 + (b^2 + 2ac)x^8 + 2bcx^{10} + c^2x^{12}) dx \\ &= \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^2*x^13)/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (ax + bx^3 + cx^5)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^2*(a*x + b*x^3 + c*x^5)^2, x]

fricas [A] time = 1.06, size = 46, normalized size = 0.85

$$\frac{1}{13}x^{13}c^2 + \frac{2}{11}x^{11}cb + \frac{1}{9}x^9b^2 + \frac{2}{9}x^9ca + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/13*x^13*c^2 + 2/11*x^11*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 2/7*x^7*b*a + 1/5*x^5*a^2

giac [A] time = 0.41, size = 46, normalized size = 0.85

$$\frac{1}{13}c^2x^{13} + \frac{2}{11}bcx^{11} + \frac{1}{9}b^2x^9 + \frac{2}{9}acx^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{13}}{13} + \frac{2bcx^{11}}{11} + \frac{2abx^7}{7} + \frac{(2ac + b^2)x^9}{9} + \frac{a^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(c*x^5+b*x^3+a*x)^2,x)`

[Out] $1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^{11}+1/13*c^2*x^{13}$

maxima [A] time = 0.42, size = 44, normalized size = 0.81

$$\frac{1}{13}c^2x^{13} + \frac{2}{11}bcx^{11} + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")`

[Out] $1/13*c^2*x^{13} + 2/11*b*c*x^{11} + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5$

mupad [B] time = 0.03, size = 45, normalized size = 0.83

$$x^9 \left(\frac{b^2}{9} + \frac{2ac}{9} \right) + \frac{a^2x^5}{5} + \frac{c^2x^{13}}{13} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a*x + b*x^3 + c*x^5)^2,x)`

[Out] $x^9*((2*a*c)/9 + b^2/9) + (a^2*x^5)/5 + (c^2*x^{13})/13 + (2*a*b*x^7)/7 + (2*b*c*x^{11})/11$

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11} + \frac{c^2x^{13}}{13} + x^9 \left(\frac{2ac}{9} + \frac{b^2}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)`

[Out] $a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/9 + b**2/9)$

$$3.74 \quad \int x (ax + bx^3 + cx^5)^2 dx$$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4} + \frac{1}{8}x^8(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{5}bcx^{10} + \frac{c^2x^{12}}{12}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1585, 1114, 631}

$$\frac{a^2x^4}{4} + \frac{1}{8}x^8(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{5}bcx^{10} + \frac{c^2x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5 + (c^2*x^12)/12

Rule 631

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || EqQ[a, 0])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
\int x(ax + bx^3 + cx^5)^2 dx &= \int x^3(a + bx^2 + cx^4)^2 dx \\
&= \frac{1}{2} \text{Subst}\left(\int x(a + bx + cx^2)^2 dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int (a^2x + 2abx^2 + (b^2 + 2ac)x^3 + 2bcx^4 + c^2x^5) dx, x, x^2\right) \\
&= \frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{12}}{12}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{120}x^4(30a^2 + 15x^4(2ac + b^2) + 40abx^2 + 24bcx^6 + 10c^2x^8)$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8)/120

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(ax + bx^3 + cx^5)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x*(a*x + b*x^3 + c*x^5)^2, x]

fricas [A] time = 0.78, size = 46, normalized size = 0.85

$$\frac{1}{12}x^{12}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{8}x^8b^2 + \frac{1}{4}x^8ca + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/12*x^12*c^2 + 1/5*x^10*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 1/3*x^6*b*a + 1/4*x^4*a^2

giac [A] time = 0.38, size = 46, normalized size = 0.85

$$\frac{1}{12}c^2x^{12} + \frac{1}{5}bcx^{10} + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{12}}{12} + \frac{bcx^{10}}{5} + \frac{abx^6}{3} + \frac{(2ac + b^2)x^8}{8} + \frac{a^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12

maxima [A] time = 0.45, size = 44, normalized size = 0.81

$$\frac{1}{12}c^2x^{12} + \frac{1}{5}bcx^{10} + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^8 \left(\frac{b^2}{8} + \frac{ac}{4} \right) + \frac{a^2x^4}{4} + \frac{c^2x^{12}}{12} + \frac{abx^6}{3} + \frac{bcx^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x + b*x^3 + c*x^5)^2,x)

[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^4)/4 + (c^2*x^12)/12 + (a*b*x^6)/3 + (b*c*x^10)/5

sympy [A] time = 0.08, size = 46, normalized size = 0.85

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{bcx^{10}}{5} + \frac{c^2x^{12}}{12} + x^8 \left(\frac{ac}{4} + \frac{b^2}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**5+b*x**3+a*x)**2,x)

[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**2/8)

$$3.75 \quad \int (ax + bx^3 + cx^5)^2 dx$$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1594, 1108}

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

Rule 1108

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int (ax + bx^3 + cx^5)^2 dx &= \int x^2 (a + bx^2 + cx^4)^2 dx \\ &= \int (a^2x^2 + 2abx^4 + (b^2 + 2ac)x^6 + 2bcx^8 + c^2x^{10}) dx \\ &= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{1}{7}(b^2 + 2ac)x^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11} \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax + bx^3 + cx^5)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^2, x]

fricas [A] time = 1.03, size = 46, normalized size = 0.85

$$\frac{1}{11}x^{11}c^2 + \frac{2}{9}x^9cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5*b*a + 1/3*x^3*a^2

giac [A] time = 0.36, size = 46, normalized size = 0.85

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2x^{11}}{11} + \frac{2bcx^9}{9} + \frac{2abx^5}{5} + \frac{(2ac + b^2)x^7}{7} + \frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^5+b*x^3+a*x)^2,x)`

[Out] $1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11$

maxima [A] time = 0.43, size = 48, normalized size = 0.89

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{1}{3}a^2x^3 + \frac{2}{35}(5cx^7 + 7bx^5)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")`

[Out] $1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5*c*x^7 + 7*b*x^5)*a$

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^7 \left(\frac{b^2}{7} + \frac{2ac}{7} \right) + \frac{a^2x^3}{3} + \frac{c^2x^{11}}{11} + \frac{2abx^5}{5} + \frac{2bcx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x + b*x^3 + c*x^5)^2,x)`

[Out] $x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*b*c*x^9)/9$

sympy [A] time = 0.07, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7 \left(\frac{2ac}{7} + \frac{b^2}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)**2,x)`

[Out] $a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 + b**2/7)$

$$3.76 \quad \int \frac{(ax+bx^3+cx^5)^2}{x} dx$$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2} + \frac{1}{6}x^6(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1585, 1107, 611}

$$\frac{a^2x^2}{2} + \frac{1}{6}x^6(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^2/x,x]

[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10)/10

Rule 611

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && (EqQ[a, 0] || !PerfectSquareQ[b^2 - 4*a*c])

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3 + cx^5)^2}{x} dx &= \int x(a + bx^2 + cx^4)^2 dx \\
&= \frac{1}{2} \text{Subst} \left(\int (a + bx + cx^2)^2 dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(a^2 + 2abx + b^2 \left(1 + \frac{2ac}{b^2} \right) x^2 + 2bcx^3 + c^2x^4 \right) dx, x, x^2 \right) \\
&= \frac{a^2x^2}{2} + \frac{1}{2}abx^4 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2(30a^2 + 10x^4(2ac + b^2) + 30abx^2 + 15bcx^6 + 6c^2x^8)$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]

[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))/60

IntegrateAlgebraic [A] time = 0.02, size = 50, normalized size = 0.93

$$\frac{1}{60}x^2(30a^2 + 30abx^2 + 20acx^4 + 10b^2x^4 + 15bcx^6 + 6c^2x^8)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^2/x,x]

[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*b^2*x^4 + 20*a*c*x^4 + 15*b*c*x^6 + 6*c^2*x^8))/60

fricas [A] time = 0.95, size = 44, normalized size = 0.81

$$\frac{1}{10}c^2x^{10} + \frac{1}{4}bcx^8 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")

[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2

giac [A] time = 0.52, size = 46, normalized size = 0.85

$$\frac{1}{10} c^2 x^{10} + \frac{1}{4} b c x^8 + \frac{1}{6} b^2 x^6 + \frac{1}{3} a c x^6 + \frac{1}{2} a b x^4 + \frac{1}{2} a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")

[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{c^2 x^{10}}{10} + \frac{b c x^8}{4} + \frac{a b x^4}{2} + \frac{(2 a c + b^2) x^6}{6} + \frac{a^2 x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^2/x,x)

[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10

maxima [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{1}{10} c^2 x^{10} + \frac{1}{4} b c x^8 + \frac{1}{6} (b^2 + 2 a c) x^6 + \frac{1}{2} a b x^4 + \frac{1}{2} a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")

[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2

mupad [B] time = 0.02, size = 45, normalized size = 0.83

$$x^6 \left(\frac{b^2}{6} + \frac{a c}{3} \right) + \frac{a^2 x^2}{2} + \frac{c^2 x^{10}}{10} + \frac{a b x^4}{2} + \frac{b c x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + b*x^3 + c*x^5)^2/x,x)

[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^8)/4

sympy [A] time = 0.08, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6 \left(\frac{ac}{3} + \frac{b^2}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**2/x,x)

[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/6)

$$3.77 \quad \int \frac{(ax+bx^3+cx^5)^2}{x^2} dx$$

Optimal. Leaf size=49

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1585, 1090}

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]

[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{(ax + bx^3 + cx^5)^2}{x^2} dx &= \int (a + bx^2 + cx^4)^2 dx \\ &= \int \left(a^2 + 2abx^2 + b^2 \left(1 + \frac{2ac}{b^2} \right) x^4 + 2bcx^6 + c^2x^8 \right) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]

[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9

IntegrateAlgebraic [A] time = 0.02, size = 53, normalized size = 1.08

$$a^2x + \frac{2}{3}abx^3 + \frac{2}{5}acx^5 + \frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^2/x^2,x]

[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5 + (2*a*c*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9

fricas [A] time = 1.28, size = 41, normalized size = 0.84

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")

[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x

giac [A] time = 0.51, size = 43, normalized size = 0.88

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")

[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x

maple [A] time = 0.00, size = 42, normalized size = 0.86

$$\frac{c^2x^9}{9} + \frac{2bcx^7}{7} + \frac{2abx^3}{3} + \frac{(2ac + b^2)x^5}{5} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^5+b*x^3+a*x)^2/x^2,x)`

[Out] $a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(2ac + b^2)x^5 + \frac{2}{7}bcx^7 + \frac{1}{9}c^2x^9$

maxima [A] time = 0.43, size = 41, normalized size = 0.84

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")`

[Out] $\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{3}abx^3 + a^2x$

mupad [B] time = 0.02, size = 42, normalized size = 0.86

$$a^2x + x^5 \left(\frac{b^2}{5} + \frac{2ac}{5} \right) + \frac{c^2x^9}{9} + \frac{2abx^3}{3} + \frac{2bcx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x + b*x^3 + c*x^5)^2/x^2,x)`

[Out] $a^2x + x^5 \left(\frac{2ac}{5} + \frac{b^2}{5} \right) + \frac{c^2x^9}{9} + \frac{2abx^3}{3} + \frac{2bcx^7}{7}$

sympy [A] time = 0.08, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5 \left(\frac{2ac}{5} + \frac{b^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)**2/x**2,x)`

[Out] $a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5 \left(\frac{2ac}{5} + \frac{b^2}{5} \right)$

$$3.78 \quad \int \frac{x^8}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=100

$$\frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1585, 1114, 701, 634, 618, 206, 628}

$$\frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x + b*x^3 + c*x^5), x]

[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{ax + bx^3 + cx^5} dx &= \int \frac{x^7}{a + bx^2 + cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{ab + (b^2 - ac)x}{c^2(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} + \frac{\text{Subst} \left(\int \frac{ab + (b^2 - ac)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c^2} \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} - \frac{(b(b^2 - 3ac)) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} + \frac{(b^2 - ac) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} + \frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3} + \frac{(b(b^2 - 3ac)) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b \right)}{2c^3} \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} + \frac{b(b^2 - 3ac) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2c^3 \sqrt{b^2 - 4ac}} + \frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.93

$$\frac{-\frac{2b(b^2 - 3ac) \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + (b^2 - ac) \log(a + bx^2 + cx^4) + cx^2 (cx^2 - 2b)}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x + b*x^3 + c*x^5), x]

[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4]/(4*c^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x^8/(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 1.15, size = 313, normalized size = 3.13

$$\frac{(b^2c^2 - 4ac^3)x^4 - 2(b^3c - 4abc^2)x^2 - (b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bx^2 + b^2 - 2ac - (2c^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^4 + bx^2 + a)}{4(b^2c^3 - 4ac^4)} + \frac{(b^2c^2 - 4ac^3)x^4 - 2(b^3c - 4abc^2)x^2 + 2(b^3 - 3abc)\sqrt{-b^2 + 4ac} \arctan\left(\frac{(2c^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) + (b^4 - 5ab^2c + 4a^2c^2) \log(cx^4 + bx^2 + a)}{4(b^2c^3 - 4ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]

giac [A] time = 0.47, size = 92, normalized size = 0.92

$$\frac{cx^4 - 2bx^2}{4c^2} + \frac{(b^2 - ac) \log(cx^4 + bx^2 + a)}{4c^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)

maple [A] time = 0.00, size = 142, normalized size = 1.42

$$\frac{x^4}{4c} + \frac{3ab \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}c^2} - \frac{b^3 \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}c^3} - \frac{bx^2}{2c^2} - \frac{a \ln(cx^4 + bx^2 + a)}{4c^2} + \frac{b^2 \ln(cx^4 + bx^2 + a)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(c*x^5+b*x^3+a*x),x)

[Out] 1/4/c*x^4-1/2*b/c^2*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^4 - 2bx^2}{4c^2} - \int \frac{(b^2 - ac)x^3 + abx}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/c^2

mupad [B] time = 2.20, size = 842, normalized size = 8.42

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a*x + b*x^3 + c*x^5),x)

[Out] $x^4/(4*c) - (\log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*((b*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))))/(8*c^3*(4*a*c - b^2)^{(1/2)}) - (a*b*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^{(1/2)}*(16*a*c^4 - 4*b^2*c^3)))/a - x^2*((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(8*c^3*(4*a*c - b^2)^{(1/2)}) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*c*(4*a*c - b^2)^{(1/2)}*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*c - b^2)))/((2*a*(4*a*c - b^2)^{(1/2)})) + (b*(((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2)))/((2*a*(4*a*c - b^2)^{(1/2)})))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c)*(3*a*c - b^2)/(2*c^3*(4*a*c - b^2)^{(1/2)})$

sympy [B] time = 2.95, size = 391, normalized size = 3.91

$$\frac{bx^2}{2c^2} + \left(\frac{b\sqrt{-4ac + b^2} (3ac - b^2)}{4c^3(4ac - b^2)} - \frac{ac - b^2}{4c^3} \right) \log \left(x^2 + \frac{2a^2c - ab^2 + 8ac^2 \left(\frac{b\sqrt{-4ac + b^2} (3ac - b^2)}{4c^3(4ac - b^2)} - \frac{ac - b^2}{4c^3} \right) - 2b^2c^2 \left(\frac{b\sqrt{-4ac + b^2} (3ac - b^2)}{4c^3(4ac - b^2)} - \frac{ac - b^2}{4c^3} \right)}{3abc - b^3} \right) + \left(\frac{b\sqrt{-4ac + b^2} (3ac - b^2)}{4c^3(4ac - b^2)} - \frac{ac - b^2}{4c^3} \right) \log \left(x^2 + \frac{2a^2c - ab^2 + 8ac^2 \left(\frac{b\sqrt{-4ac + b^2} (3ac - b^2)}{4c^3(4ac - b^2)} - \frac{ac - b^2}{4c^3} \right) - 2b^2c^2 \left(\frac{b\sqrt{-4ac + b^2} (3ac - b^2)}{4c^3(4ac - b^2)} - \frac{ac - b^2}{4c^3} \right)}{3abc - b^3} \right) + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**5+b*x**3+a*x),x)

[Out]
$$\begin{aligned} & -b*x**2/(2*c**2) + (-b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/(4*c**3*(4*a*c - \\ & b**2)) - (a*c - b**2)/(4*c**3))*\log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(- \\ & b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2) \\ & / (4*c**3)) - 2*b**2*c**2*(-b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/(4*c**3*(4* \\ & a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*\sqrt{-4*a*c + \\ & b**2}*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*\log(\\ & x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*\sqrt{-4*a*c + b**2}*(3*a*c - b**2)/ \\ & (4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*\sqrt{-4*a \\ & *c + b**2}*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) \\ & / (3*a*b*c - b**3)) + x**4/(4*c) \end{aligned}$$

$$3.79 \quad \int \frac{x^7}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{bx}{c^2} + \frac{x^3}{3c}$$

Rubi [A] time = 0.60, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{bx}{c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x + b*x^3 + c*x^5), x]

[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c)))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_
))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{ax + bx^3 + cx^5} dx &= \int \frac{x^6}{a + bx^2 + cx^4} dx \\
&= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4} dx}{3c} \\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4} dx}{3c^2} \\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2 - ac - \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c^2} + \frac{\left(b^2 - ac + \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c^2} \\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2 - ac - \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2 - ac + \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 250, normalized size = 1.23

$$\frac{(b^2\sqrt{b^2-4ac} - ac\sqrt{b^2-4ac} + 3abc - b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{(b^2\sqrt{b^2-4ac} - ac\sqrt{b^2-4ac} - 3abc + b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{bx}{c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x + b*x^3 + c*x^5),x]

[Out] $-\frac{(b*x)}{c^2} + \frac{x^3}{3*c} + \frac{((-b^3 + 3*a*b*c + b^2*\text{Sqrt}[b^2 - 4*a*c] - a*c*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]}{(\text{Sqrt}[2]*c^{5/2}*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])} + \frac{((b^3 - 3*a*b*c + b^2*\text{Sqrt}[b^2 - 4*a*c] - a*c*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]}{(\text{Sqrt}[2]*c^{5/2}*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(a*x + b*x^3 + c*x^5),x]

[Out] IntegrateAlgebraic[x^7/(a*x + b*x^3 + c*x^5), x]

fricas [B] time = 1.60, size = 1564, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*c*x^3 - 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^{10} - 4*a*c^{11}))))/(b^2*c^5 - 4*a*c^6))*\log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + \text{sqrt}(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^{10} - 4*a*c^{11}))))*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^{10} - 4*a*c^{11}))))/(b^2*c^5 - 4*a*c^6)) + 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^{10} - 4*a*c^{11}))))/(b^2*c^5 - 4*a*c^6))*\log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - \text{sqrt}(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^{10} - 4*a*c^{11}))))/(b^2*c^5 - 4*a*c^6))$

$$\begin{aligned}
& ^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*\log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + \text{sqrt}(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*\text{sqrt}(1/2)*c^2*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*\log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - \text{sqrt}(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*\text{sqrt}(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
\end{aligned}$$

giac [B] time = 2.04, size = 2457, normalized size = 12.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
& *\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^6*c^2 + 7*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt} \\
& (b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b* \\
& c - \text{sqrt}(b^2 - 4*a*c)*c)*b^5*c^3 - 12*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - s \\
& \text{qrt}(b^2 - 4*a*c)*c)*a*b^3*c^4 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 \\
& - 4*a*c)*c)*b^4*c^4 + 3*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - \\
& 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - \\
& (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - \text{sqrt}(2)*\text{sqrt}(b^2 \\
& - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^6 + 9*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
& *\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c)*c)*a*b^4*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt} \\
& (b*c - \text{sqrt}(b^2 - 4*a*c)*c)*b^5*c - 24*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \\
& \text{sqrt}(b^2 - 4*a*c)*c)*a*b^3*c^2 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt} \\
& (b^2 - 4*a*c)*c)*b^4*c^2 + 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 \\
& - 4*a*c)*c)*a^3*c^3 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4* \\
& a*c)*c)*a^2*b*c^3 + 5*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c
\end{aligned}$$

$$\begin{aligned}
&) * c) * a * b^2 * c^3 - 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c \\
&) * a^2 * c^4 - 2 * (b^2 - 4 * a * c) * b^4 * c^2 + 10 * (b^2 - 4 * a * c) * a * b^2 * c^3 - 8 * (b^2 - \\
& 4 * a * c) * a^2 * c^4) * c^2 - 2 * (\sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a * b^5 * c^2 \\
& - 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^3 * c^3 - 2 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) \\
& * a * b^4 * c^3 + 2 * a * b^5 * c^3 + 16 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b * c^4 + 8 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a \\
& ^2 * b^2 * c^4 + \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a * b^3 * c^4 - 16 * a^2 * b^3 \\
& * c^4 - 4 * \sqrt{2} * \sqrt{b * c - \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b * c^5 + 32 * a^3 * b * c^5 - \\
& 2 * (b^2 - 4 * a * c) * a * b^3 * c^3 + 8 * (b^2 - 4 * a * c) * a^2 * b * c^4) * \text{abs}(c) * \arctan(2 * \sqrt{2} * \\
& \sqrt{1/2} * x / \sqrt{(b * c^3 + \sqrt{b^2 * c^6 - 4 * a * c^7}) / c^4}) / ((a * b^4 * c^4 - 8 * a^2 * \\
& b^2 * c^5 - 2 * a * b^3 * c^5 + 16 * a^3 * c^6 + 8 * a^2 * b * c^6 + a * b^2 * c^6 - 4 * a^2 * c^7) * c \\
& ^2) + 1/8 * (2 * b^6 * c^4 - 14 * a * b^4 * c^5 + 24 * a^2 * b^2 * c^6 - \sqrt{2} * \sqrt{b^2 - 4 * \\
& a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^6 * c^2 + 7 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} \\
& * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^4 * c^3 + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * b^5 * c^3 - 12 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^2 * c^4 - 6 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * a * b^3 * c^4 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^4 * c^4 + 3 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * a * b^2 * c^5 - 2 * (b^2 - 4 * a * c) * b^4 * c^4 + 6 * (b^2 - 4 * a * c) * a * b^2 * \\
& c^5 - (2 * b^6 * c^2 - 18 * a * b^4 * c^3 + 48 * a^2 * b^2 * c^4 - 32 * a^3 * c^5 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^6 \\
& + 9 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^4 * c + 2 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * b^5 * c - 24 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^2 * c^2 - 10 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * a * b^3 * c^2 - \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * b^4 * c^2 + 16 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * a^3 * c^3 + 8 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b * c^3 + 5 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * a * b^2 * c^3 - 4 * \sqrt{2} * \sqrt{b^2 - 4 * a * c} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * c^4 - 2 * (b^2 - 4 * a * c) * b^4 * c^2 + 10 * (b^2 - 4 * a * c) * a * b^2 * c^3 - 8 * (\\
& b^2 - 4 * a * c) * a^2 * c^4) * c^2 + 2 * (\sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^5 * c^2 - 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^3 * c^3 - 2 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) \\
& * a * b^4 * c^3 - 2 * a * b^5 * c^3 + 16 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^3 * b * c^4 + 8 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b^2 * c^4 + \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a * b^3 * c^4 + 16 * a^2 * b^3 * c^4 - 4 * \sqrt{2} * \sqrt{b * c + \sqrt{b^2 - 4 * a * c}} * c) * a^2 * b * c^5 - 32 * a^3 * b * c^5 + 2 * (b^2 - 4 * a * c) * a * b^3 * c^3 - 8 * (b^2 - 4 * a * c) * a^2 * b * c^4) * \text{abs}(c) * \arctan(2 * \sqrt{2} * \sqrt{1/2} * x / \sqrt{(b * c^3 - \sqrt{b^2 * c^6 - 4 * a * c^7}) / c^4}) / ((a * b^4 * c^4 - 8 * a^2 * b^2 * c^5 - 2 * a * b^3 * c^5 + 16 * a^3 * c^6 + 8 * a^2 * b * c^6 + a * b^2 * c^6 - 4 * a^2 * c^7) * c^2) + 1/3 * (c^2 * x^3 - 3 * b * c * x) / c^3
\end{aligned}$$

maple [B] time = 0.02, size = 467, normalized size = 2.30

$$\frac{3\sqrt{2} ab \operatorname{arctanh}\left(\frac{\sqrt{2} \alpha}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} - \frac{3\sqrt{2} ab \operatorname{arctan}\left(\frac{\sqrt{2} \alpha}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} b^3 \operatorname{arctanh}\left(\frac{\sqrt{2} \alpha}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} b^3 \operatorname{arctan}\left(\frac{\sqrt{2} \alpha}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} a \operatorname{arctanh}\left(\frac{\sqrt{2} \alpha}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} a \operatorname{arctan}\left(\frac{\sqrt{2} \alpha}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} b^2 \operatorname{arctanh}\left(\frac{\sqrt{2} \alpha}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} b^2 \operatorname{arctan}\left(\frac{\sqrt{2} \alpha}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{bx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(c*x^5+b*x^3+a*x),x)`

[Out] $\frac{1}{3} \frac{x^3 - b/c^2 x + 1/2 c^2 (1/2) / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) a - 1/2 c^2 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) b^2 - 3/2 c / (-4ac + b^2)^{1/2} 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) a b + 1/2 c^2 / (-4ac + b^2)^{1/2} 2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctanh}(2^{1/2} / ((-b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) b^3 - 1/2 c^2 (1/2) / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) a + 1/2 c^2 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) b^2 - 3/2 c / (-4ac + b^2)^{1/2} 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) a b + 1/2 c^2 / (-4ac + b^2)^{1/2} 2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} \operatorname{arctan}(2^{1/2} / ((b + (-4ac + b^2)^{1/2}) c)^{1/2} c x) b^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^3 - 3bx}{3c^2} - \frac{-\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] $\frac{1}{3} \frac{(c x^3 - 3 b x)}{c^2} - \frac{\operatorname{integrate}(-((b^2 - a c) x^2 + a b) / (c x^4 + b x^2 + a), x)}{c^2}$

mupad [B] time = 2.72, size = 4127, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(a*x + b*x^3 + c*x^5),x)`

[Out] $x^3 / (3c) - \operatorname{atan}\left(\frac{((4ab^3c^3 - 16a^2b^2c^4) / c^3 - (2x(4b^3c^5 - 16ab^2c^6) * (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2}}{(8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2}}\right) / c^3 * (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} - (2x(b^6 - 2a^3c^3 + 9a^2b^2c^2 - 6ab^4c)) / c^3 * (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c -$

$$\begin{aligned}
& 3ab^2c \cdot (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * i - (((4ab^3c^3 - 16a^2b^2c^4)/c^3 + (2x(4b^3c^5 - 16ab^2c^6) \\
& * (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (2x(b^6 - 2a^3c^3 + 9a^2b^2c^2 - 6ab^4c)) / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * i) / (((4ab^3c^3 - 16a^2b^2c^4)/c^3 - (2x(4b^3c^5 - 16ab^2c^6) * (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} - (2x(b^6 - 2a^3c^3 + 9a^2b^2c^2 - 6ab^4c)) / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (((4ab^3c^3 - 16a^2b^2c^4)/c^3 + (2x(4b^3c^5 - 16ab^2c^6) * (-b^7 + b^4(-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (2x(b^6 - 2a^3c^3 + 9a^2b^2c^2 - 6ab^4c)) / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} + (2(a^4c - a^3b^2)) / c^3 * (-b^7 + b^4 * (-4ac - b^2)^3)^{1/2} - 20a^3b^2c^3 + 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} - 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * 2i - \operatorname{atan}((((4ab^3c^3 - 16a^2b^2c^4)/c^3 - (2x(4b^3c^5 - 16ab^2c^6) * (b^4 * (-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^2c^3 - 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} / c^3 * ((b^4 * (-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^2c^3 - 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} - (2x(b^6 - 2a^3c^3 + 9a^2b^2c^2 - 6ab^4c)) / c^3 * ((b^4 * (-4ac - b^2)^3)^{1/2} - b^7 + 20a^3b^2c^3 - 25a^2b^3c^2 + a^2c^2 * (-4ac - b^2)^3)^{1/2} + 9ab^5c - 3ab^2c * (-4ac - b^2)^3)^{1/2} / (8(16a^2c^7 + b^4c^5 - 8ab^2c^6))^{1/2} * i - (((4ab^3c^3 - 16a^2b^2c^4)/c^3 + (2x(4b^3c^5 - 16ab^2c^6) * ((
\end{aligned}$$

$$\begin{aligned}
& b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)} \\
&) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} + (2 * x * (b^6 - 2 * a^3 * c^3 + 9 * a^2 * b^2 * c^2 - 6 * a * b^4 * c)) / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} * i) / ((4 * a * b^3 * c^3 - 16 * a^2 * b * c^4) / c^3 - (2 * x * (4 * b^3 * c^5 - 16 * a * b * c^6)) * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)}) / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} - (2 * x * (b^6 - 2 * a^3 * c^3 + 9 * a^2 * b^2 * c^2 - 6 * a * b^4 * c)) / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} + ((4 * a * b^3 * c^3 - 16 * a^2 * b * c^4) / c^3 + (2 * x * (4 * b^3 * c^5 - 16 * a * b * c^6)) * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)}) / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} + (2 * x * (b^6 - 2 * a^3 * c^3 + 9 * a^2 * b^2 * c^2 - 6 * a * b^4 * c)) / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} + (2 * (a^4 * c - a^3 * b^2)) / c^3 * ((b^4 * (- (4 * a * c - b^2)^3)^{(1/2)} - b^7 + 20 * a^3 * b * c^3 - 25 * a^2 * b^3 * c^2 + a^2 * c^7 \\
& 2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 9 * a * b^5 * c - 3 * a * b^2 * c * (- (4 * a * c - b^2)^3)^{(1/2)}) / (8 * (16 * a^2 * c^7 + b^4 * c^5 - 8 * a * b^2 * c^6))^{(1/2)} * 2i - (b * x) / c^2
\end{aligned}$$

sympy [A] time = 4.25, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \text{RootSum}\left(t^4(256a^2c^7 - 128ab^2c^6 + 16b^4c^5) + t^2(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7) + a^5, \left(t \mapsto t \log\left(x + \frac{-64t^3a^2c^7 + 48t^3ab^2c^6 - 8t^3b^4c^5 + 14ta^3bc^3 - 28ta^2b^3c^2 + 14tab^5c - 2tb^7}{a^4c^2 - 3a^3b^2c + a^2b^4}\right)\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**5+b*x**3+a*x), x)

[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5) + _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c - 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)

$$3.80 \quad \int \frac{x^6}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^2 + cx^4)}{4c^2} + \frac{x^2}{2c}$$

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1585, 1114, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^2 + cx^4)}{4c^2} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x + b*x^3 + c*x^5), x]

[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2 *Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[(d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]]/(a + b*x + c*x^2), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1114

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^6}{ax + bx^3 + cx^5} dx &= \int \frac{x^5}{a + bx^2 + cx^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{a + bx + cx^2} dx, x, x^2 \right) \\
 &= \frac{x^2}{2c} + \frac{\text{Subst} \left(\int \frac{-a - bx}{a + bx + cx^2} dx, x, x^2 \right)}{2c} \\
 &= \frac{x^2}{2c} - \frac{b \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^2} + \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4c^2} \\
 &= \frac{x^2}{2c} - \frac{b \log(a + bx^2 + cx^4)}{4c^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2c^2} \\
 &= \frac{x^2}{2c} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx^2 + cx^4)}{4c^2}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.96

$$\frac{2(b^2-2ac) \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right) - b \log(a + bx^2 + cx^4) + 2cx^2}{4c^2 \sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x + b*x^3 + c*x^5), x]

[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x^6/(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 1.46, size = 254, normalized size = 3.14

$$\left[\frac{2(b^2c - 4ac^2)x^2 - (b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (b^3 - 4abc) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3)}, \frac{2(b^2c - 4ac^2)x^2 - 2(b^2 - 2ac)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^3 - 4abc) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x), x, algorithm="fricas")

[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]

giac [A] time = 0.63, size = 75, normalized size = 0.93

$$\frac{x^2}{2c} - \frac{b \log(cx^4 + bx^2 + a)}{4c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $\frac{1}{2}x^2/c - \frac{1}{4}b \log(cx^4 + bx^2 + a)/c^2 + \frac{1}{2}(b^2 - 2ac) \arctan((2cx^2 + b)/\sqrt{-b^2 + 4ac})/(\sqrt{-b^2 + 4ac}c^2)$

maple [A] time = 0.00, size = 111, normalized size = 1.37

$$-\frac{a \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{b^2 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c^2} + \frac{x^2}{2c} - \frac{b \ln(cx^4 + bx^2 + a)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^5+b*x^3+a*x),x)

[Out] $\frac{1}{2}/cx^2 - \frac{1}{4}b \ln(cx^4 + bx^2 + a)/c^2 - \frac{1}{c}/(4ac - b^2)^{(1/2)} \arctan((2cx^2 + b)/(4ac - b^2)^{(1/2)}) + \frac{1}{2}/c^2/(4ac - b^2)^{(1/2)} \arctan((2cx^2 + b)/(4ac - b^2)^{(1/2)}) * b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2c} - \frac{b \log(cx^4 + bx^2 + a)}{4c} - \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2/c - \text{integrate}((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c$

mupad [B] time = 2.44, size = 655, normalized size = 8.09

$$\frac{\frac{x^2}{2c} + \frac{\ln(cx^4 + bx^2 + a)(2b^3 - 8abc)}{2(16ac^3 - 4b^2c^2)} + \frac{\operatorname{atan}\left(\frac{\frac{\frac{8ab^2(2b^3 - 8abc)}{16c^3 - 4b^2c^2} \sqrt{2c - b^2}}{\frac{8b^2(2b^3 - 8abc)}{16c^3 - 4b^2c^2} \sqrt{4ac - b^2}}}{\frac{2(2ac - b^2)}{8c^2 \sqrt{4ac - b^2}}}, \frac{\frac{2(2ac - b^2)}{8c^2 \sqrt{4ac - b^2}}}{\frac{2(2ac - b^2)}{8c^2 \sqrt{4ac - b^2}}}, \frac{\frac{2(2b^3 - 8abc)}{16c^3 - 4b^2c^2} \sqrt{2c - b^2}}{\frac{2(2b^3 - 8abc)}{16c^3 - 4b^2c^2} \sqrt{4ac - b^2}}}, \frac{\frac{2(2b^3 - 8abc)}{16c^3 - 4b^2c^2} \sqrt{2c - b^2}}{\frac{2(2b^3 - 8abc)}{16c^3 - 4b^2c^2} \sqrt{4ac - b^2}}}\right)}{4b^2c^2 - 4ab^2c + b^4}}{(2ac - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a*x + b*x^3 + c*x^5),x)

[Out] $\frac{x^2}{2c} + \frac{(\log(a + b*x^2 + c*x^4) * (2*b^3 - 8*a*b*c)) / (2 * (16*a*c^3 - 4*b^2*c^2)) + (\operatorname{atan}((2*c^2 * (4*a*c - b^2) * (((8*a*b + (8*a*c^2 * (2*b^3 - 8*a*b*c)) / (16*a*c^3 - 4*b^2*c^2)) * (2*a*c - b^2)) / (8*c^2 * (4*a*c - b^2)^{(1/2)})) + (a * (2*b^3 - 8*a*b*c) * (2*a*c - b^2)) / ((4*a*c - b^2)^{(1/2)} * (16*a*c^3 - 4*b^2*c^2)))$

$$\begin{aligned} &)/a - x^2 * (((2*a*c - b^2) * ((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2 * (2*b^3 - 8*a*b*c)) / (16*a*c^3 - 4*b^2*c^2))) / (8*c^2 * (4*a*c - b^2)^{(1/2)}) - (b * (2*b^3 - 8*a*b*c) * (2*a*c - b^2)) / (2 * (4*a*c - b^2)^{(1/2)} * (16*a*c^3 - 4*b^2*c^2))) / a \\ & + (b * (((2*b^3 - 8*a*b*c) * ((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2 * (2*b^3 - 8*a*b*c)) / (16*a*c^3 - 4*b^2*c^2))) / (2 * (16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c) / c^2 + (b * (2*a*c - b^2)^2) / (2 * c^2 * (4*a*c - b^2)))) / (2 * a * (4*a*c - b^2)^{(1/2)}) \\ &) + (b * ((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c) * (8*a*b + (8*a*c^2 * (2*b^3 - 8*a*b*c)) / (16*a*c^3 - 4*b^2*c^2))) / (2 * (16*a*c^3 - 4*b^2*c^2)) - (a * (2*a*c - b^2)^2) / (c^2 * (4*a*c - b^2)))) / (2 * a * (4*a*c - b^2)^{(1/2)})) / (b^4 + 4*a^2*c^2 - 4*a*b^2*c) * (2*a*c - b^2) / (2 * c^2 * (4*a*c - b^2)^{(1/2)}) \end{aligned}$$

sympy [B] time = 1.95, size = 316, normalized size = 3.90

$$\left(\frac{-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4c^2(4ac - b^2)}}{\log\left(x^2 + \frac{-ab - 8ac^2\left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4c^2(4ac - b^2)}\right) + 2b^2c\left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4c^2(4ac - b^2)}\right)}{2ac - b^2}\right)} \right) + \left(\frac{-\frac{b}{4c^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4c^2(4ac - b^2)}}{\log\left(x^2 + \frac{-ab - 8ac^2\left(-\frac{b}{4c^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4c^2(4ac - b^2)}\right) + 2b^2c\left(-\frac{b}{4c^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4c^2(4ac - b^2)}\right)}{2ac - b^2}\right)} \right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(c*x**5+b*x**3+a*x), x)

[Out] $(-b/(4*c**2) - \text{sqrt}(-4*a*c + b**2) * (2*a*c - b**2) / (4*c**2 * (4*a*c - b**2))) * \log(x**2 + (-a*b - 8*a*c**2 * (-b/(4*c**2) - \text{sqrt}(-4*a*c + b**2) * (2*a*c - b**2) / (4*c**2 * (4*a*c - b**2)))) + 2*b**2*c * (-b/(4*c**2) - \text{sqrt}(-4*a*c + b**2) * (2*a*c - b**2) / (4*c**2 * (4*a*c - b**2)))) / (2*a*c - b**2)) + (-b/(4*c**2) + \text{sqrt}(-4*a*c + b**2) * (2*a*c - b**2) / (4*c**2 * (4*a*c - b**2))) * \log(x**2 + (-a*b - 8*a*c**2 * (-b/(4*c**2) + \text{sqrt}(-4*a*c + b**2) * (2*a*c - b**2) / (4*c**2 * (4*a*c - b**2)))) + 2*b**2*c * (-b/(4*c**2) + \text{sqrt}(-4*a*c + b**2) * (2*a*c - b**2) / (4*c**2 * (4*a*c - b**2)))) / (2*a*c - b**2)) + x**2 / (2*c)$

$$3.81 \quad \int \frac{x^5}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=179

$$-\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Rubi [A] time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1122, 1166, 205}

$$-\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x + b*x^3 + c*x^5), x]

[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1122

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{ax + bx^3 + cx^5} dx &= \int \frac{x^4}{a + bx^2 + cx^4} dx \\ &= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4} dx}{c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 202, normalized size = 1.13

$$\frac{\left(b\sqrt{b^2-4ac} + 2ac - b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac} - 2ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x + b*x^3 + c*x^5), x]

[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00


$$\int \frac{x^5}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(a*x + b*x^3 + c*x^5),x]

[Out] IntegrateAlgebraic[x^5/(a*x + b*x^3 + c*x^5), x]

fricas [B] time = 1.27, size = 1059, normalized size = 5.92



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/2*(\sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)*\log(-2*(a*b^2 - a^2*c)*x + \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))}*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - \sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)*\log(-2*(a*b^2 - a^2*c)*x - \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))}*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) + \sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)*\log(-2*(a*b^2 - a^2*c)*x + \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))}*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - \sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)*\log(-2*(a*b^2 - a^2*c)*x - \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))}*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - 2*x)/c \end{aligned}$$

giac [B] time = 1.50, size = 2109, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*\sqrt{1/2}*x/\sqrt{(b*c + \sqrt{b^2*c^2 - 4*a*c^3})/c^2}))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c^3 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 - 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^4 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5 + 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 8*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^2 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)$

$$*a^3*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*\text{abs}(c))*\arctan(2*\sqrt{1/2}*x/\sqrt{(b*c - \sqrt{b^2*c^2 - 4*a*c^3})/c^2}))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)$$

maple [B] time = 0.01, size = 343, normalized size = 1.92

$$\frac{\sqrt{2} a \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2} \sqrt{(b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} a \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{\sqrt{-4ac+b^2} \sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} b^2 \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2} \sqrt{(b+\sqrt{-4ac+b^2})c} c} - \frac{\sqrt{2} b^2 \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{-4ac+b^2} \sqrt{(b+\sqrt{-4ac+b^2})c} c} + \frac{\sqrt{2} b \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(b+\sqrt{-4ac+b^2})c} c} - \frac{\sqrt{2} b \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2\sqrt{(b+\sqrt{-4ac+b^2})c} c} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^5+b*x^3+a*x), x)

[Out] $\frac{1}{c*x} + \frac{1}{2*c^2} \sqrt{\frac{1}{(-b+(-4*a*c+b^2)^{(1/2)})c}} \sqrt{\frac{1}{(-b+(-4*a*c+b^2)^{(1/2)})c}} \operatorname{arctanh}\left(\frac{2^{(1/2)}}{(-b+(-4*a*c+b^2)^{(1/2)})c} * x\right) * b + \frac{1}{(-4*a*c+b^2)^{(1/2)} * 2^{(1/2)}} \sqrt{\frac{1}{(-b+(-4*a*c+b^2)^{(1/2)})c}} \sqrt{\frac{1}{(-b+(-4*a*c+b^2)^{(1/2)})c}} \operatorname{arctanh}\left(\frac{2^{(1/2)}}{(-b+(-4*a*c+b^2)^{(1/2)})c} * x\right) * a - \frac{1}{2*c} \sqrt{\frac{1}{(-b+(-4*a*c+b^2)^{(1/2)})c}} \sqrt{\frac{1}{(-b+(-4*a*c+b^2)^{(1/2)})c}} \operatorname{arctanh}\left(\frac{2^{(1/2)}}{(-b+(-4*a*c+b^2)^{(1/2)})c} * x\right) * b^2 - \frac{1}{2*c^2} \sqrt{\frac{1}{(b+(-4*a*c+b^2)^{(1/2)})c}} \sqrt{\frac{1}{(b+(-4*a*c+b^2)^{(1/2)})c}} \operatorname{arctan}\left(\frac{2^{(1/2)}}{(b+(-4*a*c+b^2)^{(1/2)})c} * x\right) * b + \frac{1}{(-4*a*c+b^2)^{(1/2)} * 2^{(1/2)}} \sqrt{\frac{1}{(b+(-4*a*c+b^2)^{(1/2)})c}} \sqrt{\frac{1}{(b+(-4*a*c+b^2)^{(1/2)})c}} \operatorname{arctan}\left(\frac{2^{(1/2)}}{(b+(-4*a*c+b^2)^{(1/2)})c} * x\right) * a - \frac{1}{2*c} \sqrt{\frac{1}{(b+(-4*a*c+b^2)^{(1/2)})c}} \sqrt{\frac{1}{(b+(-4*a*c+b^2)^{(1/2)})c}} \operatorname{arctan}\left(\frac{2^{(1/2)}}{(b+(-4*a*c+b^2)^{(1/2)})c} * x\right) * b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x), x, algorithm="maxima")

[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c

mupad [B] time = 2.58, size = 3026, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*x + b*x^3 + c*x^5), x)

[Out] $\frac{x}{c} - \operatorname{atan}\left(\frac{((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4) * (-b^5 + b^2*(-4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-4*a*c - b^2)^3)^{(1/2)}}{(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))^{(1/2)}}\right)/c * ($

$$\begin{aligned}
& - (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} - (2x \\
& * (b^4 + 2a^2c^2 - 4ab^2c) / c) * (- (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b \\
& ^4c^3 - 8ab^2c^4))^{1/2} * 1i - (((16a^2c^3 - 4ab^2c^2) / c + (2x * (4 \\
& * b^3c^3 - 16ab^3c^4) * (- (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 \\
& - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}) / c) * (- (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 \\
& - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (2x * (b^4 + 2a^2c^2 - 4ab^2c) / c) * (- (b^5 + b^2(-4 \\
& * ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} * 1i) / (((16a^2c^3 - 4 \\
& * ab^2c^2) / c - (2x * (4b^3c^3 - 16ab^3c^4) * (- (b^5 + b^2(-4ac - b^2)^ \\
& 3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}) / c) * (- (b^5 + b^2(-4ac - b^2)^3 \\
&)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} - (2x * (b^4 + 2a^2c^2 - 4ab^2c \\
&)) / c) * (- (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} \\
& + (((16a^2c^3 - 4ab^2c^2) / c + (2x * (4b^3c^3 - 16ab^3c^4) * (- (b^5 + \\
& b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}) / c) * (- (b^5 + b \\
& ^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (2x * (b^4 + 2 \\
& * a^2c^2 - 4ab^2c) / c) * (- (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - \\
& 8ab^2c^4))^{1/2} + (2a^2b) / c) * (- (b^5 + b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c - a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + \\
& b^4c^3 - 8ab^2c^4))^{1/2} * 2i - \operatorname{atan}((((16a^2c^3 - 4ab^2c^2) / c - \\
& (2x * (4b^3c^3 - 16ab^3c^4) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}) / c) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2 \\
& 2b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} - (2x * (b^4 + 2a^2c^2 - 4ab^2c) / c) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2 \\
& b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} * 1i - (((16a^2c^3 - 4ab^2c^2) / c + (2x * (4b^3c^3 - 16ab^3c^4) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2 \\
& - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}) / c) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2 \\
& b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (2x * (b^4 + 2a^2c^2 - 4 \\
& * ab^2c) / c) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4) \\
&))^{1/2} * 1i) / (((16a^2c^3 - 4ab^2c^2) / c - (2x * (4b^3c^3 - 16ab^3c^4) \\
&) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}) / c) * (- (b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2 - 7ab^3c + a(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2}
\end{aligned}$$

$$\begin{aligned} & ((4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} / c \\ & * (-b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2bc^2 - 7ab^3c + ac(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} - (2 \\ & * x(b^4 + 2a^2c^2 - 4ab^2c)) / c * (-b^5 - b^2(-4ac - b^2)^3)^{1/2} \\ & + 12a^2bc^2 - 7ab^3c + ac(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (((16a^2c^3 - 4ab^2c^2) / c + (2x(4 \\ & b^3c^3 - 16ab^2c^4) * (-b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2bc^2 \\ & - 7ab^3c + ac(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} / c * (-b^5 - b^2(-4ac - b^2)^3)^{1/2} + 12a^2bc^2 - \\ & 7ab^3c + ac(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (2x(b^4 + 2a^2c^2 - 4ab^2c)) / c * (-b^5 - b^2(-4ac - b^2)^3)^{1/2} \\ & + 12a^2bc^2 - 7ab^3c + ac(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} + (2a^2b) / c) * (-b^5 - b^2(-4ac - b^2)^3)^{1/2} \\ & + 12a^2bc^2 - 7ab^3c + ac(-4ac - b^2)^3)^{1/2} / (8(16a^2c^5 + b^4c^3 - 8ab^2c^4))^{1/2} * 2i \end{aligned}$$

sympy [A] time = 2.18, size = 129, normalized size = 0.72

$$\text{RootSum}\left(t^4(256a^2c^5 - 128ab^2c^4 + 16b^4c^3) + t^2(48a^2bc^2 - 28ab^3c + 4b^5) + a^3, \left(t \mapsto t \log\left(x + \frac{32t^3abc^4 - 8t^3b^3c^3 - 4ta^2c^2 + 8tab^2c - 2tb^4}{a^2c - ab^2}\right)\right)\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**5+b*x**3+a*x), x)

[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)/(a**2*c - a*b**2)))) + x/c

$$3.82 \quad \int \frac{x^4}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)}{4c}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1585, 1114, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)}{4c}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x + b*x^3 + c*x^5),x]

[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*c*Sqrt[b^2 - 4*a*c])) + Log[a + b*x^2 + c*x^4]/(4*c)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1114

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \ :> \ \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x + c*x^2)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 1585

$\text{Int}[(u_.)*(x_)^{(m_.)}*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(n_.)}, x_Symbol] \ :> \ \text{Int}[u*x^{(m + n*p)}*(a + b*x^{(q - p)} + c*x^{(r - p)})^n, x] /; \text{FreeQ}[\{a, b, c, m, p, q, r\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p] \ \&\& \ \text{PosQ}[r - p]$

Rubi steps

$$\begin{aligned} \int \frac{x^4}{ax + bx^3 + cx^5} dx &= \int \frac{x^3}{a + bx^2 + cx^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{a + bx + cx^2} dx, x, x^2 \right) \\ &= \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4c} \\ &= \frac{\log(a + bx^2 + cx^4)}{4c} + \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2c} \\ &= \frac{b \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2c\sqrt{b^2-4ac}} + \frac{\log(a + bx^2 + cx^4)}{4c} \end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.98

$$\frac{\log(a + bx^2 + cx^4) - \frac{2b \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x + b*x^3 + c*x^5), x]

[Out] $((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + b*x^2 + c*x^4])/(4*c)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a*x + b*x^3 + c*x^5),x]

[Out] IntegrateAlgebraic[x^4/(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 1.42, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) + (b^2 - 4ac) \log(cx^4 + bx^2 + a)}{4(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) + (b^2 - 4ac) \log(cx^4 + bx^2 + a)}{4(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $[1/4*(\sqrt{b^2 - 4ac})*b*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*\sqrt{b^2 - 4ac}))/ (c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*\log(c*x^4 + b*x^2 + a))/ (b^2*c - 4*a*c^2), 1/4*(2*\sqrt{-b^2 + 4ac})*b*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4ac})/ (b^2 - 4*a*c)) + (b^2 - 4*a*c)*\log(c*x^4 + b*x^2 + a))/ (b^2*c - 4*a*c^2)]$

giac [A] time = 0.42, size = 59, normalized size = 0.94

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}c} + \frac{\log(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $-1/2*b*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*c) + 1/4*\log(c*x^4 + b*x^2 + a)/c$

maple [A] time = 0.00, size = 60, normalized size = 0.95

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}c} + \frac{\ln(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(c*x^5+b*x^3+a*x),x)`

[Out] $\frac{1}{4} \ln(c x^4 + b x^2 + a) / c - \frac{1}{2} b / c / (4 a^2 c - b^2)^{1/2} \arctan((2 c x^2 + b) / (4 a^2 c - b^2)^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{c x^5 + b x^3 + a x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] `integrate(x^4/(c*x^5 + b*x^3 + a*x), x)`

mupad [B] time = 0.17, size = 118, normalized size = 1.87

$$\frac{4 a c \ln(c x^4 + b x^2 + a)}{16 a c^2 - 4 b^2 c} - \frac{b^2 \ln(c x^4 + b x^2 + a)}{16 a c^2 - 4 b^2 c} - \frac{b \operatorname{atan}\left(\frac{b}{\sqrt{4 a c - b^2}} + \frac{2 c x^2}{\sqrt{4 a c - b^2}}\right)}{2 c \sqrt{4 a c - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(a*x + b*x^3 + c*x^5),x)`

[Out] $(4 a^2 c \log(a + b x^2 + c x^4) / (16 a^2 c^2 - 4 b^2 c) - (b^2 \log(a + b x^2 + c x^4) / (16 a^2 c^2 - 4 b^2 c) - (b \operatorname{atan}(b / (4 a^2 c - b^2)^{1/2} + (2 c x^2) / (4 a^2 c - b^2)^{1/2})) / (2 c (4 a^2 c - b^2)^{1/2}))) / (2 c (4 a^2 c - b^2)^{1/2})$

sympy [B] time = 0.92, size = 223, normalized size = 3.54

$$\left(\frac{b \sqrt{-4 a c + b^2}}{4 c (4 a c - b^2)} + \frac{1}{4 c} \right) \log \left(x^2 + \frac{-8 a c \left(-\frac{b \sqrt{-4 a c + b^2}}{4 c (4 a c - b^2)} + \frac{1}{4 c} \right) + 2 a + 2 b^2 \left(-\frac{b \sqrt{-4 a c + b^2}}{4 c (4 a c - b^2)} + \frac{1}{4 c} \right)}{b} \right) + \left(\frac{b \sqrt{-4 a c + b^2}}{4 c (4 a c - b^2)} + \frac{1}{4 c} \right) \log \left(x^2 + \frac{-8 a c \left(\frac{b \sqrt{-4 a c + b^2}}{4 c (4 a c - b^2)} + \frac{1}{4 c} \right) + 2 a + 2 b^2 \left(\frac{b \sqrt{-4 a c + b^2}}{4 c (4 a c - b^2)} + \frac{1}{4 c} \right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(c*x**5+b*x**3+a*x),x)`

[Out] $(-b \sqrt{-4 a c + b^2} / (4 c (4 a^2 c - b^2)) + 1 / (4 c)) \log(x^2 + (-8 a^2 c (-b \sqrt{-4 a c + b^2} / (4 c (4 a^2 c - b^2)) + 1 / (4 c)) + 2 a + 2 b^2 (-b \sqrt{-4 a c + b^2} / (4 c (4 a^2 c - b^2)) + 1 / (4 c))) / b) + (b \sqrt{-4 a c + b^2} / (4 c (4 a^2 c - b^2)) + 1 / (4 c)) \log(x^2 + (-8 a^2 c (b \sqrt{-4 a c + b^2} / (4 c (4 a^2 c - b^2)) + 1 / (4 c)) + 2 a + 2 b^2 (b \sqrt{-4 a c + b^2} / (4 c (4 a^2 c - b^2)) + 1 / (4 c))) / b)$

$$3.83 \quad \int \frac{x^3}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1585, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x + b*x^3 + c*x^5),x]

[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1130

Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(-n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^(-n), x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos

Q[r - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{ax + bx^3 + cx^5} dx &= \int \frac{x^2}{a + bx^2 + cx^4} dx \\
&= -\left(\frac{1}{2} \left(-1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx\right) + \frac{1}{2} \left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx \\
&= -\frac{\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} + \frac{\sqrt{b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 165, normalized size = 1.10

$$\frac{\left(\sqrt{b^2 - 4ac} - b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{b^2 - 4ac} + b \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x + b*x^3 + c*x^5), x]

[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x^3/(a*x + b*x^3 + c*x^5), x]

fricas [B] time = 1.39, size = 559, normalized size = 3.73

$$\frac{1}{2}\sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log\left(\frac{\sqrt{\frac{1}{2}(b^2 - 4ac^2)}\sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^3}} + x\right) - \frac{1}{2}\sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log\left(\frac{\sqrt{\frac{1}{2}(b^2 - 4ac^2)}\sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^3}} + x\right) - \frac{1}{2}\sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log\left(\frac{\sqrt{\frac{1}{2}(b^2 - 4ac^2)}\sqrt{\frac{b - \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^3}} + x\right) + \frac{1}{2}\sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}} \log\left(\frac{\sqrt{\frac{1}{2}(b^2 - 4ac^2)}\sqrt{\frac{b + \sqrt{b^2 - 4ac}}{b^2 - 4ac^2}}}{\sqrt{b^2 - 4ac^3}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")`

[Out] $\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\left(b + \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)*\log\left(\sqrt{\frac{1}{2}}*\left(b^2c - 4ac^2\right)*\sqrt{-\left(b + \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)} + x - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\left(b + \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)*\log\left(-\sqrt{\frac{1}{2}}*\left(b^2c - 4ac^2\right)*\sqrt{-\left(b + \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)} + x - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\left(b - \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)*\log\left(\sqrt{\frac{1}{2}}*\left(b^2c - 4ac^2\right)*\sqrt{-\left(b - \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)} + x + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\left(b - \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)*\log\left(-\sqrt{\frac{1}{2}}*\left(b^2c - 4ac^2\right)*\sqrt{-\left(b - \left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)}}/\left(b^2c - 4ac^2\right)\right)/\sqrt{\left(b^2c^2 - 4ac^3\right)} + x$

giac [B] time = 1.81, size = 503, normalized size = 3.35

$$\frac{\left(2b^2 - 8ac^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}}c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}}c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}c - 2(b^2 - 4ac)^2\right)\arctan\left(\frac{\sqrt{2}cx}{\sqrt{-b + \sqrt{-4ac + b^2}}c}\right) + \left(2b^2 - 8ac^2 - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}}c^2 + 4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}c^2 + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}}c - \sqrt{2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}c - 2(b^2 - 4ac)^2\right)\arctan\left(\frac{\sqrt{2}cx}{\sqrt{-b + \sqrt{-4ac + b^2}}c}\right)}{2(b^2 - 8ac^2 - 2b^2c + 16ac^2 + 8ab^2 + b^2c^2 - 4ac^3)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")`

[Out] $-\frac{1}{2}*(2*b^2*c^2 - 8*a*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c^2 - 2*(b^2 - 4*a*c)*c^2*\arctan(2*\sqrt{\frac{1}{2}}*x/\sqrt{(b + \sqrt{b^2 - 4*a*c})/c})/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*\text{abs}(c)) + \frac{1}{2}*(2*b^2*c^2 - 8*a*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c*b^2 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c^2 - 2*(b^2 - 4*a*c)*c^2*\arctan(2*\sqrt{\frac{1}{2}}*x/\sqrt{(b - \sqrt{b^2 - 4*a*c})/c})/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*\text{abs}(c))$

maple [A] time = 0.01, size = 208, normalized size = 1.39

$$\frac{\sqrt{2} b \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{-b + \sqrt{-4ac + b^2}} c}\right)}{2\sqrt{-4ac + b^2} \sqrt{-b + \sqrt{-4ac + b^2}} c} + \frac{\sqrt{2} b \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b + \sqrt{-4ac + b^2}} c}\right)}{2\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2}} c}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{-b + \sqrt{-4ac + b^2}} c}\right)}{2\sqrt{-b + \sqrt{-4ac + b^2}} c} + \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b + \sqrt{-4ac + b^2}} c}\right)}{2\sqrt{(b + \sqrt{-4ac + b^2}} c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3/(c*x^5+b*x^3+a*x), x)$

[Out] $-1/2*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctanh}(2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b+1/2*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)+1/2/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\text{arctan}(2^{(1/2)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3/(c*x^5+b*x^3+a*x), x, \text{algorithm}="maxima")$

[Out] $\text{integrate}(x^3/(c*x^5 + b*x^3 + a*x), x)$

mupad [B] time = 2.21, size = 416, normalized size = 2.77

$$-2*\text{atanh}\left(\frac{\left(x(4a^2 - 2b^2c) + \frac{x(8b^2c^2 - 32abc)\left(b^2 + \sqrt{(4ac - b^2)^3 - 4abc}\right)}{8(16a^2c^3 - 8ab^2c^2 + b^4c)}\right)\sqrt{\frac{b^2 + \sqrt{(4ac - b^2)^3 - 4abc}}{8(16a^2c^3 - 8ab^2c^2 + b^4c)}}}{ac}\right)\sqrt{\frac{b^2 + \sqrt{(4ac - b^2)^3 - 4abc}}{8(16a^2c^3 - 8ab^2c^2 + b^4c)}}} - 2*\text{atanh}\left(\frac{\left(x(4a^2 - 2b^2c) - \frac{x(8b^2c^2 - 32abc)\left(\sqrt{(4ac - b^2)^3 - b^3 + 4abc}\right)}{8(16a^2c^3 - 8ab^2c^2 + b^4c)}\right)\sqrt{\frac{-(4ac - b^2)^3 - b^3 + 4abc}{8(16a^2c^3 - 8ab^2c^2 + b^4c)}}}{ac}\right)\sqrt{\frac{-(4ac - b^2)^3 - b^3 + 4abc}{8(16a^2c^3 - 8ab^2c^2 + b^4c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3/(a*x + b*x^3 + c*x^5), x)$

[Out] $-2*\text{atanh}\left(\frac{\left(x(4a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c)\right)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))\right)*(- (b^3 + (-4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))\right)^{(1/2)}/(a*c)\right)*(- (b^3 + (-4*a*c - b^2)^3)^{(1/2)} - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))\right)^{(1/2)} - 2*\text{atanh}\left(\frac{\left(x(4a*c^2 - 2*b^2*c) - (x*(8*b^3*c^2 - 32*a*b*c^3)*((-4*a*c - b^2)^3)^{(1/2)} - b^3 + 4*a*b*c)\right)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))\right)*\left(\frac{-(4*a*c - b^2)^3 - b^3 + 4*a*b*c}{8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)}\right)^{(1/2)}/(a*c)\right)*\left(\frac{-(4*a*c - b^2)^3 - b^3 + 4*a*b*c}{8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)}\right)^{(1/2)}$

sympy [A] time = 0.82, size = 75, normalized size = 0.50

$$\text{RootSum}\left(t^4\left(256a^2c^3 - 128ab^2c^2 + 16b^4c\right) + t^2\left(-16abc + 4b^3\right) + a, \left(t \mapsto t \log\left(64t^3ac^2 - 16t^3b^2c - 2tb + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**3/(c*x**5+b*x**3+a*x), x)$


```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*  
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*  
_t*b + x)))
```

$$3.84 \quad \int \frac{x^2}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x + b*x^3 + c*x^5),x]

[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{ax + bx^3 + cx^5} dx &= \int \frac{x}{a + bx^2 + cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right) \\
&= -\frac{\tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.08

$$\frac{\tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x + b*x^3 + c*x^5), x]

[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x^2/(a*x + b*x^3 + c*x^5), x]

fricas [A] time = 1.23, size = 129, normalized size = 3.58

$$\left[\frac{\log \left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a} \right)}{2\sqrt{b^2 - 4ac}}, -\frac{\sqrt{-b^2 + 4ac} \arctan \left(-\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac} \right)}{b^2 - 4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]

giac [A] time = 0.49, size = 35, normalized size = 0.97

$$\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^5+b*x^3+a*x),x)

[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] integrate(x^2/(c*x^5 + b*x^3 + a*x), x)

mupad [B] time = 2.04, size = 41, normalized size = 1.14

$$\frac{\operatorname{atan}\left(\frac{2acx^2+ab}{a\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a*x + b*x^3 + c*x^5),x)`

[Out] `atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)`

sympy [B] time = 0.50, size = 131, normalized size = 3.64

$$\frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(x^2 + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(x^2 + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**5+b*x**3+a*x),x)`

[Out] `-sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2`

$$3.85 \quad \int \frac{x}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac} \sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1585, 1093, 205}

$$\frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac} \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2} \sqrt{c} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac} \sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3 + c*x^5), x]

[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos

Q[r - p]

Rubi steps

$$\begin{aligned}
\int \frac{x}{ax + bx^3 + cx^5} dx &= \int \frac{1}{a + bx^2 + cx^4} dx \\
&= \frac{c \int \frac{1}{\frac{b}{2} - \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{\sqrt{b^2 - 4ac}} - \frac{c \int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2 - 4ac} + cx^2} dx}{\sqrt{b^2 - 4ac}} \\
&= \frac{\sqrt{2} \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2} \sqrt{c} \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 4ac} \sqrt{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.86

$$\frac{\sqrt{2} \sqrt{c} \left(\frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x + b*x^3 + c*x^5), x]

[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a*x + b*x^3 + c*x^5), x]

[Out] IntegrateAlgebraic[x/(a*x + b*x^3 + c*x^5), x]

fricas [B] time = 1.31, size = 613, normalized size = 4.09

$$-\frac{1}{2} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log \left(2cx + \sqrt{\frac{b^2 - 4ac - ab^2 - 4a^2c}{\sqrt{b^2 - 4ac}}} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \right) - \frac{1}{2} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log \left(2cx - \sqrt{\frac{b^2 - 4ac - ab^2 - 4a^2c}{\sqrt{b^2 - 4ac}}} \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \right) - \frac{1}{2} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log \left(2cx + \sqrt{\frac{b^2 - 4ac - ab^2 - 4a^2c}{\sqrt{b^2 - 4ac}}} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \right) + \frac{1}{2} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \log \left(2cx - \sqrt{\frac{b^2 - 4ac - ab^2 - 4a^2c}{\sqrt{b^2 - 4ac}}} \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{ab^2 - 4a^2c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))))
```

giac [B] time = 1.86, size = 1026, normalized size = 6.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c))*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c))*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c))*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
```



```
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
- 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

maple [A] time = 0.01, size = 116, normalized size = 0.77

$$\frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}\right)}{\sqrt{-4 a c+b^2} \sqrt{\left(-b+\sqrt{-4 a c+b^2}\right) c}}-\frac{\sqrt{2} c \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}\right)}{\sqrt{-4 a c+b^2} \sqrt{\left(b+\sqrt{-4 a c+b^2}\right) c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^5+b*x^3+a*x),x)

[Out] $-c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)-c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{c x^5 + b x^3 + a x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] integrate(x/(c*x^5 + b*x^3 + a*x), x)

mupad [B] time = 2.48, size = 763, normalized size = 5.09

$$\frac{-\operatorname{atan}\left(\frac{b^2 x^2 + b \sqrt{-4 a c + b^2} x + 48 a^2 c^2 - 12 a b^2 c + 16 a^2 c^2}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}\right) \sqrt{\frac{b^2 + \sqrt{-4 a c + b^2}}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}} - \operatorname{atan}\left(\frac{b^2 x^2 - b \sqrt{-4 a c + b^2} x + 48 a^2 c^2 - 12 a b^2 c + 16 a^2 c^2}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}\right) \sqrt{\frac{b^2 - \sqrt{-4 a c + b^2}}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}}}{4 a^2 b^2 \sqrt{\frac{b^2 + \sqrt{-4 a c + b^2}}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}} + 64 a^2 c^2 \sqrt{\frac{b^2 - \sqrt{-4 a c + b^2}}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}} - 32 a^2 b^2 \sqrt{\frac{b^2 + \sqrt{-4 a c + b^2}}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}} + 32 a^2 b^2 \sqrt{\frac{b^2 - \sqrt{-4 a c + b^2}}{12 a^2 c^2 + 48 a^2 c^2 + 4 a^2 c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*x + b*x^3 + c*x^5),x)

[Out] $-\operatorname{atan}\left(\frac{b^4 x^2 + b x (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)^{(1/2)} + a^2 c^2 x^2 + 16 a^2 c^2 x + 8 a^2 c^2}{4 a^2 b^4 (-b^3 + (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)^{(1/2)} - 4 a^2 b^2 c)}\right) / (8 a^2 b^4 + 128 a^3 c^2 - 64 a^2 b^2 c)^{(1/2)} + 64 a^3 c^2 (-b^3 + (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)^{(1/2)} - 4 a^2 b^2 c)^{(1/2)}$

$$\begin{aligned}
& - 12*a*b^4*c)^{(1/2)} - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)} \\
&) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)})) * (-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)} * 2i - \operatorname{atan}((b^4*x*1i - b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} * 1i + a^2*c^2*x*16i - a*b^2*c*x*8i) / (4*a*b^4 * ((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - b^3 + 4*a*b*c) / (8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)} + 64*a^3*c^2 * ((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - b^3 + 4*a*b*c) / (8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)} - 32*a^2*b^2*c * ((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - b^3 + 4*a*b*c) / (8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)})) * ((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - b^3 + 4*a*b*c) / (8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^{(1/2)} * 2i
\end{aligned}$$

sympy [A] time = 1.19, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4(256a^3c^2 - 128a^2b^2c + 16ab^4) + t^2(-16abc + 4b^3) + c, \left(t \mapsto t \log\left(x + \frac{32t^3a^2bc - 8t^3ab^3 + 4tac - 2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**5+b*x**3+a*x),x)

[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**3 + 4*_t*a*c - 2*_t*b**2)/c)))

$$3.86 \quad \int \frac{1}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=69

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{\log(a+bx^2+cx^4)}{4a} + \frac{\log(x)}{a}$$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1594, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{\log(a+bx^2+cx^4)}{4a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^(-1), x]

[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/(2*a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax + bx^3 + cx^5} dx &= \int \frac{1}{x(a + bx^2 + cx^4)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2a} + \frac{\text{Subst} \left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, x^2 \right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4a} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4a} \\
&= \frac{\log(x)}{a} - \frac{\log(a + bx^2 + cx^4)}{4a} + \frac{b \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^2 \right)}{2a} \\
&= \frac{b \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx^2 + cx^4)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac} + b\right) \log\left(-\sqrt{b^2-4ac} + b + 2cx^2\right) + \left(b - \sqrt{b^2-4ac}\right) \log\left(\sqrt{b^2-4ac} + b + 2cx^2\right) + 4 \log(x) \sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(-1), x]

[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax + bx^3 + cx^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^(-1), x]

[Out] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^(-1), x]

fricas [A] time = 1.35, size = 223, normalized size = 3.23

$$\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (b^2 - 4ac) \log(cx^4 + bx^2 + a) + 4(b^2 - 4ac) \log(x)}{4(ab^2 - 4a^2c)} - \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^2 - 4ac) \log(cx^4 + bx^2 + a) + 4(b^2 - 4ac) \log(x)}{4(ab^2 - 4a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

giac [A] time = 0.43, size = 68, normalized size = 0.99

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}a} - \frac{\log(cx^4 + bx^2 + a)}{4a} + \frac{\log(x^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a

maple [A] time = 0.01, size = 66, normalized size = 0.96

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}a} + \frac{\ln(x)}{a} - \frac{\ln(cx^4 + bx^2 + a)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^5+b*x^3+a*x),x)

[Out] 1/a*ln(x)-1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{1}{4} \frac{\log(cx^4 + bx^2 + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] `-integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a`

mupad [B] time = 2.70, size = 1014, normalized size = 14.70

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*x + b*x^3 + c*x^5),x)`

[Out]
$$\log(x)/a + (\log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)) + (b*\operatorname{atan}((16*a^3*x^2*((3*b^3 - 8*a*b*c))*((8*a*c - 2*b^2)^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)))/(2*(4*a*b^2 - 16*a^2*c)))))/(4*(4*a*b^2 - 16*a^2*c)^2 - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)))/(2*(4*a*b^2 - 16*a^2*c)))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2)))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2))*(4*a*c - b^2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*((8*a*c - 2*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c - b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2)))/(b^2*c^4*(25*a*c - 6*b^2)))/(2*a*(4*a*c - b^2)^(1/2))$$

sympy [B] time = 4.28, size = 253, normalized size = 3.67

$$\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) \log\left(x^2 + \frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) + 2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) - 2ac + b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) \log\left(x^2 + \frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) + 2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right) - 2ac + b^2}{bc}\right) + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**5+b*x**3+a*x),x)

[Out]
$$\begin{aligned} & \left(\frac{-b\sqrt{-4ac + b^2}}{4a(4ac - b^2)} - \frac{1}{4a} \right) \log(x^2 + (-8a^2c - b^2)\sqrt{-4ac + b^2}) \\ & + 2ab^2 \left(\frac{-b\sqrt{-4ac + b^2}}{4a(4ac - b^2)} - \frac{1}{4a} \right) - \frac{2ac + b^2}{bc} \\ & + \left(\frac{b\sqrt{-4ac + b^2}}{4a(4ac - b^2)} - \frac{1}{4a} \right) \log(x^2 + (-8a^2c + b^2)\sqrt{-4ac + b^2}) \\ & + 2ab^2 \left(\frac{b\sqrt{-4ac + b^2}}{4a(4ac - b^2)} - \frac{1}{4a} \right) - \frac{2ac + b^2}{bc} + \log(x)/a \end{aligned}$$

$$3.87 \quad \int \frac{1}{x(ax+bx^3+cx^5)} dx$$

Optimal. Leaf size=174

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{ax}$$

Rubi [A] time = 0.20, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1123, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{b}{\sqrt{b^2-4ac}} + 1 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(1 - \frac{b}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x + b*x^3 + c*x^5)),x]

[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1123

Int[((d_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*x^2 + c*x^4)^(p+1))/(a*d*(m+1)), x] - Dist[1/(a*d^2*(m+1)), Int[(d*x)^(m+2)*(b*(m+2*p+3) + c*(m+4*p+5)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(ax + bx^3 + cx^5)} dx &= \int \frac{1}{x^2(a + bx^2 + cx^4)} dx \\ &= -\frac{1}{ax} + \frac{\int \frac{-b-cx^2}{a+bx^2+cx^4} dx}{a} \\ &= -\frac{1}{ax} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2a} - \frac{\left(c\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2a} \\ &= -\frac{1}{ax} - \frac{\sqrt{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.40, size = 191, normalized size = 1.10

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a} + \frac{2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]

[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]

$x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]]/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])/a$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(ax + bx^3 + cx^5)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a*x + b*x^3 + c*x^5)),x]

[Out] IntegrateAlgebraic[1/(x*(a*x + b*x^3 + c*x^5)), x]

fricas [B] time = 1.39, size = 1116, normalized size = 6.41



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out]
$$-1/2*(\text{sqrt}(1/2)*a*x*\text{sqrt}(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x + \text{sqrt}(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*\text{sqrt}(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - \text{sqrt}(1/2)*a*x*\text{sqrt}(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x - \text{sqrt}(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*\text{sqrt}(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + \text{sqrt}(1/2)*a*x*\text{sqrt}(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x + \text{sqrt}(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*\text{sqrt}(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - \text{sqrt}(1/2)*a*x*\text{sqrt}(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x - \text{sqrt}(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*\text{sqrt}(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\text{sqrt}((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + 2)/(a*x)$$

giac [B] time = 1.88, size = 1839, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) - 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
+ 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
```

$1/2) * x / \sqrt{(a * b - \sqrt{(a^2 * b^2 - 4 * a^3 * c)}) / (a * c))} / ((a^3 * b^4 - 8 * a^4 * b^2 * c - 2 * a^3 * b^3 * c + 16 * a^5 * c^2 + 8 * a^4 * b * c^2 + a^3 * b^2 * c^2 - 4 * a^4 * c^3) * \text{abs}(a) * \text{abs}(c)) - 1 / (a * x)$

maple [A] time = 0.02, size = 232, normalized size = 1.33

$$\frac{\sqrt{2} bc \operatorname{arctanh}\left(\frac{\sqrt{2} cx}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c} a} + \frac{\sqrt{2} bc \operatorname{arctan}\left(\frac{\sqrt{2} cx}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c} a} + \frac{\sqrt{2} c \operatorname{arctanh}\left(\frac{\sqrt{2} cx}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(-b + \sqrt{-4ac + b^2})c} a} - \frac{\sqrt{2} c \operatorname{arctan}\left(\frac{\sqrt{2} cx}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{(b + \sqrt{-4ac + b^2})c} a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^5+b*x^3+a*x),x)`

[Out] $-1/a/x + 1/2/a * c * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) + 1/2/a * c / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b - 1/2/a * c * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) + 1/2/a * c / (-4 * a * c + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] `-integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)`

mupad [B] time = 2.86, size = 2997, normalized size = 17.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a*x + b*x^3 + c*x^5)),x)`

[Out] $- \operatorname{atan}\left(\left(x * (4 * a^4 * c^4 - 2 * a^3 * b^2 * c^3) + (-b^5 + b^2 * (-4 * a * c - b^2)^3)\right)^{(1/2)} + 12 * a^2 * b * c^2 - 7 * a * b^3 * c - a * c * (-4 * a * c - b^2)^3\right)^{(1/2)} / (8 * (a^3 * b^4 + 16 * a^5 * c^2 - 8 * a^4 * b^2 * c))^{(1/2)} * (4 * a^4 * b^3 * c^2 - 16 * a^5 * b * c^3 + x * (32 * a^6 * b * c^3 - 8 * a^5 * b^3 * c^2)) * (-b^5 + b^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 12 * a^2 * b * c^2 - 7 * a * b^3 * c - a * c * (-4 * a * c - b^2)^3)^{(1/2)} / (8 * (a^3 * b^4 + 16 * a^5 * c^2 - 8 * a^4 * b^2 * c))^{(1/2)} * (-b^5 + b^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 12 * a^2 * b * c^2 - 7 * a * b^3 * c - a * c * (-4 * a * c - b^2)^3)^{(1/2)} / (8 * (a^3 * b^4 + 16 * a^5 * c^2 - 8 * a^4 * b^2 * c))^{(1/2)} * i + (x * (4 * a^4 * c^4 - 2 * a^3 * b^2 * c^3) + (-b^5 + b^2 * (-4 * a * c - b^2)^3)^{(1/2)} + 12 * a^2 * b * c^2 - 7 * a * b^3 * c - a * c * (-4 * a * c - b^2)^3)^{(1/2)}$

$$\begin{aligned} & *(- (4ac - b^2)^3)^{1/2} / (8(a^3b^4 + 16a^5c^2 - 8a^4b^2c))^{1/2} * \\ & (4a^4b^3c^2 - 16a^5b^3c^3 + x(32a^6b^3c^3 - 8a^5b^3c^2)) * (- (b^5 - b^2 * \\ & (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2 - 7ab^3c + a * (- (4ac - b^2)^3)^{1/2}) / \\ & (8(a^3b^4 + 16a^5c^2 - 8a^4b^2c))^{1/2} * (- (b^5 - b^2 * (- (4ac - b^2)^3)^{1/2} + \\ & 12a^2b^3c^2 - 7ab^3c + a * (- (4ac - b^2)^3)^{1/2}) / (8(a^3b^4 + 16a^5c^2 - 8a^4b^2c))^{1/2} + \\ & 2a^3c^4) * (- (b^5 - b^2 * (- (4ac - b^2)^3)^{1/2} + 12a^2b^3c^2 - 7ab^3c + a * (- (4ac - b^2)^3)^{1/2}) / \\ & (8(a^3b^4 + 16a^5c^2 - 8a^4b^2c))^{1/2} * 2i - 1/(ax) \end{aligned}$$

sympy [A] time = 2.62, size = 148, normalized size = 0.85

$$\text{RootSum}\left(t^4(256a^5c^2 - 128a^4b^2c + 16a^3b^4) + t^2(48a^2bc^2 - 28ab^3c + 4b^5) + c^3, \left(t \mapsto t \log\left(x + \frac{-64t^3a^5c^2 + 48t^3a^4b^2c - 8t^3a^3b^4 - 10ta^2bc^2 + 10tab^3c - 2tb^5}{ac^3 - b^2c^2}\right)\right)\right) - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**5+b*x**3+a*x), x)

[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2 + 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)

$$3.88 \quad \int \frac{1}{x^2(ax+bx^3+cx^5)} dx$$

Optimal. Leaf size=89

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^2 + cx^4)}{4a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1585, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^2 + cx^4)}{4a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]

[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\int \frac{(b + 2cx)(a + bx + cx^2)}{(a + bx + cx^2)^2}, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

Rule 709

$\text{Int}[\frac{(d + ex)^m}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Simp}[e(d + ex)^{m+1}/((m+1)(cd^2 - bde + ae^2)), x] + \text{Dist}[1/(cd^2 - bde + ae^2), \text{Int}[\frac{(d + ex)^{m+1} \text{Simp}[cd - be - cex, x]}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + ex)^m (f + gx)}{(a + bx + cx^2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 1114

$\text{Int}[(x^m)((a + bx + cx^2)^p), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m-1)/2}(a + bx + cx^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, p, x\} \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rule 1585

$\text{Int}[(u + vx)^m (ax^p + bx^q + cx^r)^n, x_Symbol] \rightarrow \text{Int}[u^m x^{m+np} (a + bx^{q-p} + cx^{r-p})^n, x] /; \text{FreeQ}\{a, b, c, m, p, q, r, x\} \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q-p] \ \&\& \ \text{PosQ}[r-p]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax + bx^3 + cx^5)} dx &= \int \frac{1}{x^3(a + bx^2 + cx^4)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a + bx + cx^2)} dx, x, x^2 \right) \\
&= -\frac{1}{2ax^2} + \frac{\text{Subst} \left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, x^2 \right)}{2a} \\
&= -\frac{1}{2ax^2} + \frac{\text{Subst} \left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx, x, x^2 \right)}{2a} \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, x^2 \right)}{2a^2} \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4a^2} + \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, b + \dots \right)}{4a^2} \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx^2 + cx^4)}{4a^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + \dots \right)}{2a^2} \\
&= -\frac{1}{2ax^2} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a^2 \sqrt{b^2 - 4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx^2 + cx^4)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 135, normalized size = 1.52

$$\frac{\frac{(b\sqrt{b^2-4ac}-2ac+b^2)\log(-\sqrt{b^2-4ac}+b+2cx^2)}{\sqrt{b^2-4ac}} + \frac{(b\sqrt{b^2-4ac}+2ac-b^2)\log(\sqrt{b^2-4ac}+b+2cx^2)}{\sqrt{b^2-4ac}}}{4a^2} - \frac{2a}{x^2} - 4b \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]

[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(ax + bx^3 + cx^5)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3 + c*x^5)), x]

[Out] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3 + c*x^5)), x]

fricas [A] time = 1.37, size = 293, normalized size = 3.29

$$\frac{\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac}x^2 \log\left(\frac{2c^2x^4 + 2bx^2 + a^2 - 2ac(2c^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (b^3 - 4abc)x^2 \log(cx^4 + bx^2 + a) + 4(b^3 - 4abc)x^2 \log(x) + 2ab^2 - 8a^2c}{4(a^2b^2 - 4a^3c)x^2} - \frac{2(b^2 - 2ac)\sqrt{-b^2 + 4ac}x^2 \arctan\left(\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right) - (b^3 - 4abc)x^2 \log(cx^4 + bx^2 + a) + 4(b^3 - 4abc)x^2 \log(x) + 2ab^2 - 8a^2c}{4(a^2b^2 - 4a^3c)x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x), x, algorithm="fricas")

[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]

giac [A] time = 0.43, size = 94, normalized size = 1.06

$$\frac{b \log(cx^4 + bx^2 + a)}{4a^2} - \frac{b \log(x^2)}{2a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}a^2} + \frac{bx^2 - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x), x, algorithm="giac")

[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x^2 - a)/(a^2*x^2)

maple [A] time = 0.01, size = 119, normalized size = 1.34

$$-\frac{c \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}a} + \frac{b^2 \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}a^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^4 + bx^2 + a)}{4a^2} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^5+b*x^3+a*x), x)

[Out] $-1/2/a/x^2-1/a^2*b*\ln(x)+1/4*b*\ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*c+1/2/a^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b \log(x)}{a^2} + \frac{\frac{1}{4} b \log(cx^4 + bx^2 + a)}{a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] $-b*\log(x)/a^2 + \text{integrate}((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2/(a*x^2)$

mupad [B] time = 3.91, size = 2033, normalized size = 22.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a*x + b*x^3 + c*x^5)),x)`

[Out] $(\text{atan}((16*a^6*x^2*((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))))/(2*(16*a^3*c - 4*a^2*b^2)))))/(2*(16*a^3*c - 4*a^2*b^2)) - (((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^{(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^{(1/2)}) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (((2*b^3 - 8*a*b*c)*((2*a*c - b^2)*(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^{(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^{(3/2)}) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^{(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^{(1/2)}*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^{(3/2)))/(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)$

$$\begin{aligned}
& *(((2*b^3 - 8*a*b*c)*(((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^{(1/2)}*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))/(4*a^2*(4*a*c - b^2)^{(1/2)}) - (b^2*c^2*(2*a*c - b^2)^3)/(16*a^5*(4*a*c - b^2)^{(3/2)))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^{(3/2)}*(3*b^4 + a^2*c^2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*(((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^{(1/2)}*(16*a^3*c - 4*a^2*b^2)))/(4*a^2*(4*a*c - b^2)^{(1/2)}) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3))*(2*a*c - b^2)/(2*a^2*(4*a*c - b^2)^{(1/2)}) - (b*log(x))/a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) - 1/(2*a*x^2)
\end{aligned}$$

sympy [B] time = 123.75, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2(4ac - b^2)} \right) \log \left(x^2 + \frac{-8a^3c \left(\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2(4ac - b^2)} \right) + 2a^2b^2 \left(\frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2(4ac - b^2)} \right) + 3abc - b^3}{2ac^2 - b^2c} \right) + \left(\frac{b}{4a^2} \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2(4ac - b^2)} \right) \log \left(x^2 + \frac{-8a^3c \left(\frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2(4ac - b^2)} \right) + 2a^2b^2 \left(\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2(4ac - b^2)} \right) + 3abc - b^3}{2ac^2 - b^2c} \right) - \frac{1}{2ax^2} - \frac{b \log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**5+b*x**3+a*x), x)

[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/(2*a*x**2) - b*log(x)/a**2

$$3.89 \quad \int \frac{x^{11}}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=166

$$-\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{x^2(b^2-3ac)}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b \log(a+bx^2+cx^4)}{2c^3}$$

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1585, 1114, 738, 800, 634, 618, 206, 628}

$$-\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{x^2(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bx^4}{2c(b^2-4ac)} - \frac{b \log(a+bx^2+cx^4)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^9}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{x^2(6a+2bx)}{a+bx+cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)} \right) dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2} dx, x, x^2 \right)}{c^2(b^2 - 4ac)} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{2c^3} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \log(a + bx^2 + cx^4)}{2c^3} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 151, normalized size = 0.91

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \frac{a^2c(3b-2cx^2) - ab^2(b-4cx^2) + b^4(-x^2)}{(b^2-4ac)(a+bx^2+cx^4)} - b \log(a + bx^2 + cx^4) + cx^2}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (c*x^2 + (-b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b +

$$\frac{2cx^2/\sqrt{-b^2 + 4ac}}{(-b^2 + 4ac)^{3/2}} - b\log[a + bx^2 + cx^4]/(2c^3)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^11/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^11/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.39, size = 868, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}((b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^6 - ab^5 + 7a^2b^3c - 12a^3bc^2 + (b^5c - 8ab^3c^2 + 16a^2b^2c^3)x^4 - (b^6 - 9a^2b^4c + 26a^2b^2c^2 - 24a^3c^3)x^2 - (ab^4 - 6a^2b^2c + 6a^3c^2 + (b^4c - 6ab^2c^2 + 6a^2c^3)x^4 + (b^5 - 6ab^3c + 6a^2b^2c^2)x^2) \sqrt{b^2 - 4ac}) \log((2c^2x^4 + 2b^2cx^2 + b^2 - 2ac + (2cx^2 + b) \sqrt{b^2 - 4ac}) / (cx^4 + bx^2 + a)) - (ab^5 - 8a^2b^3c + 16a^3bc^2 + (b^5c - 8ab^3c^2 + 16a^2b^2c^3)x^4 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)x^2) \log(cx^4 + bx^2 + a) / (ab^4c^3 - 8a^2b^2c^4 + 16a^3c^5 + (b^4c^4 - 8a^2b^2c^5 + 16a^2c^6)x^4 + (b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5)x^2), \frac{1}{2}((b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^6 - ab^5 + 7a^2b^3c - 12a^3bc^2 + (b^5c - 8ab^3c^2 + 16a^2b^2c^3)x^4 - (b^6 - 9a^2b^4c + 26a^2b^2c^2 - 24a^3c^3)x^2 - 2(ab^4 - 6a^2b^2c + 6a^3c^2 + (b^4c - 6ab^2c^2 + 6a^2c^3)x^4 + (b^5 - 6ab^3c + 6a^2b^2c^2)x^2) \sqrt{-b^2 + 4ac}) \arctan(-(2cx^2 + b) \sqrt{-b^2 + 4ac}) / (b^2 - 4ac) - (ab^5 - 8a^2b^3c + 16a^3bc^2 + (b^5c - 8ab^3c^2 + 16a^2b^2c^3)x^4 + (b^6 - 8a^2b^4c + 16a^2b^2c^2)x^2) \log(cx^4 + bx^2 + a) / (ab^4c^3 - 8a^2b^2c^4 + 16a^3c^5 + (b^4c^4 - 8a^2b^2c^5 + 16a^2c^6)x^4 + (b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5)x^2)]$

giac [A] time = 1.98, size = 161, normalized size = 0.97

$$\frac{(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2c^3 - 4ac^4)\sqrt{-b^2 + 4ac}} + \frac{x^2}{2c^2} + \frac{b^3x^4 - 4abcx^4 - 2a^2cx^2 - a^2b}{2(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)} - \frac{b \log(cx^4 + bx^2 + a)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(c*x⁵+b*x³+a*x)²,x, algorithm="giac")

[Out] (b⁴ - 6*a*b²*c + 6*a²*c²)*arctan((2*c*x² + b)/sqrt(-b² + 4*a*c))/(b²*c³ - 4*a*c⁴)*sqrt(-b² + 4*a*c) + 1/2*x²/c² + 1/2*(b³*x⁴ - 4*a*b*c*x⁴ - 2*a²*c*x² - a²*b)/((c*x⁴ + b*x² + a)*(b²*c² - 4*a*c³)) - 1/2*b*log(c*x⁴ + b*x² + a)/c³

maple [B] time = 0.01, size = 383, normalized size = 2.31

$$\frac{a^2 x^2}{(c x^4 + b x^2 + a)(4ac - b^2)c} - \frac{2ab^2 x^2}{(c x^4 + b x^2 + a)(4ac - b^2)c^2} + \frac{b^4 x^2}{2(c x^4 + b x^2 + a)(4ac - b^2)c^3} - \frac{6a^2 \arctan\left(\frac{2c^2 x^2 + b}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^2 c} + \frac{6ab^2 \arctan\left(\frac{2c^2 x^2 + b}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^2 c^2} - \frac{b^4 \arctan\left(\frac{2c^2 x^2 + b}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^2 c^3} - \frac{3a^2 b}{2(c x^4 + b x^2 + a)(4ac - b^2)c^2} + \frac{ab^3}{2(c x^4 + b x^2 + a)(4ac - b^2)c^3} - \frac{2ab \ln(c x^4 + b x^2 + a)}{(4ac - b^2)c^2} + \frac{b^3 \ln(c x^4 + b x^2 + a)}{2(4ac - b^2)c^3} + \frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(c*x⁵+b*x³+a*x)²,x)

[Out] 1/2*x²/c²+1/c/(c*x⁴+b*x²+a)/(4*a*c-b²)*x²*a²-2/c²/(c*x⁴+b*x²+a)/(4*a*c-b²)*x²*a*b²+1/2/c³/(c*x⁴+b*x²+a)/(4*a*c-b²)*x²*b⁴-3/2/c²/(c*x⁴+b*x²+a)/(4*a*c-b²)*a²*b+1/2/c³/(c*x⁴+b*x²+a)/(4*a*c-b²)*a*b³-2/c²/(4*a*c-b²)*ln(c*x⁴+b*x²+a)*a*b+1/2/c³/(4*a*c-b²)*ln(c*x⁴+b*x²+a)*b³-6/c/(4*a*c-b²)^(3/2)*arctan((2*c*x²+b)/(4*a*c-b²)^(1/2))*a²+6/c²/(4*a*c-b²)^(3/2)*arctan((2*c*x²+b)/(4*a*c-b²)^(1/2))*a*b²-1/c³/(4*a*c-b²)^(3/2)*arctan((2*c*x²+b)/(4*a*c-b²)^(1/2))*b⁴

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ab^3 - 3a^2bc + (b^4 - 4ab^2c + 2a^2c^2)x^2}{2(ab^2c^3 - 4a^2c^4 + (b^2c^4 - 4ac^5)x^4 + (b^3c^3 - 4abc^4)x^2)} + \frac{x^2}{2c^2} + \frac{-2 \int \frac{(b^3 - 4abc)x^3 + (ab^2 - 3a^2c)x}{cx^4 + bx^2 + a} dx}{b^2c^2 - 4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(c*x⁵+b*x³+a*x)²,x, algorithm="maxima")

[Out] -1/2*(a*b³ - 3*a²*b*c + (b⁴ - 4*a*b²*c + 2*a²*c²)*x²)/(a*b²*c³ - 4*a²*c⁴ + (b²*c⁴ - 4*a*c⁵)*x⁴ + (b³*c³ - 4*a*b*c⁴)*x²) + 1/2*x²/c² + 2*integrate(-((b³ - 4*a*b*c)*x³ + (a*b² - 3*a²*c)*x)/(c*x⁴ + b*x² + a), x)/(b²*c² - 4*a*c³)

mupad [B] time = 0.53, size = 1473, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹¹/(a*x + b*x³ + c*x⁵)²,x)

$$\begin{aligned}
& **2)****3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2* \\
& b**2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) - \text{sqrt}(-(4*a*c - \\
& b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2* \\
& 2*b**2*c**2 + 12*a*b**4*c - b**6)))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (\\
& -b/(2*c**3) + \text{sqrt}(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2 \\
& *c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*\log(x**2 + \\
& (-5*a**2*b*c - 16*a**2*c**4*(-b/(2*c**3) + \text{sqrt}(-(4*a*c - b**2)**3)*(6*a**2 \\
& *c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a \\
& *b**4*c - b**6)))) + a*b**3 + 8*a*b**2*c**3*(-b/(2*c**3) + \text{sqrt}(-(4*a*c - b \\
& *2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b \\
& **2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) + \text{sqrt}(-(4*a*c - \\
& b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2 \\
& *b**2*c**2 + 12*a*b**4*c - b**6)))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (- \\
& 3*a**2*b*c + a*b**3 + x**2*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(8*a**2*c**4 \\
& - 2*a*b**2*c**3 + x**4*(8*a*c**5 - 2*b**2*c**4) + x**2*(8*a*b*c**4 - 2*b**3 \\
& *c**3)) + x**2/(2*c**2)
\end{aligned}$$

$$3.90 \quad \int \frac{x^{10}}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=331

$$\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}} - 13abc + 3b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}} - 13abc + 3b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} - 2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.70, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1120, 1279, 1166, 205}

$$\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}} - 13abc + 3b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}} - 13abc + 3b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{x(3b^2-10ac)}{2c^2(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bx^3}{2c(b^2-4ac)}}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} - 2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((3*b^2 - 10*a*c)*x)/(2*c*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2-4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2-4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2-4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_))*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_
))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^8}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{x^4(10a + 3bx^2)}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{x^2(9ab + 3(3b^2 - 10ac)x^2)}{a + bx^2 + cx^4} dx}{6c(b^2 - 4ac)} \\
&= \frac{(3b^2 - 10ac)x}{2c^2(b^2 - 4ac)} - \frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{3a(3b^2 - 10ac) + 3b(3b^2 - 10ac)x^2}{a + bx^2 + cx^4} dx}{6c^2(b^2 - 4ac)} \\
&= \frac{(3b^2 - 10ac)x}{2c^2(b^2 - 4ac)} - \frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(3b^3 - 13abc - \frac{3b^4}{c})}{6c^2(b^2 - 4ac)} \\
&= \frac{(3b^2 - 10ac)x}{2c^2(b^2 - 4ac)} - \frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(3b^3 - 13abc - \frac{3b^4}{c})}{2\sqrt{2}c^{5/2}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.66, size = 327, normalized size = 0.99

$$\frac{-\frac{2\sqrt{c}x(2a^2c - ab(b - 3cx^2) + b^3(-x^2))}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\sqrt{2}(-20a^2c^2 + 19ab^2c - 13abc\sqrt{b^2 - 4ac} + 3b^3\sqrt{b^2 - 4ac} - 3b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}}{4c^{5/2}} - \frac{\sqrt{2}(20a^2c^2 - 19ab^2c - 13abc\sqrt{b^2 - 4ac} + 3b^3\sqrt{b^2 - 4ac} + 3b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}} + 4\sqrt{c}x$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (4*sqrt[c]*x - (2*sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2 + 3*b^3*sqrt[b^2 - 4*a*c] - 13*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) - (sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*sqrt[b^2 - 4*a*c] - 13*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]])/(4*c^(5/2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^10/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^10/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.72, size = 2856, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + \sqrt{1/2}*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*\log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x + 1/2*\sqrt{1/2}*(27*b^{10} - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))} - \sqrt{1/2}*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*\log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x - 1/2*\sqrt{1/2}*(27*b^{10} - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*\sqrt{-(9*b^7 - 105*a*b^5*c +$

$$\begin{aligned}
& 385a^2b^3c^2 - 420a^3b^2c^3 + (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8) \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13})) / (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8)) + \sqrt{1/2} * (\\
& ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4) * x^4 + (b^3c^2 - 4ab^2c^3) * x^2) * \sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8) * \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8)) * \log(-(189a^2b^6 - 1971a^3b^4c + 5625a^4b^2c^2 - 2500a^5c^3) * x + 1/2 * \sqrt{1/2} * (27b^{10} - 459ab^8c + 2961a^2b^6c^2 - 8818a^3b^4c^3 + 11360a^4b^2c^4 - 4000a^5c^5 + (3b^9c^5 - 52ab^7c^6 + 336a^2b^5c^7 - 960a^3b^3c^8 + 1024a^4b^2c^9) * \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} * \sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8) * \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8)) - \sqrt{1/2} * (ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4) * x^4 + (b^3c^2 - 4ab^2c^3) * x^2) * \sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8) * \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8)) * \log(-(189a^2b^6 - 1971a^3b^4c + 5625a^4b^2c^2 - 2500a^5c^3) * x - 1/2 * \sqrt{1/2} * (27b^{10} - 459ab^8c + 2961a^2b^6c^2 - 8818a^3b^4c^3 + 11360a^4b^2c^4 - 4000a^5c^5 + (3b^9c^5 - 52ab^7c^6 + 336a^2b^5c^7 - 960a^3b^3c^8 + 1024a^4b^2c^9) * \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} * \sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8) * \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))} / (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8)) + 2 * (3ab^2 - 10a^2c) * x) / (ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4) * x^4 + (b^3c^2 - 4ab^2c^3) * x^2)
\end{aligned}$$

giac [B] time = 3.71, size = 3339, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2

$$\begin{aligned}
& *c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 \\
& - 928*a^3*b^3*c^9 + 640*a^4*b*c^{10} - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c} \\
& - \sqrt{b^2 - 4*a*c}*c)*b^9*c^4 + 43*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a*b^7*c^5 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^8*c^5 \\
& - 220*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^6 \\
& - 62*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^6 \\
& - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^7*c^6 + 464*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c^7 + 192*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b^4*c^7 + 31*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^7 \\
& - 320*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*b*c^8 - 160*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^8 - 96*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b^3*c^8 + 80*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^9 \\
& - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 \\
& + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^5 + 25*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a*b^3*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4*c - 52*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^2 - 26*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^2 \\
& - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c^2 + 13*\sqrt{2}*\sqrt{b^2 - 4*a*c} \\
& *\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3) \\
& *(b^2*c^2 - 4*a*c^3)^2 - 2*(3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^3 - 34*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b^4*c^4 - 6*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c^4 + 6*a*b^6*c^4 + 128*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a^3*b^2*c^5 + 44*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3*c^5 + 3*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*c^6 - 80*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a^3*b*c^6 - 22*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c} \\
& *a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c) \\
& *a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*\sqrt{1/2}*x/\sqrt{((b^3*c^2 - 4*a*b*c^3 + \sqrt{(b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9))*abs(c)) - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^{10} - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^9*c^4 + 43*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^7*c^5 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^8*c^5 - 220*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^6 - 62*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^6*c^6 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^7*c^6 +
\end{aligned}$$

$$\begin{aligned}
& 464\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^7 + \\
& 192\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^7 + \\
& 31\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^7 - \\
& 320\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^8 - 1 \\
& 60\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^8 - \\
& 96\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^8 + \\
& 80\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}c^9 - 6 \\
& (b^2 - 4ac)b^7c^6 + 62(b^2 - 4ac)ab^5c^7 - 192(b^2 - 4ac)a^2 \\
& b^3c^8 + 160(b^2 - 4ac)a^3b^3c^9 - (6b^5c^2 - 50ab^3c^3 + 104a^2 \\
& *bc^4 - 3\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}b^5 + \\
& 25\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}ab^3c + 6\sqrt{2} \\
& \sqrt{b^2 - 4ac}\sqrt{bc + \sqrt{b^2 - 4ac}}b^4c - 52\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^2b^2c^2 - 26\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}ab^2c^2 - 3\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}b^3c^2 + 13\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}ab^3c^2 - 6(b^2 - 4ac)b^3c^2 \\
& + 26(b^2 - 4ac)ab^3c^3)(b^2c^2 - 4ac^3)^2 + 2(3\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}ab^6c^3 - 34\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^2b^4c^4 - 6\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}ab^5c^4 - 6ab^6c^4 + 128\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^3b^2c^5 + 44\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^2b^3c^5 + 3\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}ab^4c^5 + 68a^2b^4c^5 - 160\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^4c^6 - 80\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^3b^2c^6 - 256a^3b^2c^6 + 40\sqrt{2}\sqrt{b^2 - 4ac} \\
& \sqrt{bc + \sqrt{b^2 - 4ac}}a^3c^7 + 320a^4c^7 + 6(b^2 - 4ac) \\
& ab^4c^4 - 44(b^2 - 4ac)a^2b^2c^5 + 80(b^2 - 4ac)a^3 \\
& c^6) \operatorname{abs}(-b^2c^2 + 4ac^3) \operatorname{arctan}(2\sqrt{1/2}x/\sqrt{(b^3c^2 - 4ab^2c^3 - \sqrt{(b^3c^2 - 4ab^2c^3)^2 - 4(a^2b^2c^2 - 4a^2c^3)(b^2c^3 - 4ac^4))})/(b^2c^3 - 4ac^4)))/((ab^6c^5 - 12a^2b^4c^6 - 2ab^5c^6 + 48a^3b^2c^7 + 16a^2b^3c^7 + ab^4c^7 - 64a^4c^8 - 32a^3b^3c^8 - 8a^2b^2c^8 + 16a^3c^9) \operatorname{abs}(-b^2c^2 + 4ac^3) \operatorname{abs}(c))
\end{aligned}$$

maple [B] time = 0.04, size = 844, normalized size = 2.55

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^{10}/(c*x^5+b*x^3+a*x)^2, x)$

[Out] $1/c^2x^3/2c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2+13/4/c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b-3/4/c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*$

$$2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * a^2 - 19/4/c / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * a * b^2 + 3/4/c^2 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctanh}(2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^4 - 13/4/c / (4ac - b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * a * b + 3/4/c^2 / (4ac - b^2) * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^3 + 5/4/c / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * a^2 - 19/4/c / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * a * b^2 + 3/4/c^2 / (4ac - b^2) / (-4ac + b^2)^{(1/2)} * 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * \operatorname{arctan}(2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)}) * c)^{(1/2)} * c * x) * b^4$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^3 - 3abc)x^3 + (ab^2 - 2a^2c)x}{2(b^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^4 + (b^3c^2 - 4abc^3)x^2)} + \frac{-\int \frac{3ab^2 - 10a^2c + (3b^3 - 13abc)x^2}{cx^4 + bx^2 + a} dx}{2(b^2c^2 - 4ac^3)} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3) + x/c^2

mupad [B] time = 3.76, size = 7599, normalized size = 22.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10/(a*x + b*x^3 + c*x^5)^2,x)

[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9))^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9))^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9))^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3

$$\begin{aligned}
& *b^8 + 768a^2b^3c^7)) / (2*(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - \\
& 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) \\
& / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} - (x*(9b^8 + 200a^4c^4 + 481a^2b^4c^2 - 718a^3b^2c^3 - 114ab^6c)) / (2*(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} * i - (((10240a^5c^7 + 48ab^8c^3 - 736a^2b^6c^4 + 4224a^3b^4c^5 - 10752a^4b^2c^6) / (8*(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5)) + (x*(- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} * (16b^7c^5 - 192ab^5c^6 - 1024a^3b^8c^8 + 768a^2b^3c^7)) / (2*(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} + (x*(9b^8 + 200a^4c^4 + 481a^2b^4c^2 - 718a^3b^2c^3 - 114ab^6c)) / (2*(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} + (x*(9b^8 + 200a^4c^4 + 481a^2b^4c^2 - 718a^3b^2c^3 - 114ab^6c)) / (2*(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} * i) / (((10240a^5c^7 + 48ab^8c^3 - 736a^2b^6c^4 + 4224a^3b^4c^5 - 10752a^4b^2c^6) / (8*(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5)) - (x*(- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} * (16b^7c^5 - 192ab^5c^6 - 1024a^3b^8c^8 + 768a^2b^3c^7)) / (2*(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (- (9b^{13} + 9b^4 * (- (4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 * (- (4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c * (- (4ac - b^2)^9)^{1/2})) / (32*(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} * i)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(x(9b^8 + 200a^4c^4 + 481a^2b^4c^2 - 718a^3b^2c^3 - 114ab^6c))}{(2(16a^2c^5 + b^4c^3 - 8ab^2c^4))}(-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2}} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2(-4ac - b^2)^9)^{1/2} \\
& - 213ab^{11}c - 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& + (((10240a^5c^7 + 48ab^8c^3 - 736a^2b^6c^4 + 4224a^3b^4c^5 - 10752a^4b^2c^6) / (8(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5))) + (x(-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2} \\
& + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2(-4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& * (16b^7c^5 - 192ab^5c^6 - 1024a^3b^8c^8 + 768a^2b^3c^7)) / (2(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2} \\
& + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2(-4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& + (x(9b^8 + 200a^4c^4 + 481a^2b^4c^2 - 718a^3b^2c^3 - 114ab^6c)) / (2(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2} \\
& + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2(-4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& + (63a^3b^5 - 573a^4b^3c + 1300a^5b^2c^2) / (4(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5))) * (-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2} \\
& + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2(-4ac - b^2)^9)^{1/2} - 213ab^{11}c - 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& * 2i - \operatorname{atan}\left(\frac{(10240a^5c^7 + 48ab^8c^3 - 736a^2b^6c^4 + 4224a^3b^4c^5 - 10752a^4b^2c^6)}{(8(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5))}\right) \\
& - (x(-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-4ac - b^2)^9)^{1/2} \\
& - 213ab^{11}c + 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2} \\
& * (16b^7c^5 - 192ab^5c^6 - 1024a^3b^8c^8 + 768a^2b^3c^7)) / (2(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (-9b^{13} + 9b^4(-4ac - b^2)^9)^{1/2} \\
& + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-4ac - b^2)^9)^{1/2} - 213ab^{11}c + 51ab^2c(-4ac - b^2)^9)^{1/2} / (32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10}))^{1/2}
\end{aligned}$$

$$\begin{aligned}
& *b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} - (x*(9*b^8 + 200* \\
& a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c)) / (2*(16*a^2*c^5 \\
& + b^4*c^3 - 8*a*b^2*c^4)) * (- (9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 268 \\
& 80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4 \\
& 4800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51* \\
& a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10} \\
& *c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2 \\
& *c^{10}))^{(1/2)} * i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422 \\
& 4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6) / (8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 \\
& - 48*a^2*b^2*c^5)) + (x*(-(9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880* \\
& a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480 \\
& 0*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b \\
& ^2*c*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^ \\
& 6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^ \\
& 10)))^{(1/2)} * (16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7) \\
&) / (2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) * (- (9*b^{13} - 9*b^4*(-(4*a*c - b^ \\
& 2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024 \\
& 0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 2 \\
& 13*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^1 \\
& 2*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c \\
& ^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 \\
& - 718*a^3*b^2*c^3 - 114*a*b^6*c)) / (2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) \\
&) * (- (9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b \\
& ^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2 \\
& *c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9 \\
&)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - \\
& 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} * i) / (((10 \\
& 240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4 \\
& *b^2*c^6) / (8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(\\
& - (9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9* \\
& c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^ \\
& 2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(\\
& 1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 240*a^2*b^8*c^7 - 128 \\
& 0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10}))^{(1/2)} * (16*b^7*c^5 - \\
& 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7) / (2*(16*a^2*c^5 + b^4*c^ \\
& 3 - 8*a*b^2*c^4)) * (- (9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b \\
& *c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5 \\
& *b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c* \\
& (- (4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^{11} + b^{12}*c^5 - 24*a*b^{10}*c^6 + 2 \\
& 40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^{10})) \\
&)^{(1/2)} - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114* \\
& a*b^6*c)) / (2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)) * (- (9*b^{13} - 9*b^4*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^ \\
& 3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(\\
& 1/2)} - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(4096*a^6*c^
\end{aligned}$$

$$\begin{aligned}
& 11 + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10})^{(1/2)} + (((10240a^5c^7 + 48ab^8c^3 - 736a^2b^6c^4 + 4224a^3b^4c^5 - 10752a^4b^2c^6)/(8(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5))) + (x(-(9b^{13} - 9b^4(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-(4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2c(-(4ac - b^2)^9)^{(1/2)}))/(32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10})))^{(1/2)} * (16b^7c^5 - 192ab^5c^6 - 1024a^3b^6c^8 + 768a^2b^3c^7))/(2(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (-(9b^{13} - 9b^4(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-(4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2c(-(4ac - b^2)^9)^{(1/2)}))/(32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10})))^{(1/2)} + (x(9b^8 + 200a^4c^4 + 481a^2b^4c^2 - 718a^3b^2c^3 - 114ab^6c))/(2(16a^2c^5 + b^4c^3 - 8ab^2c^4)) * (-(9b^{13} - 9b^4(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-(4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2c(-(4ac - b^2)^9)^{(1/2)}))/(32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10})))^{(1/2)} + (63a^3b^5 - 573a^4b^3c + 1300a^5b^2c^2)/(4(64a^3c^6 - b^6c^3 + 12ab^4c^4 - 48a^2b^2c^5)) * (-(9b^{13} - 9b^4(-(4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 - 25a^2c^2(-(4ac - b^2)^9)^{(1/2)} - 213ab^{11}c + 51ab^2c(-(4ac - b^2)^9)^{(1/2)}))/(32(4096a^6c^{11} + b^{12}c^5 - 24ab^{10}c^6 + 240a^2b^8c^7 - 1280a^3b^6c^8 + 3840a^4b^4c^9 - 6144a^5b^2c^{10})))^{(1/2)} * 2i + x/c^2
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)

[Out] Timed out

$$3.91 \quad \int \frac{x^9}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=132

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2 - 4ac)^{3/2}} - \frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(a + bx^2 + cx^4)}{4c^2}$$

Rubi [A] time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1585, 1114, 738, 773, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bx^2}{2c(b^2 - 4ac)} + \frac{\log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*\text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^{(3/2)}) + \text{Log}[a + b*x^2 + c*x^4]/(4*c^2)$

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_)^m)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int((((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1114

```
Int[(x_)^m*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^p, x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^m*((a_.)*(x_)^p + (b_.)*(x_)^q + (c_.)*(x_)^r)^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^7}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{x(4a+bx)}{a+bx+cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2} dx, x, x^2 \right)}{2c(b^2 - 4ac)} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c^2} - \frac{(b(b^2 - 6ac))}{4c^2} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(a + bx^2 + cx^4)}{4c^2} + \frac{(b(b^2 - 6ac))}{4c^2} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b(b^2 - 6ac) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{\log(a + bx^2 + cx^4)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 121, normalized size = 0.92

$$\frac{2(-2a^2c + ab(b - 3cx^2) + b^3x^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2b(b^2 - 6ac) \tan^{-1} \left(\frac{b + 2cx}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}} + \log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^9/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^9/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.29, size = 663, normalized size = 5.02

$$\frac{3ab^2 - 12ab^2c + 36a^2c^2 + 2(b^3c - 6a^2b^2c + 16a^3c^2) \sqrt{b^2 - 4ac} \arctan\left(\frac{2cx^2 + b}{\sqrt{b^2 - 4ac}}\right) + (b^3c - 6a^2b^2c + 16a^3c^2) \log\left(\frac{cx^4 + bx^2 + a}{b^2 - 4ac}\right) + (b^4c - 8a^2b^2c + 16a^3c^2) \sqrt{b^2 - 4ac} \arctan\left(\frac{2cx^2 + b}{\sqrt{b^2 - 4ac}}\right) + (b^4c - 8a^2b^2c + 16a^3c^2) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3) \sqrt{b^2 - 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]

giac [A] time = 1.95, size = 152, normalized size = 1.15

$$-\frac{(b^3 - 6abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) - \frac{b^2cx^4 - 4ac^2x^4 - b^3x^2 + 2abcx^2 - ab^2}{4(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)} + \frac{\log(cx^4 + bx^2 + a)}{4c^2}}{2(b^2c^2 - 4ac^3)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4 + b*x^2 + a)/c^2

maple [A] time = 0.01, size = 222, normalized size = 1.68

$$-\frac{3ab \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{\frac{3}{2}}c} + \frac{b^3 \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)}{2(4ac - b^2)^{\frac{3}{2}}c^2} + \frac{a \ln(cx^4 + bx^2 + a)}{(4ac - b^2)c} - \frac{b^2 \ln(cx^4 + bx^2 + a)}{4(4ac - b^2)c^2} + \frac{\frac{(3ac - b^2)bx^2}{(4ac - b^2)c^2} + \frac{(2ac - b^2)a}{(4ac - b^2)c^2}}{2cx^4 + 2bx^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^9/(c*x^5+b*x^3+a*x)^2,x)$

[Out] $\frac{1}{2} * \left(\frac{(3*a*c - b^2)}{(4*a*c - b^2)} * \frac{b}{c^2} * x^2 + \frac{(2*a*c - b^2)}{(4*a*c - b^2)} * \frac{a}{c^2} \right) / (c*x^4 + b*x^2 + a) + \frac{1}{c} / (4*a*c - b^2) * \ln(c*x^4 + b*x^2 + a) * a - \frac{1}{4} / c^2 / (4*a*c - b^2) * \ln(c*x^4 + b*x^2 + a) * b^2 - \frac{3}{c} / (4*a*c - b^2)^{(3/2)} * \arctan\left(\frac{(2*c*x^2 + b)}{(4*a*c - b^2)^{(1/2)}}\right) * a * b + \frac{1}{2} / c^2 / (4*a*c - b^2)^{(3/2)} * \arctan\left(\frac{(2*c*x^2 + b)}{(4*a*c - b^2)^{(1/2)}}\right) * b^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ab^2 - 2a^2c + (b^3 - 3abc)x^2}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^4 + (b^3c^2 - 4abc^3)x^2)} - \int \frac{(b^2 - 4ac)x^3 + abx}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^9/(c*x^5+b*x^3+a*x)^2,x, \text{algorithm}="maxima")$

[Out] $\frac{1}{2} * (a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2) / (a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - \text{integrate}(-((b^2 - 4*a*c)*x^3 + a*b*x) / (c*x^4 + b*x^2 + a), x) / (b^2*c - 4*a*c^2)$

mupad [B] time = 2.94, size = 1336, normalized size = 10.12



Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^9/(a*x + b*x^3 + c*x^5)^2,x)$

[Out] $\left(\frac{a*(2*a*c - b^2)}{(2*c^2*(4*a*c - b^2))} + \frac{b*x^2*(3*a*c - b^2)}{(2*c^2*(4*a*c - b^2))} \right) / (a + b*x^2 + c*x^4) - \frac{(\log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) + (b*\text{atan}\left(\frac{(8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*x^2 + ((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + (8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))) * (6*a*c - b^2)) / (8*c^2*(4*a*c - b^2)^{(3/2)}) + (b*(8*b^3*c^4 - 32*a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (16*c^2*(4*a*c - b^2)^{(3/2)}*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))) / (a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c) / (4*a*c^3 - b^2*c^2) + ((6*b^3*c^2 - 28*a*b*c^3) / (4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))$

$$\begin{aligned} & (2*c^4)) * (2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c) / (2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a*b*c^5)*(6*a*c - b^2)^2) / (c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))) / (2*a*(4*a*c - b^2)^(3/2)) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))) / (8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / ((4*a*c - b^2)^(3/2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))) / (a*(4*a*c - b^2)) + (b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) * (2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)) / (2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2) / (c^2*(4*a*c - b^2)^3))) / (2*a*(4*a*c - b^2)^(3/2))) / (b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c)) * (6*a*c - b^2) / (2*c^2*(4*a*c - b^2)^(3/2)) \end{aligned}$$

sympy [B] time = 19.76, size = 745, normalized size = 5.64

$$\left(\frac{\sqrt{-4ac - b^2} (ac - b^2)}{c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} \right)^{\frac{1}{2}} \log \left(\frac{-32a^2 \left(\frac{\sqrt{-4ac - b^2} (ac - b^2)}{c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} + \frac{1}{2} \right) + 8a^2 + 16a^2c}{2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} \right) - 2a^2 \left(\frac{\sqrt{-4ac - b^2} (ac - b^2)}{c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} + \frac{1}{2} \right) \right) \left(\frac{\sqrt{-4ac - b^2} (ac - b^2)}{c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} \right)^{\frac{1}{2}} \log \left(\frac{-32a^2 \left(\frac{\sqrt{-4ac - b^2} (ac - b^2)}{c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} + \frac{1}{2} \right) + 8a^2 + 16a^2c}{2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} \right) - 2a^2 \left(\frac{\sqrt{-4ac - b^2} (ac - b^2)}{c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)} + \frac{1}{2} \right) \right) \left(\frac{2a^2 - ab^2 + c^2(64a^3 - 48ab^2c + 12a^2c^2 - b^3)}{64a^3 - 48ab^2c + 12a^2c^2 - b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)

[Out] $(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*\log(x**2 + (-32*a**2*c**3*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*\log(x**2 + (-32*a**2*c**3*(b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2))$

$$3.92 \quad \int \frac{x^8}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} + 2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.53, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bx}{2c(b^2-4ac)}}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}} + 2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*c^{3/2}*(b^2 - 4*a*c)*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*c^{3/2}*(b^2 - 4*a*c)*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_))*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_
))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^6}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{x^2(6a + bx^2)}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{bx}{2c(b^2 - 4ac)} + \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{ab + (b^2 - 6ac)x^2}{a + bx^2 + cx^4} dx}{2c(b^2 - 4ac)} \\
&= -\frac{bx}{2c(b^2 - 4ac)} + \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(b^2 - 6ac - \frac{b(b^2 - 8ac)}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2}}{4c(b^2 - 4ac)} \\
&= -\frac{bx}{2c(b^2 - 4ac)} + \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(b^2 - 6ac - \frac{b(b^2 - 8ac)}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.51, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}x(a(b-2cx^2)+b^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/2}} + \frac{\sqrt{2}(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $\frac{((-2*\text{Sqrt}[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[2]*(-b^3 + 8*a*b*c + b^2*\text{Sqrt}[b^2 - 4*a*c] - 6*a*c*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*(b^3 - 8*a*b*c + b^2*\text{Sqrt}[b^2 - 4*a*c] - 6*a*c*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])}{(4*c^{(3/2)})}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(ax + bx^3 + cx^5)^2} dx$$

$$\frac{((b^4 - 18ab^2c + 81a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}{(b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))} + \sqrt{\frac{1}{2}} \cdot ((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4ab^2c^2)x^2) \cdot \sqrt{-(b^5 - 15ab^3c + 60a^2b^2c^2 - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))} \cdot \sqrt{\frac{(b^4 - 18ab^2c + 81a^2c^2)}{(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} \cdot \log\left(\frac{(5ab^4 - 81a^2b^2c + 324a^3c^2)x - 1/2\sqrt{\frac{1}{2}}(b^7 - 17ab^5c + 88a^2b^3c^2 - 144a^3b^2c^3 + (b^8c^3 - 24ab^6c^4 + 192a^2b^4c^5 - 640a^3b^2c^6 + 768a^4c^7)) \cdot \sqrt{\frac{(b^4 - 18ab^2c + 81a^2c^2)}{(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} \cdot \sqrt{-(b^5 - 15ab^3c + 60a^2b^2c^2 - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))} \cdot \sqrt{\frac{(b^4 - 18ab^2c + 81a^2c^2)}{(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}} \cdot \sqrt{-(b^5 - 15ab^3c + 60a^2b^2c^2 - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))}\right) / ((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4ab^2c^2)x^2)$$

giac [B] time = 4.02, size = 2736, normalized size = 10.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out]
$$-1/2*(b^2x^3 - 2acx^3 + abx)/((cx^4 + bx^2 + a)(b^2c - 4ac^2)) - 1/16*(2b^8c^4 - 32ab^6c^5 + 160a^2b^4c^6 - 256a^3b^2c^7 - \sqrt{2}) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot b^8c^2 + 16 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot ab^6c^3 + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot b^7c^3 - 80 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a^2b^4c^4 - 24 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a^3b^2c^5 + 64 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a^2b^3c^5 + 12 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot ab^4c^5 - 32 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a^2b^2c^6 - 2 \cdot (b^2 - 4ac) \cdot b^6c^4 + 24 \cdot (b^2 - 4ac) \cdot ab^4c^5 - 64 \cdot (b^2 - 4ac) \cdot a^2b^2c^6 - (2b^4c^2 - 20ab^2c^3 + 48a^2c^4 - \sqrt{2}) \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot b^4 + 10 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot ab^2c + 2 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot b^3c - 24 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a^2c^2 - 12 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot abc^2 - \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot b^2c^2 + 6 \cdot \sqrt{2} \cdot \sqrt{b^2 - 4ac} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot ac^3 - 2 \cdot (b^2 - 4ac) \cdot b^2c^2 + 12 \cdot (b^2 - 4ac) \cdot ac^3 \cdot (b^2c - 4ac^2)^2 - 2 \cdot (\sqrt{2}) \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot ab^5c^2 - 8 \cdot \sqrt{2} \cdot \sqrt{bc + \sqrt{b^2 - 4ac}} \cdot c \cdot a$$

$$\begin{aligned}
& ^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 - 2*a*b^5*c^3 \\
& + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}} \\
& (b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}* \\
& c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a \\
& ^2*b*c^4)*\text{abs}(b^2*c - 4*a*c^2))*\arctan(2*\sqrt{1/2}*x/\sqrt{(b^3*c - 4*a*b*c^2 \\
& + \sqrt{(b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)} \\
&))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a \\
& ^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2 \\
& *b^2*c^6 + 16*a^3*c^7)*\text{abs}(b^2*c - 4*a*c^2)*\text{abs}(c)) + 1/16*(2*b^8*c^4 - 32* \\
& a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^8*c^2 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^6*c^3 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^7*c^3 - 80*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^4*c^4 - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^6*c^4 + 128*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b^2*c^5 + 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^5 + 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^5 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c - 24*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^2 - 12*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^2 + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 + 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c^2 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^3 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^4 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^4 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*\text{abs}(b^2*c - 4*a*c^2))*\arctan(2*\sqrt{1/2}*x/\sqrt{(b^3*c - 4*a*b*c^2 - \sqrt{(b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)})))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*\text{abs}(b^2*c - 4*a*c^2)*\text{abs}(c))
\end{aligned}$$

maple [B] time = 0.03, size = 602, normalized size = 2.22

$$\frac{2\sqrt{2} a b \arctanh\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{(4ac - b^2)\sqrt{4ac + b^2}\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{2\sqrt{2} a b \arctan\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{(4ac - b^2)\sqrt{4ac + b^2}\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{\sqrt{2} b^3 \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{4(4ac - b^2)\sqrt{4ac + b^2}\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{\sqrt{2} b^3 \arctan\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{4(4ac - b^2)\sqrt{4ac + b^2}\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{3\sqrt{2} a \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{2(4ac - b^2)\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{3\sqrt{2} a \arctan\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{2(4ac - b^2)\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{\sqrt{2} b^2 \operatorname{arctanh}\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{4(4ac - b^2)\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{\sqrt{2} b^2 \arctan\left(\frac{\sqrt{2} x}{\sqrt{(b + \sqrt{4ac + b^2})}}\right)}{4(4ac - b^2)\sqrt{(b + \sqrt{4ac + b^2})}} + \frac{a x}{c^2 x^2 + 2 a x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(c*x^5+b*x^3+a*x)^2,x)`

[Out]
$$\begin{aligned} & (-1/2*(2*a*c-b^2)/(4*a*c-b^2)/c*x^3+1/2/(4*a*c-b^2)*a*b/c*x)/(c*x^4+b*x^2+a) \\ & -3/2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}) \\ & /((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a+1/4/(4*a*c-b^2)/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*a \\ & \operatorname{rctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3+3/2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a-1/4/(4*a*c-b^2)/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^2 - 2ac)x^3 + abx}{2((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4abc^2)x^2)} - \frac{-\int \frac{(b^2-6ac)x^2+ab}{cx^4+bx^2+a} dx}{2(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*\operatorname{integrate}(-((b^2 - 6*a*c)*x^2 + a*b)/ \\ & (c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2) \end{aligned}$$

mupad [B] time = 3.86, size = 6293, normalized size = 23.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(a*x + b*x^3 + c*x^5)^2,x)`

[Out]
$$\begin{aligned} & -((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/ \\ & (a + b*x^2 + c*x^4) - \operatorname{atan}((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)}))/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 120*a^3*b^6*c^4 - 120*a^4*b^4*c^3 + 120*a^5*b^2*c^2 - 120*a^6*c))) \end{aligned}$$

$$\begin{aligned}
& (8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*(16 \\
& *b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5)/(2*(b^4*c + 1 \\
& 6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^ \\
& 5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c \\
& - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10} \\
& 0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^ \\
& 2*c^8))^{(1/2)} - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b \\
& ^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2 \\
& 7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - \\
& 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614 \\
& 4*a^5*b^2*c^8))^{(1/2)}*i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c \\
& ^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c \\
& ^3)) + (x*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2 \\
& *b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a* \\
& c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b \\
& ^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*(1 \\
& 6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + \\
& 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a \\
& ^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9 \\
& *c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10} \\
& ^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)} + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(\\
& b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - \\
& 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - \\
& 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61 \\
& 44*a^5*b^2*c^8))^{(1/2)}*i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5 \\
& *c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2* \\
& c^3)) - (x*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^ \\
& 2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a \\
& *c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2* \\
& b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8))^{(1/2)}*(\\
& 16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + \\
& 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840* \\
& a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^ \\
& 9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b \\
& ^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5* \\
& b^2*c^8))^{(1/2)} - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2* \\
& (b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - \\
& 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(4096*a^6*c^9 + b^{12}*c^3 \\
& - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6 \\
& 144*a^5*b^2*c^8))^{(1/2)} + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c \\
& ^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^ \\
& ^3))
\end{aligned}$$

$$\begin{aligned}
& 3)) + (x*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{1/2}) - 3840*a^5*b*c^5 + 288*a^2* \\
& b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c \\
& - b^2)^9)^{1/2})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8* \\
& c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{1/2}*(16 \\
& *b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1 \\
& 6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{1/2}) - 3840*a^ \\
& 5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9* \\
& c - 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a*b^{1 \\
& 0}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^ \\
& 2*c^8)))^{1/2} + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b \\
& ^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{1/2}) - \\
& 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2 \\
& 7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^9 + b^{12}*c^3 - \\
& 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614 \\
& 4*a^5*b^2*c^8)))^{1/2} + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c \\
& - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))*(-(b^{11} + b^2*(-(4*a*c - \\
& b^2)^9)^{1/2}) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840* \\
& a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^ \\
& 9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^ \\
& 4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{1/2}*2i - \operatorname{atan}((((16*a*b^7*c^2 - 1024*a^4 \\
& *b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b \\
& ^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{1/2}) - 384 \\
& 0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a* \\
& b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24*a \\
& *b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^ \\
& 5*b^2*c^8)))^{1/2}*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b \\
& ^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} - b^2*(-(4*a*c - b \\
& ^2)^9)^{1/2}) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a \\
& ^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^9 \\
& + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4 \\
& *b^4*c^7 - 6144*a^5*b^2*c^8)))^{1/2} - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^ \\
& 2 - 16*a*b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} - b^2*(-(4 \\
& *a*c - b^2)^9)^{1/2}) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 \\
& + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096 \\
& *a^6*c^9 + b^{12}*c^3 - 24*a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + \\
& 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^{1/2}*1i - (((16*a*b^7*c^2 - 1024*a^ \\
& 4*b*c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a* \\
& b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{1/2}) - 38 \\
& 40*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a \\
& *b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^9 + b^{12}*c^3 - 24* \\
& a*b^{10}*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a \\
& ^5*b^2*c^8)))^{1/2}*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2* \\
& b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^{11} - b^2*(-(4*a*c - \\
& b^2)^9)^{1/2}) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840* \\
& a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{1/2})/(32*(4096*a^6*c^
\end{aligned}$$

$$\begin{aligned}
& 9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} + (x*(b^6 - 72a^3c^3 + 74a^2b^2c^2 - 16ab^4c)) / (2*(b^4c + 16a^2c^3 - 8ab^2c^2)) * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} * i) / (((16ab^7c^2 - 1024a^4b^5c^5 - 192a^2b^5c^3 + 768a^3b^3c^4) / (8*(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)) - (x * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} * (16b^7c^3 - 192ab^5c^4 - 1024a^3b^3c^6 + 768a^2b^3c^5)) / (2*(b^4c + 16a^2c^3 - 8ab^2c^2)) * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} - (x*(b^6 - 72a^3c^3 + 74a^2b^2c^2 - 16ab^4c)) / (2*(b^4c + 16a^2c^3 - 8ab^2c^2)) * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} + (((16ab^7c^2 - 1024a^4b^5c^5 - 192a^2b^5c^3 + 768a^3b^3c^4) / (8*(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)) + (x * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} * (16b^7c^3 - 192ab^5c^4 - 1024a^3b^3c^6 + 768a^2b^3c^5)) / (2*(b^4c + 16a^2c^3 - 8ab^2c^2)) * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} + (x*(b^6 - 72a^3c^3 + 74a^2b^2c^2 - 16ab^4c)) / (2*(b^4c + 16a^2c^3 - 8ab^2c^2)) * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} + (5a^2b^4 + 216a^4c^2 - 66a^3b^2c) / (4*(b^6c - 64a^3c^4 - 12ab^4c^2 + 48a^2b^2c^3)) * (-(b^{11} - b^2 * (-(4ac - b^2)^9))^{(1/2)} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c + 9ac * (-(4ac - b^2)^9))^{(1/2)}) / (32*(4096a^6c^9 + b^{12}c^3 - 24ab^{10}c^4 + 240a^2b^8c^5 - 1280a^3b^6c^6 + 3840a^4b^4c^7 - 6144a^5b^2c^8))^{(1/2)} * 2i
\end{aligned}$$

sympy [A] time = 33.31, size = 379, normalized size = 1.40

$$\frac{abx + b^2(-2ac + b^2)}{8b^2c^2 - 2ab^2c + a^2(8ac^2 - 2b^2c^2) + b^2(8ab^2 - 2b^2c^2)} + \text{RootSum}\left(\left(\frac{1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c^7 - 327680a^3b^6c^6 + 61440a^2b^8c^5 - 6144ab^{10}c^4 + 256b^{12}c^3}{(49152at^3 + 40960a^2t^2 + 12288at^3 + 64b^3c^3 - 1728at^2 + 656a^2t^2 - 88ab^2c + 4b^2c^2)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)

[Out] (a*b*x + x**3*(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*c**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c + 25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))

$$3.93 \quad \int \frac{x^7}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=78

$$\frac{2a \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} + \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1114, 722, 618, 206}

$$\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 722

Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m-1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p+1))/((p+1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p+3)*(c*d^2 - b*d*e + a*e^2))/((p+1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m-2)*(a + b*x + c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,

0] && LtQ[p, -1]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^7}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^5}{(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{x^2(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{a \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{b^2 - 4ac} \\
 &= \frac{x^2(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2a) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\
 &= \frac{x^2(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2a \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 1.19

$$\frac{2a \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}} + \frac{a(b - 2cx^2) + b^2x^2}{2c(4ac - b^2)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^7/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.38, size = 407, normalized size = 5.22

$$\left[\frac{ab^3 - 4a^2bc + (b^4 - 6ab^2c + 8a^2c^2)x^2 + 2(ac^2x^4 + abcx^2 + a^2c)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x^2)} - \frac{ab^3 - 4a^2bc + (b^4 - 6ab^2c + 8a^2c^2)x^2 - 4(ac^2x^4 + abcx^2 + a^2c)\sqrt{-b^2 + 4ac} \arctan\left(\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right)}{2(ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]

giac [A] time = 2.00, size = 96, normalized size = 1.23

$$\frac{2a \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{b^2x^2 - 2acx^2 + ab}{2(cx^4 + bx^2 + a)(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))

maple [A] time = 0.01, size = 104, normalized size = 1.33

$$\frac{2a \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{\frac{ab}{(4ac-b^2)c} - \frac{(2ac-b^2)x^2}{(4ac-b^2)c}}{2cx^4 + 2bx^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/2*(-(2*a*c-b^2)/(4*a*c-b^2)/c*x^2+1/(4*a*c-b^2)*a*b/c)/(c*x^4+b*x^2+a)+2*a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.20, size = 187, normalized size = 2.40

$$\frac{\frac{x^2(2ac-b^2)}{2c(4ac-b^2)} - \frac{ab}{2c(4ac-b^2)}}{cx^4 + bx^2 + a} - \frac{2a \operatorname{atan}\left(\frac{b^3-4abc}{(4ac-b^2)^{3/2}} - \frac{x^2\left(\frac{4ac^2}{(4ac-b^2)^{7/2}} + \frac{4a(b^3c^2-4abc^3)(b^3-4abc)}{(4ac-b^2)^{13/2}}\right)}{8a^2c^2}\right)(4ac-b^2)^4}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(a*x + b*x^3 + c*x^5)^2,x)

[Out] -((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)

sympy [B] time = 1.52, size = 282, normalized size = 3.62

$$-a \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x^2 + \frac{-16a^2c^2\sqrt{\frac{1}{(4ac-b^2)^3}} + 8a^2b^2c\sqrt{\frac{1}{(4ac-b^2)^3}} - ab^4\sqrt{\frac{1}{(4ac-b^2)^3}} + ab}{2ac}\right) + a \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x^2 + \frac{16a^3c^2\sqrt{\frac{1}{(4ac-b^2)^3}} - 8a^2b^2c\sqrt{\frac{1}{(4ac-b^2)^3}} + ab^4\sqrt{\frac{1}{(4ac-b^2)^3}} + ab}{2ac}\right) + \frac{ab + x^2(-2ac + b^2)}{8a^2c^2 - 2ab^2c + x^4(8ac^3 - 2b^2c^2) + x^2(8ab^2c - 2b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)`

[Out]
$$\begin{aligned} & -a\sqrt{-1/(4ac - b^2)^3}\log(x^2 + (-16a^3c^2\sqrt{-1/(4ac - b^2)^3} \\ & + 8a^2b^2c\sqrt{-1/(4ac - b^2)^3} - ab^4\sqrt{-1/(4ac - b^2)^3} \\ & + ab)/(2ac)) + a\sqrt{-1/(4ac - b^2)^3}\log(x^2 + (16a^3c^2\sqrt{-1/(4ac - b^2)^3} \\ & - 8a^2b^2c\sqrt{-1/(4ac - b^2)^3} + ab^4\sqrt{-1/(4ac - b^2)^3} \\ & + ab)/(2ac)) + (ab + x^2(-2ac + b^2))/(8a^2c^2 - 2ab^2c + x^4(8ac^3 - 2b^2c^2) + x^2(8ab^2c^2 - 2b^3c)) \end{aligned}$$

$$3.94 \quad \int \frac{x^6}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b - \frac{4ac+b^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac} + 4ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Rubi [A] time = 0.36, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1120, 1166, 205}

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b - \frac{4ac+b^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac} + 4ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1120

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(d^3*(d*x)^(m-3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*(p+1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p+1)*(b^2 - 4*a*c)), Int[(d*x)^(m-4)*(2*a*(m-3) + b*(m+4*p+3)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^6}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^4}{(a + bx^2 + cx^4)^2} dx \\ &= \frac{x(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{2a - bx^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\ &= \frac{x(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^2 + 4ac - b\sqrt{b^2 - 4ac}) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4(b^2 - 4ac)^{3/2}} + \frac{(b^2 + 4ac + b\sqrt{b^2 - 4ac}) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4(b^2 - 4ac)^{3/2}} \\ &= \frac{x(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^2 + 4ac - b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{(b^2 + 4ac + b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.41, size = 235, normalized size = 0.99

$$\frac{1}{4} \left(\frac{2(2ax + bx^3)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} - 4ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} + 4ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
```


- 4*a*c]]]/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])]/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^6/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 0.89, size = 1668, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))

$$\begin{aligned} & ^2*c^4 - 64*a^3*c^5)) * \sqrt{-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} / (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) - \sqrt{1/2} \\ & * ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) * \sqrt{-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} / (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4) \\ & * \log((3*b^2 + 4*a*c)*x - \sqrt{1/2}*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)) / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}) * \sqrt{-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) / \sqrt{b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5}} / (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) + 4*a*x / ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) \end{aligned}$$

giac [B] time = 3.39, size = 2132, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $\frac{1}{2} \frac{(b*x^3 + 2*a*x)}{((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))} - \frac{1}{16} \frac{(2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^6*c + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 + 4*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^4*c - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*c^3 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*\text{abs}(b^2 - 4*a*c)) * \arctan(2*\sqrt{1/2}*x/\sqrt{((b^3 - 4*a*b*c + \sqrt{((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2))})/(b^2*c - 4*a*c^2))})/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^$

$$\begin{aligned}
& 3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^7 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^5*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^6*c + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5*c^2 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c^3 - 32*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^3 + 16*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc tan(2*\sqrt{1/2}*x/\sqrt{(b^3 - 4*a*b*c - \sqrt{(b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2))})/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
\end{aligned}$$

maple [B] time = 0.03, size = 452, normalized size = 1.91

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{4ac+b^2})c}}\right)}{(4ac-b^2)\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{4ac+b^2})c}}\right)}{(4ac-b^2)\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} b^2 \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{4ac+b^2})c}}\right)}{4(4ac-b^2)\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} b^2 \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{4ac+b^2})c}}\right)}{4(4ac-b^2)\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2} b \operatorname{arctanh}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{4ac+b^2})c}}\right)}{4(4ac-b^2)\sqrt{(-b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2} b \operatorname{arctan}\left(\frac{\sqrt{2} c x}{\sqrt{(b+\sqrt{4ac+b^2})c}}\right)}{4(4ac-b^2)\sqrt{(b+\sqrt{-4ac+b^2})c}} + \frac{b^3}{2(4ac-b^2)} \frac{ax}{4ac-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^5+b*x^3+a*x)^2,x)

[Out] $(-1/2/(4*a*c-b^2)*b*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})*c)^{1/2}*\operatorname{arctanh}(2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})*c)^{1/2})*c*x)*b-c/(4*a*c-b^2)/(-4*a*c+b^2)^{1/2}*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})*c)^{1/2}*\operatorname{arctanh}(2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})*c)^{1/2})*c*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{1/2}*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})*c)^{1/2}*\operatorname{arctanh}(2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})*c)^{1/2})*c*x)*b^2-1/4/(4*a*c-b^2)*2^{1/2}/((b+(-4*a*c+b^2)^{1/2})*c)^{1/2}*\operatorname{arctan}(2^{1/2}/((b+(-4*a*c+b^2)^{1/2})*c)^{1/2})*c*x)*b-c/(4*a*c-b^2)/(-4*a*c+b^2)^{1/2}*2^{1/2}/((b+(-4*a*c+b^2)^{1/2})*c)^{1/2}*\operatorname{arctan}(2^{1/2}/((b+(-4*a*c+b^2)^{1/2})*c)^{1/2})*c*x)*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^{1/2}*2^{1/2}/((b+(-4*a*c+b^2)^{1/2})*c)^{1/2}*\operatorname{arctan}(2^{1/2}/((b+(-4*a*c+b^2)^{1/2})*c)^{1/2})*c*x)*b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) + \frac{1}{2}*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)$

mupad [B] time = 3.64, size = 4973, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a*x + b*x^3 + c*x^5)^2,x)

[Out]
$$- \operatorname{atan}\left(\frac{(2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*((-(4*a*c - b^2)^9)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{1/2}*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))\right) * \left(\frac{((-(4*a*c - b^2)^9)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{1/2} - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))\right) * \left(\frac{((-(4*a*c - b^2)^9)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{1/2} * i - \left(\frac{(2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*((-(4*a*c - b^2)^9)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{1/2}*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))\right) * \left(\frac{((-(4*a*c - b^2)^9)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{1/2} + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))\right) * \left(\frac{((-(4*a*c - b^2)^9)^{1/2} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^{12}*c + 4096*a^6*c^7 - 24*a*b^{10}*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{1/2} * i\right) / \left(\frac{(2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))\right)$$

$$\begin{aligned}
& 3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * \\
& (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2)) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} * i) / (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4) / (8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} * (16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4)) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2)) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} + (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4) / (8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} * (16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4)) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2)) / (2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} - (4*a^2*b*c^2 + 3*a*b^3*c) / (4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))) * (- (b^9 + (- (4*a*c - b^2)^9)^{(1/2)} - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3) / (32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^{(1/2)} * 2i - ((a*x) / (4*a*c - b^2) + (b*x^3) / (2*(4*a*c - b^2))) / (a + b*x^2 + c*x^4)
\end{aligned}$$

sympy [A] time = 4.55, size = 296, normalized size = 1.25

$\frac{-2x - 3a^2}{8c^2 - 2ab^2 + 4(8ac^2 - 29c^2) + 4(6abc - 29b^2)} + \text{RootSum}\left(x^6 \left((1048576a^2c^2 - 157284a^2b^2c^2 + 98304a^2b^4c^2 - 32768a^2b^6c^2 + 6144a^2b^8c^2 - 6144a^2b^{10}c^2 + 256a^{12}c^2) + x^6 \left(-12288a^4c^4 + 8192a^4b^2c^4 - 1536a^4b^4c^4 + 16a^4c^2 + 24a^2b^2c^2 + 9ab^6 \right) \left(x + \frac{16384a^3b^4c^4 - 12288a^2b^6c^4 + 3072a^2b^8c^4 - 256a^2b^{10}c^4 + 64a^2b^{12}c^4 - 128a^2b^{14}c^4 - 4a^6}{4c - 3c^2} \right) \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
- 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
+ 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
- 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

$$3.95 \quad \int \frac{x^5}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=75

$$\frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1114, 638, 618, 206}

$$\frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
 &= \frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\
 &= \frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 1.05

$$\frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $(2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*\text{ArcTan}[(b + 2*c*x^2)/\text{Sqrt}[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^{(3/2)}$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(a*x + b*x^3 + c*x^5)^2, x]

[Out] IntegrateAlgebraic[x^5/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.47, size = 360, normalized size = 4.80

$$\left[\frac{2ab^2 - 8a^2c + (b^3 - 4abc)x^2 - (bcx^4 + b^2x^2 + ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)}, \frac{2ab^2 - 8a^2c + (b^3 - 4abc)x^2 - 2(bcx^4 + b^2x^2 + ab)\sqrt{-b^2 + 4ac} \arctan\left(-\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $[1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c))*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c))*x^2 - 2*(b*c*x^4 + b^2*x^2 + a*b)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]$

giac [A] time = 2.05, size = 82, normalized size = 1.09

$$\frac{b \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{bx^2 + 2a}{2(cx^4 + bx^2 + a)(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $b*\arctan((2*c*x^2 + b)/\text{sqrt}(-b^2 + 4*a*c))/((b^2 - 4*a*c)*\text{sqrt}(-b^2 + 4*a*c)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))$

maple [A] time = 0.01, size = 77, normalized size = 1.03

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{-bx^2-2a}{2(4ac-b^2)(cx^4+bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 0.14, size = 178, normalized size = 2.37

$$\frac{b \operatorname{atan}\left(\frac{b^3-4abc}{(4ac-b^2)^{3/2}} - \frac{x^2(4ac-b^2)^4 \left(\frac{b^2c^2}{a(4ac-b^2)^{7/2}} + \frac{b^2(2b^3c^2-8abc^3)(b^3-4abc)}{2a(4ac-b^2)^{13/2}}\right)}{2b^2c^2}\right)}{(4ac-b^2)^{3/2}} - \frac{\frac{a}{4ac-b^2} + \frac{bx^2}{2(4ac-b^2)}}{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*x + b*x^3 + c*x^5)^2,x)

[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c - b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)

sympy [B] time = 1.36, size = 269, normalized size = 3.59

$$\frac{b \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x^2 + \frac{-16a^2b^2 \sqrt{\frac{1}{(4ac-b^2)^3}} + 8ab^3c \sqrt{\frac{1}{(4ac-b^2)^3}} - b^5 \sqrt{\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right)}{2} - \frac{b \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x^2 + \frac{16a^2b^2 \sqrt{\frac{1}{(4ac-b^2)^3}} - 8ab^3c \sqrt{\frac{1}{(4ac-b^2)^3}} + b^5 \sqrt{\frac{1}{(4ac-b^2)^3}} + b^2}{2bc}\right)}{2} + \frac{-2a - bx^2}{8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)

[Out] $b\sqrt{-1/(4ac - b^2)^3} \log(x^2 + (-16a^2bc^2\sqrt{-1/(4ac - b^2)^3} + 8ab^3c\sqrt{-1/(4ac - b^2)^3} - b^5\sqrt{-1/(4ac - b^2)^3} + b^2)/(2bc)) / 2 - b\sqrt{-1/(4ac - b^2)^3} \log(x^2 + (16a^2bc^2\sqrt{-1/(4ac - b^2)^3} - 8ab^3c\sqrt{-1/(4ac - b^2)^3} + b^5\sqrt{-1/(4ac - b^2)^3} + b^2)/(2bc)) / 2 + (-2a - bx^2)/(8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3))$

$$3.96 \quad \int \frac{x^4}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=221

$$\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Rubi [A] time = 0.24, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1119, 1166, 205}

$$\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[c]*(2*b - \text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1119

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(d*x)^(m-1)*(b+2*c*x^2)*(a+b*x^2+c*x^4)^(p+1))/(2*(p+1)*(b^2-4*a*c)), x] - Dist[d^2/(2*(p+1)*(b^2-4*a*c)), Int[(d*x)^(m-2)*(b*(m-1)+2*c*(m+4*p+5)*x^2)*(a+b*x^2+c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2-4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^2}{(a + bx^2 + cx^4)^2} dx \\ &= -\frac{x(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4} dx}{2(b^2 - 4ac)} \\ &= -\frac{x(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(c\left(1 + \frac{2b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2(b^2 - 4ac)} + \frac{c(2b - \sqrt{b^2-4ac})}{2(b^2 - 4ac)} \\ &= -\frac{x(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} (2b - \sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2} (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} (2b - \sqrt{b^2 - 4ac})}{\sqrt{2} (b^2 - 4ac)} \end{aligned}$$

Mathematica [A] time = 0.43, size = 222, normalized size = 1.00

$$\frac{-bx - 2cx^3}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\sqrt{c} (\sqrt{b^2 - 4ac} - 2b) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2} (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} (\sqrt{b^2 - 4ac} + 2b) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{2} (b^2 - 4ac)^{3/2} \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (-b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
```

)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^4/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.74, size = 1680, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(4*c*x^3 + \text{sqrt}(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\text{sqrt}(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))) \\ &)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*\log((3*b^2*c + 4*a*c^2)*x + 1/2*\text{sqrt}(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*\text{sqrt}(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))) - \text{sqrt}(1/2)* \\ & ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\text{sqrt}(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*\log((3*b^2*c + 4*a*c^2)*x - 1/2*\text{sqrt}(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*\text{sqrt}(- \\ & (b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))) + \text{sqrt}(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\text{sqrt}(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*\log((3*b^2*c + 4*a*c^2)*x + 1/2*\text{sqrt}(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*\text{sqrt}(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))/\text{sqrt}(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))) \end{aligned}$$

$$\begin{aligned}
& - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^4*c + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c) \\
& * \text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^5*c - 32*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * a^2*b^2*c^2 - 16*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * a*b^3*c^2 - 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * b^4*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * b^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * a*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * b*c - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 - (\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * b^5 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^3*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * b^4*c + 2*b^5*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * a^2*b*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * b^3*c^2 - 16*a*b^3*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c) \\
& * a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*\text{abs}(b^2 - 4*a*c))*\text{arctan}(2*\text{sqrt}(1/2)*x/\text{sqrt}((b^3 - 4*a*b*c - \text{sqrt}((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2))))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*\text{abs}(b^2 - 4*a*c)*\text{abs}(c))
\end{aligned}$$

maple [A] time = 0.07, size = 342, normalized size = 1.55

$$\frac{\sqrt{2}bc \operatorname{arctanh}\left(\frac{\sqrt{2}cx}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{(4ac-b^2)\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2}bc \operatorname{arctan}\left(\frac{\sqrt{2}cx}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{(4ac-b^2)\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}} - \frac{\sqrt{2}c \operatorname{arctanh}\left(\frac{\sqrt{2}cx}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{2(4ac-b^2)\sqrt{(-b+\sqrt{-4ac+b^2})c}} + \frac{\sqrt{2}c \operatorname{arctan}\left(\frac{\sqrt{2}cx}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{2(4ac-b^2)\sqrt{(b+\sqrt{-4ac+b^2})c}} + \frac{x}{2(4ac-b^2)\left(x^2+\frac{b}{2c}+\frac{\sqrt{-4ac+b^2}}{2c}\right)} + \frac{x}{2(4ac-b^2)\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^5+b*x^3+a*x)^2,x)

[Out] $1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctan}(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctan}(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b+1/2/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctanh}(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*\text{arctanh}(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $-1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*\text{integrate}((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)$

mupad [B] time = 3.36, size = 4854, normalized size = 21.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a*x + b*x^3 + c*x^5)^2,x)

[Out] $\text{atan}\left(\frac{((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}\right)*\left(\frac{((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)} - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}\right)*\left(\frac{((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*i - ((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}\right)*\left(\frac{((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)} + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}\right)*\left(\frac{((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*i\right)/\left(\frac{(4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}\right)*\left(\frac{((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*i\right)/\left(\frac{(4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*((-(4*a*c - b^2)^9)^{(1/2)} - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^{12} + 4096*a^7*c^6 - 24*a^2*b^{10}*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^{(1/2)}*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)}\right)$

$$\begin{aligned}
& (a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} - (x(4a^4c^4 - 5b^2c^3))/(b^4 + 16a^2c^2 - 8ab^2c) * (((-4ac - b^2)^9)^{(1/2)} - b^9 + 768a^4b^4c^4 + 96a^2b^5c^2 - 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} + (((8b^7c^2 - 96ab^5c^3 - 512a^3b^3c^4)/(4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) - (x(((4ac - b^2)^9)^{(1/2)} - b^9 + 768a^4b^4c^4 + 96a^2b^5c^2 - 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} * (8b^7c^2 - 96ab^5c^3 - 512a^3b^3c^4)/(b^4 + 16a^2c^2 - 8ab^2c)) * (((-4ac - b^2)^9)^{(1/2)} - b^9 + 768a^4b^4c^4 + 96a^2b^5c^2 - 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} + (x(4a^4c^4 - 5b^2c^3))/(b^4 + 16a^2c^2 - 8ab^2c)) * (((-4ac - b^2)^9)^{(1/2)} - b^9 + 768a^4b^4c^4 + 96a^2b^5c^2 - 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)})) * (((-4ac - b^2)^9)^{(1/2)} - b^9 + 768a^4b^4c^4 + 96a^2b^5c^2 - 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} * 2i + \operatorname{atan}((((8b^7c^2 - 96ab^5c^3 - 512a^3b^3c^4)/(4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) + (x(-(b^9 + (-4ac - b^2)^9)^{(1/2)} - 768a^4b^4c^4 - 96a^2b^5c^2 + 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} * (8b^7c^2 - 96ab^5c^3 - 512a^3b^3c^4)/(b^4 + 16a^2c^2 - 8ab^2c)) * (-(b^9 + (-4ac - b^2)^9)^{(1/2)} - 768a^4b^4c^4 - 96a^2b^5c^2 + 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} - (x(4a^4c^4 - 5b^2c^3))/(b^4 + 16a^2c^2 - 8ab^2c)) * (-(b^9 + (-4ac - b^2)^9)^{(1/2)} - 768a^4b^4c^4 - 96a^2b^5c^2 + 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} * 1i - (((8b^7c^2 - 96ab^5c^3 - 512a^3b^3c^4)/(4(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c)) - (x(-(b^9 + (-4ac - b^2)^9)^{(1/2)} - 768a^4b^4c^4 - 96a^2b^5c^2 + 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} * (8b^7c^2 - 96ab^5c^3 - 512a^3b^3c^4)/(b^4 + 16a^2c^2 - 8ab^2c)) * (-(b^9 + (-4ac - b^2)^9)^{(1/2)} - 768a^4b^4c^4 - 96a^2b^5c^2 + 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} + (x(4a^4c^4 - 5b^2c^3))/(b^4 + 16a^2c^2 - 8ab^2c)) * (-(b^9 + (-4ac - b^2)^9)^{(1/2)} - 768a^4b^4c^4 - 96a^2b^5c^2 + 512a^3b^3c^3)/(32(a^3b^12 + 4096a^7c^6 - 24a^2b^10c + 240a^3b^8c^2 - 1280a^4b^6c^3 + 3840a^5b^4c^4 - 6144a^6b^2c^5))^{(1/2)} * 1i)/(4a^4c^4 + 3b^2c^3)/(2(b^6 - 64a^3c^3 + 48a^2b^2c^2 - 12ab^4c))
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 c^2 - 12 a b^4 c) + (((8 b^7 c^2 - 96 a b^5 c^3 - 512 a^3 b c^5 + 384 a^2 b^3 c^4) / (4 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) + (x \\
& (-b^9 + (-4 a c - b^2)^9)^{1/2} - 768 a^4 b c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (a b^{12} + 4096 a^7 c^6 - 24 a^2 b^{10} c + 240 a^3 b^8 c^2 - 1 \\
& 280 a^4 b^6 c^3 + 3840 a^5 b^4 c^4 - 6144 a^6 b^2 c^5)))^{1/2} * (8 b^7 c^2 - 96 a b^5 c^3 - 512 a^3 b c^5 + 384 a^2 b^3 c^4) / (b^4 + 16 a^2 c^2 - 8 a b^2 c)) * (-b^9 + (-4 a c - b^2)^9)^{1/2} - 768 a^4 b c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (a b^{12} + 4096 a^7 c^6 - 24 a^2 b^{10} c + 240 a^3 b^8 c^2 - 1280 a^4 b^6 c^3 + 3840 a^5 b^4 c^4 - 6144 a^6 b^2 c^5)))^{1/2} - (x \\
& (4 a c^4 - 5 b^2 c^3) / (b^4 + 16 a^2 c^2 - 8 a b^2 c)) * (-b^9 + (-4 a c - b^2)^9)^{1/2} - 768 a^4 b c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (a b^{12} + 4096 a^7 c^6 - 24 a^2 b^{10} c + 240 a^3 b^8 c^2 - 1280 a^4 b^6 c^3 + 3840 a^5 b^4 c^4 - 6144 a^6 b^2 c^5)))^{1/2} + (((8 b^7 c^2 - 96 a b^5 c^3 - 512 a^3 b c^5 + 384 a^2 b^3 c^4) / (4 (b^6 - 64 a^3 c^3 + 48 a^2 b^2 c^2 - 12 a b^4 c)) - (x * (-b^9 + (-4 a c - b^2)^9)^{1/2} - 768 a^4 b c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (a b^{12} + 4096 a^7 c^6 - 24 a^2 b^{10} c + 240 a^3 b^8 c^2 - 1280 a^4 b^6 c^3 + 3840 a^5 b^4 c^4 - 6144 a^6 b^2 c^5)))^{1/2} * (8 b^7 c^2 - 96 a b^5 c^3 - 512 a^3 b c^5 + 384 a^2 b^3 c^4) / (b^4 + 16 a^2 c^2 - 8 a b^2 c)) * (-b^9 + (-4 a c - b^2)^9)^{1/2} - 768 a^4 b c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (a b^{12} + 4096 a^7 c^6 - 24 a^2 b^{10} c + 240 a^3 b^8 c^2 - 1280 a^4 b^6 c^3 + 3840 a^5 b^4 c^4 - 6144 a^6 b^2 c^5)))^{1/2} + (x * (4 a c^4 - 5 b^2 c^3) / (b^4 + 16 a^2 c^2 - 8 a b^2 c)) * (-b^9 + (-4 a c - b^2)^9)^{1/2} - 768 a^4 b c^4 - 96 a^2 b^5 c^2 + 512 a^3 b^3 c^3) / (32 (a b^{12} + 4096 a^7 c^6 - 24 a^2 b^{10} c + 240 a^3 b^8 c^2 - 1280 a^4 b^6 c^3 + 3840 a^5 b^4 c^4 - 6144 a^6 b^2 c^5)))^{1/2} * 2i + ((b * x) / (2 * (4 a c - b^2))) + (c * x^3) / (4 a c - b^2) / (a + b * x^2 + c * x^4)
\end{aligned}$$

sympy [A] time = 13.17, size = 298, normalized size = 1.35

$\frac{ix + 2x^3}{8ac^2 - 2ab^2 + 1 + (8ac^2 - 2b^2)^2} + \sqrt{(8ac^2 - 2b^2)^2} + \text{RootSum}\left(x^7(1048576a^7c^6 - 1572864a^6b^2c^5 + 983040a^5b^4c^4 - 327680a^4b^6c^3 + 61440a^3b^8c^2 + 256ab^{12}) + x^5(-12288a^4b^2c^4 + 8192a^3b^4c^3 - 1536a^2b^6c^2 + 16b^9) + 16a^2c^3 + 24ab^2c^2 + 96c; \left(1 + \log\left(x + \frac{16384a^3b^3c^4 - 8192a^2b^5c^3 + 512a^2b^7c^2 - 64a^2b^9 - 128a^2b^2c - 16ab^7 - 4b^9}{4ac^2 + 3b^2}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)

[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2 - 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2 + 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c**2 - 16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))

$$3.97 \quad \int \frac{x^3}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=74

$$\frac{2c \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1107, 614, 618, 206}

$$\frac{2c \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]

[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

`Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]`

Rule 1585

`Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x}{(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
 &= -\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{c \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{b^2 - 4ac} \\
 &= -\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2c) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\
 &= -\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2c \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 1.07

$$-\frac{\frac{4c \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + \frac{b + 2cx^2}{a + bx^2 + cx^4}}{2(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^3/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.26, size = 361, normalized size = 4.88

$$\left[\frac{b^3 - 4abc + 2(b^2c - 4ac^2)x^2 + 2(c^2x^4 + bcx^2 + ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)}, -\frac{b^3 - 4abc + 2(b^2c - 4ac^2)x^2 - 4(c^2x^4 + bcx^2 + ac)\sqrt{-b^2 + 4ac} \arctan\left(-\frac{(2cx^2 + b)\sqrt{-b^2 + 4ac}}{b^2 - 4ac}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $[-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]$

giac [A] time = 2.13, size = 82, normalized size = 1.11

$$\frac{2c \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2cx^2 + b}{2(cx^4 + bx^2 + a)(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $-2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))$

maple [A] time = 0.01, size = 75, normalized size = 1.01

$$\frac{2c \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{2cx^2+b}{2(4ac-b^2)(cx^4+bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.16, size = 172, normalized size = 2.32

$$\frac{\frac{b}{2(4ac-b^2)} + \frac{cx^2}{4ac-b^2}}{cx^4+bx^2+a} - \frac{2c \operatorname{atan}\left(\frac{b^3-4abc}{(4ac-b^2)^{3/2}} - \frac{x^2(4ac-b^2)^4 \left(\frac{4c^4}{a(4ac-b^2)^{7/2}} + \frac{4c^2(b^3c^2-4abc^3)(b^3-4abc)}{a(4ac-b^2)^{13/2}}\right)}{8c^4}\right)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*x + b*x^3 + c*x^5)^2,x)

[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(4*a*c - b^2)^(7/2))) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a*c - b^2)^(13/2))))/(8*c^4))/(4*a*c - b^2)^(3/2)

sympy [B] time = 1.28, size = 267, normalized size = 3.61

$$-c \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x^2 + \frac{-16a^2c^3 \sqrt{\frac{1}{(4ac-b^2)}} + 8ab^2c^2 \sqrt{\frac{1}{(4ac-b^2)}} - b^4c \sqrt{\frac{1}{(4ac-b^2)}} + bc}{2c^2}\right) + c \sqrt{\frac{1}{(4ac-b^2)^3}} \log\left(x^2 + \frac{16a^2c^3 \sqrt{\frac{1}{(4ac-b^2)}} - 8ab^2c^2 \sqrt{\frac{1}{(4ac-b^2)}} + b^4c \sqrt{\frac{1}{(4ac-b^2)}} + bc}{2c^2}\right) + \frac{b+2cx^2}{8a^2c-2ab^2+x^4(8ac^2-2b^2c)+x^2(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)`

[Out]
$$-c\sqrt{-1/(4ac - b^2)^3}\log(x^2 + (-16a^2c^3\sqrt{-1/(4ac - b^2)^3} + 8ab^2c^2\sqrt{-1/(4ac - b^2)^3} - b^4c\sqrt{-1/(4ac - b^2)^3} + bc)/(2c^2)) + c\sqrt{-1/(4ac - b^2)^3}\log(x^2 + (16a^2c^3\sqrt{-1/(4ac - b^2)^3} - 8ab^2c^2\sqrt{-1/(4ac - b^2)^3} + b^4c\sqrt{-1/(4ac - b^2)^3} + bc)/(2c^2)) + (b + 2cx^2)/(8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3))$$

$$3.98 \quad \int \frac{x^2}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=252

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(-b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 0.46, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1585, 1092, 1166, 205}

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c} \left(b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(-b\sqrt{b^2 - 4ac} - 12ac + b^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{(a + bx^2 + cx^4)^2} dx \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{b^2 - 2ac - 2(b^2 - 4ac) - bcx^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)} \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(c(b^2 - 12ac - b\sqrt{b^2 - 4ac})\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4a(b^2 - 4ac)^{3/2}} + \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b^2 - 12ac + b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.42, size = 243, normalized size = 0.96

$$\frac{\frac{2x(-2ac + b^2 + bcx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}}{4a} + \frac{\sqrt{2}\sqrt{c}(b\sqrt{b^2 - 4ac} + 12ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/

$\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-b^2 + 12*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]))/ (4*a)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x^2/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 1.17, size = 2309, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*b*c*x^3 + \text{sqrt}(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*\text{sqrt}(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - \text{sqrt}(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*\text{sqrt}(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*\text{sqrt}(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\text{sqrt}((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)))/ ($

$$\begin{aligned}
& 2 - 4*a*c)*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2* \\
& b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^ \\
& 3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& *b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c \\
& + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c - \sqrt{(\\
& 2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b*c^2 - 2*(b^2 - 4*a*c \\
&)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a \\
& *b^6 - 14*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c - 2*\sqrt{2}*\sqrt{ \\
& t(b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^5*c - 2*a*b^6*c + 64*\sqrt{2}*\sqrt{b*c + \sqrt{ \\
& rt(b^2 - 4*a*c}*c})*a^3*b^2*c^2 + 20*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c} \\
& *a^2*b^3*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^2 + 28*a^2*b \\
& ^4*c^2 - 96*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^4*c^3 - 48*\sqrt{2}*\sqrt{ \\
& rt(b*c + \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^3 - 10*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}*c})*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}*c})*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c \\
&)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*\text{abs}(a*b^2 - 4*a^2*c))*\arctan(2*\sqrt{ \\
& rt(1/2)*x/\sqrt{((a*b^3 - 4*a^2*b*c + \sqrt{(a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 \\
& - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/((a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12* \\
& a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6 \\
& 4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*\text{abs}(a*b^2 - 4*a^2*c) \\
& *\text{abs}(c)) - 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5 \\
& *b*c^5 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^7 \\
& + 20*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^5*c + \\
& 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^6*c - 112 \\
& *\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*b^3*c^2 - 32 \\
& *\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^4*c^2 - \sqrt{ \\
& rt(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^5*c^2 + 192*\sqrt{ \\
& rt(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^5*b*c^3 + 96*\sqrt{ \\
& rt(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*b^2*c^3 + 16*\sqrt{ \\
& rt(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^3*c^3 - 48*\sqrt{ \\
& rt(2)*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*b*c^4 - 2*(b^2 - \\
& 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b \\
& *c^4 + (2*b^3*c^2 - 8*a*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& ^2 - 4*a*c}*c})*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a* \\
& c}*c})*a*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b \\
& ^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b*c^2 - 2* \\
& (b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}*c})*a*b^6 - 14*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^4*c - 2 \\
& *\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^5*c + 2*a*b^6*c + 64*\sqrt{2}*\sqrt{ \\
& rt(b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b^2*c^2 + 20*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - \\
& 4*a*c}*c})*a^2*b^3*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^4*c^ \\
& 2 - 28*a^2*b^4*c^2 - 96*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^4*c^3 - 4 \\
& 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c^3 - 10*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*\sqrt{2}*\sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}*c})*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
\end{aligned}$$

$$(b^2 - 4ac) \cdot a^2 b^2 c^2 - 48(b^2 - 4ac) \cdot a^3 c^3 \cdot \text{abs}(a b^2 - 4a^2 c) \cdot \arctan\left(\frac{2\sqrt{1/2} \cdot x / \sqrt{(a b^3 - 4a^2 b c - \sqrt{(a b^3 - 4a^2 b c)^2 - 4(a^2 b^2 - 4a^3 c)(a b^2 c - 4a^2 c^2)})}}{a b^2 c - 4a^2 c^2}\right) / ((a^3 b^6 - 12a^4 b^4 c - 2a^3 b^5 c + 48a^5 b^2 c^2 + 16a^4 b^3 c^2 + a^3 b^4 c^2 - 64a^6 c^3 - 32a^5 b c^3 - 8a^4 b^2 c^3 + 16a^5 c^4) \cdot \text{abs}(a b^2 - 4a^2 c) \cdot \text{abs}(c))$$

maple [B] time = 0.06, size = 733, normalized size = 2.91

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(c \cdot x^5 + b \cdot x^3 + a \cdot x)^2, x)$

[Out]
$$\begin{aligned} & -1/4/(4ac-b^2)/ax/(x^2+1/2b/c+1/2(-4ac+b^2)^{1/2}/c) \cdot b-c/(-4ac+b^2)^{1/2}/(4ac-b^2) \cdot x/(x^2+1/2b/c+1/2(-4ac+b^2)^{1/2}/c) + 1/4/(-4ac+b^2)^{1/2}/(4ac-b^2)/ax/(x^2+1/2b/c+1/2(-4ac+b^2)^{1/2}/c) \cdot b^2-1/4c/(4ac-b^2)/a^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot cx) \cdot b-3c^2/(-4ac+b^2)^{1/2}/(4ac-b^2) \cdot 2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot cx) + 1/4c/(-4ac+b^2)^{1/2}/(4ac-b^2)/a^2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot \arctan(2^{1/2}/((b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot cx) \cdot b^2-1/4/(4ac-b^2)/ax/(x^2+1/2b/c-1/2(-4ac+b^2)^{1/2}/c) \cdot b+c/(-4ac+b^2)^{1/2}/(4ac-b^2) \cdot x/(x^2+1/2b/c-1/2(-4ac+b^2)^{1/2}/c) - 1/4/(-4ac+b^2)^{1/2}/(4ac-b^2)/ax/(x^2+1/2b/c-1/2(-4ac+b^2)^{1/2}/c) \cdot b^2+1/4c/(4ac-b^2)/a^2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot cx) \cdot b-3c^2/(-4ac+b^2)^{1/2}/(4ac-b^2) \cdot 2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot cx) + 1/4c/(-4ac+b^2)^{1/2}/(4ac-b^2)/a^2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot \operatorname{arctanh}(2^{1/2}/((-b+(-4ac+b^2)^{1/2})c)^{1/2} \cdot cx) \cdot b^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2/(c \cdot x^5 + b \cdot x^3 + a \cdot x)^2, x, \text{algorithm}="maxima")$

[Out]
$$\frac{1}{2} \cdot (b \cdot c \cdot x^3 + (b^2 - 2ac) \cdot x) / ((a b^2 c - 4a^2 c^2) \cdot x^4 + a^2 b^2 - 4a^3 c + (a b^3 - 4a^2 b c) \cdot x^2) + \frac{1}{2} \cdot \text{integrate}((b \cdot c \cdot x^2 + b^2 - 6ac) / (c \cdot x^4 + b \cdot x^2 + a), x) / (a b^2 - 4a^2 c)$$

mupad [B] time = 3.85, size = 6404, normalized size = 25.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(a*x + b*x^3 + c*x^5)^2, x)$

[Out]
$$\begin{aligned} & \left(\frac{x(2ac - b^2)}{2a(4ac - b^2)} - \frac{bcx^3}{2a(4ac - b^2)} \right) / (a + bx^2 + cx^4) + \text{atan}\left(\frac{((6144a^5c^6 + 16ab^8c^2 - 288a^2b^6c^3 + 1920a^3b^4c^4 - 5632a^4b^2c^5) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - (x(-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2}) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2}}{(1024a^5b^5c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))} \right) * (-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2} + (x(72a^2c^5 + b^4c^3 - 14ab^2c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))) * (-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2} * i - \left(\frac{((6144a^5c^6 + 16ab^8c^2 - 288a^2b^6c^3 + 1920a^3b^4c^4 - 5632a^4b^2c^5) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) + (x(-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2}) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2}}{(1024a^5b^5c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))} \right) * (-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2} - (x(72a^2c^5 + b^4c^3 - 14ab^2c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))) * (-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2} / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2} * i) / \left(\frac{((6144a^5c^6 + 16ab^8c^2 - 288a^2b^6c^3 + 1920a^3b^4c^4 - 5632a^4b^2c^5) / (8(a^2b^6 - 64a^5c^3 - 12a^3b^4c + 48a^4b^2c^2)) - (x(-b^{11} + b^2(-4ac - b^2)^9)^{1/2} - 3840a^5b^5c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3c^4 - 27ab^9c - 9ac(-4ac - b^2)^9)^{1/2}) / (32(a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5))^{1/2}}{(1024a^5b^5c^5 - 16a^2b^7c^2 + 192a^3b^5c^3 - 768a^4b^3c^4) / (2(a^2b^4 + 16a^4c^2 - 8a^3b^2c))} \right) \end{aligned}$$

$$\begin{aligned}
& - 8*a^3*b^2*c)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + \\
& 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c \\
& *(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 2 \\
& 40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 \\
& - 8*a^3*b^2*c)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + \\
& 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c* \\
& (-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 24 \\
& 0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 \\
& - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (5*b^3*c^4 - 36*a*b*c^5)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))*(-(b^{11} + b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*2i + atan((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)}*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^3*b^{12} + 4096*a^9*c^6 - 24*a^4*b^{10}*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^{(1/2)} + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^{11} - b^2*(-(4*a*c - b^2)^9)^{(1/2)} - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c + 9*a*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(
\end{aligned}$$

$$\begin{aligned}
& c^4 - 6144a^8b^2c^5))^{(1/2)} - (x*(72a^2c^5 + b^4c^3 - 14ab^2c^4)) \\
& / (2*(a^2b^4 + 16a^4c^2 - 8a^3b^2c)) * (-b^{11} - b^2*(-(4ac - b^2)^9) \\
& ^{(1/2)} - 3840a^5b^2c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3 \\
& *c^4 - 27ab^9c + 9ac*(-(4ac - b^2)^9)^{(1/2)}) / (32*(a^3b^{12} + 4096a^9 \\
& *c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c \\
& ^4 - 6144a^8b^2c^5))^{(1/2)} + (5b^3c^4 - 36ab^2c^5) / (4*(a^2b^6 - 64a \\
& ^5c^3 - 12a^3b^4c + 48a^4b^2c^2))) * (-b^{11} - b^2*(-(4ac - b^2)^9) \\
&)^{(1/2)} - 3840a^5b^2c^5 + 288a^2b^7c^2 - 1504a^3b^5c^3 + 3840a^4b^3 \\
& *c^4 - 27ab^9c + 9ac*(-(4ac - b^2)^9)^{(1/2)}) / (32*(a^3b^{12} + 4096a \\
& ^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c \\
& ^4 - 6144a^8b^2c^5))^{(1/2)} * 2i
\end{aligned}$$

sympy [A] time = 165.89, size = 394, normalized size = 1.56

$$\frac{-b^2 + (2b - a)^2}{b^2 - 2a^2 + a^2(b^2 - 2a^2)} + \text{RootSum}\left(\left(\frac{1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12}}{32a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5}\right) + \log\left(\frac{22788a^2b^6 - 28672a^3b^4c + 9216a^4b^2c^2 - 1280a^5b^2c^3 + 64a^6b^2c^4 + 1728a^7b^2c^5 - 2304a^8b^2c^6 - 92a^9c^6 + 4a^{10}}{32a^3b^{12} + 4096a^9c^6 - 24a^4b^{10}c + 240a^5b^8c^2 - 1280a^6b^6c^3 + 3840a^7b^4c^4 - 6144a^8b^2c^5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)

[Out] $(-b*c*x**3 + x*(2*a*c - b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + \text{RootSum}(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b**4*c**3, \text{Lambda}(_t, _t*\log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*b**4*c**2))))$

$$3.99 \quad \int \frac{x}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=122

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx^2 + cx^4)}{4a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx^2 + cx^4)}{4a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-b^2 + 4ac - bcx}{x(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{-b^2 + 4ac}{ax} + \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a(a + bx + cx^2)} \right) dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(x)}{a^2} - \frac{\text{Subst} \left(\int \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(x)}{a^2} - \frac{\text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2} - \frac{(b(b^2 - 6ac))}{2a^2} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(x)}{a^2} - \frac{\log(a + bx^2 + cx^4)}{4a^2} + \frac{(b(b^2 - 6ac)) \text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2a^2} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b(b^2 - 6ac) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2a^2(b^2 - 4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a + bx^2 + cx^4)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 207, normalized size = 1.70

$$\frac{2a(-2ac + b^2 + bcx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^2 \sqrt{b^2 - 4ac} - 4ac \sqrt{b^2 - 4ac} - 6abc + b^3) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-b^2 \sqrt{b^2 - 4ac} + 4ac \sqrt{b^2 - 4ac} - 6abc + b^3) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + 4 \log(x)}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x + b*x^3 + c*x^5)^2, x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -

$b^2\sqrt{b^2 - 4ac} + 4ac\sqrt{b^2 - 4ac})\log[b + \sqrt{b^2 - 4ac} + 2cx^2]/(b^2 - 4ac)^{3/2}/(4a^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a*x + b*x^3 + c*x^5)^2,x]

[Out] IntegrateAlgebraic[x/(a*x + b*x^3 + c*x^5)^2, x]

fricas [B] time = 2.06, size = 813, normalized size = 6.66

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*\sqrt{b^2 - 4*a*c}))/ (c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*\log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*\log(x)] / (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c})/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*\log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*\log(x)] / (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)$

giac [A] time = 2.32, size = 166, normalized size = 1.36

$$-\frac{(b^3 - 6abc)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^2b^2 - 4a^3c)\sqrt{-b^2+4ac}} + \frac{b^2cx^4 - 4ac^2x^4 + b^3x^2 - 2abcx^2 + 3ab^2 - 8a^2c}{4(cx^4 + bx^2 + a)(a^2b^2 - 4a^3c)} - \frac{\log(cx^4 + bx^2 + a)}{4a^2} + \frac{\log(x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $-1/2*(b^3 - 6*a*b*c)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^2*b^2 - 4*a^3*c)*\sqrt{-b^2 + 4*a*c}) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/4*\log(c*x^4 + b*x^2 + a)/a^2 + 1/2*\log(x^2)/a^2$

maple [B] time = 0.02, size = 253, normalized size = 2.07

$$-\frac{bcx^2}{2(cx^4+bx^2+a)(4ac-b^2)a} - \frac{3bc\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a} + \frac{b^3\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2(4ac-b^2)^{\frac{3}{2}}a^2} - \frac{b^2}{2(cx^4+bx^2+a)(4ac-b^2)a} - \frac{c\ln(cx^4+bx^2+a)}{(4ac-b^2)a} + \frac{b^2\ln(cx^4+bx^2+a)}{4(4ac-b^2)a^2} + \frac{c}{(cx^4+bx^2+a)(4ac-b^2)} + \frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^5+b*x^3+a*x)^2,x)

[Out] $1/a^2*\ln(x) - 1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2 + 1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c - 1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2 - 1/a/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a) + 1/4/a^2/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^2 - 3/a/(4*a*c-b^2)^{(3/2)}*a*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*c + 1/2/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bcx^2 + b^2 - 2ac}{2((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)} + \frac{-\int \frac{(b^2c - 4ac^2)x^3 + (b^3 - 5abc)x}{cx^4 + bx^2 + a} dx}{a^2b^2 - 4a^3c} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + \text{integrate}(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + \log(x)/a^2$

mupad [B] time = 6.31, size = 5048, normalized size = 41.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*x + b*x^3 + c*x^5)^2,x)

[Out] $\log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (\log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +$

$$\begin{aligned}
& 192*a^4*b^2*c^2)) + (b*\operatorname{atan}((x^2*(((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + \\
& 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48 \\
& *a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560* \\
& a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6* \\
& b^3*c^5)))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))*(4*a^2* \\
& b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2 \\
& *(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2* \\
& c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056 \\
& *a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64* \\
& a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3* \\
& b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4 \\
& *c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((\\
& 6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4 \\
& *b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 \\
& - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) \\
& - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 1 \\
& 2*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5)))/(2* \\
& (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))*(4*a^2*b^6 - 256*a^5 \\
& *c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2* \\
& c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2 \\
& *c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(\\
& 2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688 \\
& *a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b \\
& ^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c) \\
&))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 7 \\
& 2*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(((6*a*b^5*c^4 + 80*a^3* \\
& b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c \\
& ^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/ \\
& (a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3* \\
& c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a \\
& ^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5)))/(2*(a^3*b^6 - 64*a^6*c^3 \\
& - 12*a^4*b^4*c + 48*a^5*b^2*c^2))*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + \\
& 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2 \\
& *(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128* \\
& a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^ \\
& 3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4* \\
& c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 \\
& + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + \\
& 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(256 \\
& 0*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^ \\
& 6*b^3*c^5)))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))*(4*a^ \\
& 2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a \\
& ^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^ \\
& 2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 10 \\
& 56*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 6
\end{aligned}$$

$$\begin{aligned}
& 4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))/(4*a^2*(4*a*c - b^2)^{(3/2)}) + (b^2*(6*a*c - b^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))/(8*a^3*c^2*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c))*(16*a^6*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^9*c^3*(4*a*c - b^2)^{(9/2)} - 192*a^7*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^8*b^2*c^2*(4*a*c - b^2)^{(9/2)))/(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^{(3/2)}) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^{(3/2)}) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b^2)^{(3/2)}*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b^3*(6*a*c - b^2)^3*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4))/(64*a^6*(4*a*c - b^2)^{(9/2)}*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^9*c^3*(4*a*c - b^2)^{(9/2)} - 192*a^7*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^8*b^2*c^2*(4*a*c - b^2)^{(9/2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^{(7/2)}*(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(16*a^6*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^9*c^3*(4*a*c - b^2)^{(9/2)} - 192*a^7*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^8*b^2*c^2*(4*a*c - b^2)^{(9/2)))*(((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (b*(6*a*c - b^2)*((b*((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32
\end{aligned}$$

$$\begin{aligned} & *a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a \\ & *b^4*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - \\ & 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2)/(4*a^2*(4*a*c - b^2)^{(3/2)} \\ &)) + (b*(6*a*c - b^2)*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2* \\ & b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(8*a^2*(4*a*c - b^2)^{(3/2)} \\ &)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^ \\ & 4*c + 192*a^4*b^2*c^2)))/(4*a^2*(4*a*c - b^2)^{(3/2)}) + (b^2*(6*a*c - b^2)^ \\ & 2*(4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + \\ & 96*a^2*b^2*c^2 - 24*a*b^4*c))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 \\ & - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) \\ &))/(8*a^3*c^2*(4*a*c - b^2)^3*(b^6*c^2 - 12*a*b^4*c^3 + 36*a^2*b^2*c^4)*(6* \\ & b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c))*(6*a*c - b^2)/(2*a^2*(\\ & 4*a*c - b^2)^{(3/2)}) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**5+b*x**3+a*x)**2,x)

[Out] Timed out

$$3.100 \quad \int \frac{1}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=308

$$\frac{3b^2 - 10ac}{2a^2x(b^2 - 4ac)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(-(3b^2 - 10ac) \sqrt{b^2 - 4ac} \right)}{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{-2ac + b^2 + bcx^2}{2ax(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Rubi [A] time = 1.35, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1594, 1121, 1281, 1166, 205}

$$\frac{3b^2 - 10ac}{2a^2x(b^2 - 4ac)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(-(3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2} a^2 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{-2ac + b^2 + bcx^2}{2ax(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^(-2), x]

[Out] $-(3b^2 - 10ac)/(2a^2(b^2 - 4ac)x) + (b^2 - 2ac + bcx^2)/(2a(b^2 - 4ac)x(a + bx^2 + cx^4)) - (\text{Sqrt}[c]*(3b^3 - 16abc + (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]])]/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]) + (\text{Sqrt}[c]*(3b^3 - 16abc - (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]])]/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]])$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> -Simp[((d*x)^(m+1)*(b^2 - 2ac + bcx^2)*(a + bx^2 + cx^4)^(p+1))/(2ad*(p+1)*(b^2 - 4ac)), x] + Dist[1/(2a*(p+1)*(b^2 - 4ac)), Int[(d*x)^m*(a + bx^2 + cx^4)^(p+1)*Simp[b^2*(m+2p+3) - 2ac*(m+4p+5) + bc*(m+4p+7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4ac, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^2 (a + bx^2 + cx^4)^2} dx \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\int \frac{-3b^2 + 10ac - 3bcx^2}{x^2(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{\int \frac{-b(3b^2 - 13ac) - c(3b^2 - 10ac)x^2}{a + bx^2 + cx^4} dx}{2a^2(b^2 - 4ac)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\left(c\left(3b^2 - 10ac + \frac{3b^3}{\sqrt{b^2 - 4ac}} - \frac{1}{\sqrt{b^2 - 4ac}}\right)\right)}{4a^2(b^2 - 4ac)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\sqrt{c}\left(3b^2 - 10ac + \frac{3b^3}{\sqrt{b^2 - 4ac}} - \frac{1}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{2}a^2(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.62, size = 302, normalized size = 0.98

$$\frac{-\frac{2x(-3abc-2ac^2x^2+b^3+b^2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{4}{x}}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(-2), x]

[Out]
$$\left(-\frac{4}{x} - \frac{(2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))}{(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)} + \frac{(\text{Sqrt}[2]*\text{Sqrt}[c]*(-3*b^3 + 16*a*b*c - 3*b^2*\text{Sqrt}[b^2 - 4*a*c] + 10*a*c*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]]}{(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]} + \frac{(\text{Sqrt}[2]*\text{Sqrt}[c]*(3*b^3 - 16*a*b*c - 3*b^2*\text{Sqrt}[b^2 - 4*a*c] + 10*a*c*\text{Sqrt}[b^2 - 4*a*c]))*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]]}{(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]}\right)/(4*a^2)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^(-2), x]

[Out] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^(-2), x]

fricas [B] time = 1.65, size = 2912, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out]
$$-\frac{1}{4}*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*x^2 - \text{sqrt}(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*\text{sqrt}(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*\text{sqrt}((8*1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*\log(-\frac{(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*\text{sqrt}(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*$$

$$\begin{aligned}
& a^9 b^2 c^4 - 1280 a^{10} c^5) \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 + (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) + \sqrt{1/2} * ((a^2 b^2 c - 4 a^3 c^2) x^5 + (a^2 b^3 - 4 a^3 b c) x^3 + (a^3 b^2 - 4 a^4 c) x) \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 + (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) * \log(-(189 b^6 c^3 - 1971 a b^4 c^4 + 5625 a^2 b^2 c^5 - 2500 a^3 c^6) x - 1/2 \sqrt{1/2} * (27 b^{11} - 486 a b^9 c + 3330 a^2 b^7 c^2 - 10549 a^3 b^5 c^3 + 14408 a^4 b^3 c^4 - 5200 a^5 b c^5 - (3 a^5 b^{10} - 55 a^6 b^8 c + 392 a^7 b^6 c^2 - 1344 a^8 b^4 c^3 + 2176 a^9 b^2 c^4 - 1280 a^{10} c^5) \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 + (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) - \sqrt{1/2} * ((a^2 b^2 c - 4 a^3 c^2) x^5 + (a^2 b^3 - 4 a^3 b c) x^3 + (a^3 b^2 - 4 a^4 c) x) \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 - (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) * \log(-(189 b^6 c^3 - 1971 a b^4 c^4 + 5625 a^2 b^2 c^5 - 2500 a^3 c^6) x + 1/2 \sqrt{1/2} * (27 b^{11} - 486 a b^9 c + 3330 a^2 b^7 c^2 - 10549 a^3 b^5 c^3 + 14408 a^4 b^3 c^4 - 5200 a^5 b c^5 + (3 a^5 b^{10} - 55 a^6 b^8 c + 392 a^7 b^6 c^2 - 1344 a^8 b^4 c^3 + 2176 a^9 b^2 c^4 - 1280 a^{10} c^5) \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 - (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) + \sqrt{1/2} * ((a^2 b^2 c - 4 a^3 c^2) x^5 + (a^2 b^3 - 4 a^3 b c) x^3 + (a^3 b^2 - 4 a^4 c) x) \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 - (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3)) * \log(-(189 b^6 c^3 - 1971 a b^4 c^4 + 5625 a^2 b^2 c^5 - 2500 a^3 c^6) x - 1/2 \sqrt{1/2} * (27 b^{11} - 486 a b^9 c + 3330 a^2 b^7 c^2 - 10549 a^3 b^5 c^3 + 14408 a^4 b^3 c^4 - 5200 a^5 b c^5 + (3 a^5 b^{10} - 55 a^6 b^8 c + 392 a^7 b^6 c^2 - 1344 a^8 b^4 c^3 + 2176 a^9 b^2 c^4 - 1280 a^{10} c^5) \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} \sqrt{-(9 b^7 - 105 a b^5 c + 385 a^2 b^3 c^2 - 420 a^3 b c^3 - (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))} \sqrt{(81 b^8 - 918 a b^6 c + 3051 a^2 b^4 c^2 - 2550 a^3 b^2 c^3 + 625 a^4 c^4) / (a^{10} b^6 - 12 a^{11} b^4 c + 48 a^{12} b^2 c^2 - 64 a^{13} c^3))} / (a^5 b^6 - 12 a^6 b^4 c + 48 a^7 b^2 c^2 - 64 a^8 c^3))
\end{aligned}$$

$$- 5200*a^5*b*c^5 + (3*a^5*b^{10} - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^{10}*c^5)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)}/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)))*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)}/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x}$$

giac [B] time = 2.43, size = 3087, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out]
$$-1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b^8 + 40*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^5*b^6*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b^7*c - 176*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^6*b^4*c^2 - 56*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^5*b^5*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b^6*c^2 + 256*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^7*b^2*c^3 + 128*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^6*b^3*c^3 + 28*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^5*b^4*c^3 - 64*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*b^4 + 22*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b^2*c + 6*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*b^3*c - 40*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*c^2 - 20*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*b*c^2 - 3*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*b^2*c^2 + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^7 - 37*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^5*c - 6*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^4*b^3*c^2 + 50*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^3*b^4*c^2 + 3*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*a^5*b*c^3 - 104*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*c)*$$

$$\begin{aligned}
& *c) *a^4 *b^2 *c^3 - 25 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} *c) *a^3 *b^3 *c^3 - \\
& 304 *a^4 *b^3 *c^3 + 52 * \sqrt{2} * \sqrt{b*c + \sqrt{b^2 - 4*a*c}} *c) *a^4 *b*c^4 + 41 \\
& 6 *a^5 *b*c^4 + 6 * (b^2 - 4*a*c) *a^2 *b^5 *c - 50 * (b^2 - 4*a*c) *a^3 *b^3 *c^2 + 10 \\
& 4 * (b^2 - 4*a*c) *a^4 *b*c^3) * \operatorname{abs}(a^2 *b^2 - 4*a^3 *c) * \arctan(2 * \sqrt{1/2} * x / \sqrt{ \\
& t((a^2 *b^3 - 4*a^3 *b*c + \sqrt{(a^2 *b^3 - 4*a^3 *b*c)^2 - 4*(a^3 *b^2 - 4*a^4 * \\
& c) * (a^2 *b^2 *c - 4*a^3 *c^2))}) / (a^2 *b^2 *c - 4*a^3 *c^2)) / ((a^5 *b^6 - 12*a^6 *b \\
& ^4 *c - 2*a^5 *b^5 *c + 48*a^7 *b^2 *c^2 + 16*a^6 *b^3 *c^2 + a^5 *b^4 *c^2 - 64*a^8 \\
& *c^3 - 32*a^7 *b*c^3 - 8*a^6 *b^2 *c^3 + 16*a^7 *c^4) * \operatorname{abs}(a^2 *b^2 - 4*a^3 *c) * \operatorname{ab} \\
& s(c)) + 1/16 * (6*a^4 *b^8 *c^2 - 80*a^5 *b^6 *c^3 + 352*a^6 *b^4 *c^4 - 512*a^7 *b^ \\
& 2 *c^5 - 3 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^4 *b^8 \\
& + 40 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^5 *b^6 *c + \\
& 6 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^4 *b^7 *c - 17 \\
& 6 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^6 *b^4 *c^2 - 5 \\
& 6 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^5 *b^5 *c^2 - 3 \\
& * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^4 *b^6 *c^2 + 25 \\
& 6 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^7 *b^2 *c^3 + 1 \\
& 28 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^6 *b^3 *c^3 + \\
& 28 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^5 *b^4 *c^3 - \\
& 64 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^6 *b^2 *c^4 - \\
& 6 * (b^2 - 4*a*c) *a^4 *b^6 *c^2 + 56 * (b^2 - 4*a*c) *a^5 *b^4 *c^3 - 128 * (b^2 - 4*a \\
& *c) *a^6 *b^2 *c^4 + (6*b^4 *c^2 - 44*a*b^2 *c^3 + 80*a^2 *c^4 - 3 * \sqrt{2} * \sqrt{b \\
& ^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *b^4 + 22 * \sqrt{2} * \sqrt{b^2 - 4*a \\
& *c} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a*b^2 *c + 6 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{ \\
& b*c - \sqrt{b^2 - 4*a*c}} *c) *b^3 *c - 40 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}} *c) *a^2 *c^2 - 20 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}} *c) *a*b*c^2 - 3 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}} *c) *b^2 *c^2 + 10 * \sqrt{2} * \sqrt{b^2 - 4*a*c} * \sqrt{b*c - \sqrt{b^2 \\
& - 4*a*c}} *c) *a*c^3 - 6 * (b^2 - 4*a*c) *b^2 *c^2 + 20 * (b^2 - 4*a*c) *a*c^3) * (a^2 * \\
& b^2 - 4*a^3 *c)^2 - 2 * (3 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^2 *b^7 - 3 \\
& 7 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^3 *b^5 *c - 6 * \sqrt{2} * \sqrt{b*c - \\
& \sqrt{b^2 - 4*a*c}} *c) *a^2 *b^6 *c + 6 *a^2 *b^7 *c + 152 * \sqrt{2} * \sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}} *c) *a^4 *b^3 *c^2 + 50 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^ \\
& 3 *b^4 *c^2 + 3 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^2 *b^5 *c^2 - 74 *a^3 * \\
& b^5 *c^2 - 208 * \sqrt{2} * \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^5 *b*c^3 - 104 * \sqrt{2} * \\
& \sqrt{b*c - \sqrt{b^2 - 4*a*c}} *c) *a^4 *b^2 *c^3 - 25 * \sqrt{2} * \sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}} *c) *a^3 *b^3 *c^3 + 304 *a^4 *b^3 *c^3 + 52 * \sqrt{2} * \sqrt{b*c - \sqrt{ \\
& b^2 - 4*a*c}} *c) *a^4 *b*c^4 - 416 *a^5 *b*c^4 - 6 * (b^2 - 4*a*c) *a^2 *b^5 *c + 50 \\
& * (b^2 - 4*a*c) *a^3 *b^3 *c^2 - 104 * (b^2 - 4*a*c) *a^4 *b*c^3) * \operatorname{abs}(a^2 *b^2 - 4*a \\
& ^3 *c) * \arctan(2 * \sqrt{1/2} * x / \sqrt{(a^2 *b^3 - 4*a^3 *b*c - \sqrt{(a^2 *b^3 - 4*a \\
& ^3 *b*c)^2 - 4*(a^3 *b^2 - 4*a^4 *c) * (a^2 *b^2 *c - 4*a^3 *c^2))}) / (a^2 *b^2 *c - 4* \\
& a^3 *c^2)) / ((a^5 *b^6 - 12*a^6 *b^4 *c - 2*a^5 *b^5 *c + 48*a^7 *b^2 *c^2 + 16*a^6 \\
& *b^3 *c^2 + a^5 *b^4 *c^2 - 64*a^8 *c^3 - 32*a^7 *b*c^3 - 8*a^6 *b^2 *c^3 + 16*a^7 \\
& *c^4) * \operatorname{abs}(a^2 *b^2 - 4*a^3 *c) * \operatorname{abs}(c))
\end{aligned}$$

[Out] - atan(((((-9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - (((-9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((((-9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((((-9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2)))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +

$$\begin{aligned}
& (240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5) \\
&)^{(1/2)}*(851968*a^{14}*b*c^8 + 192*a^8*b^{13}*c^2 - 4672*a^9*b^{11}*c^3 + 47360*a \\
& ^{10}*b^9*c^4 - 256000*a^{11}*b^7*c^5 + 778240*a^{12}*b^5*c^6 - 1261568*a^{13}*b^3* \\
& c^7 + x*(-(9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077 \\
& *a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - \\
& 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - \\
& b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8* \\
& c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(104 \\
& 8576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 - 6144*a^{11}*b^{11}*c^3 + 61440*a^{12}*b^9*c \\
& ^4 - 327680*a^{13}*b^7*c^5 + 983040*a^{14}*b^5*c^6 - 1572864*a^{15}*b^3*c^7)) + x \\
& *(204800*a^{12}*c^9 + 144*a^6*b^{12}*c^3 - 3264*a^7*b^{10}*c^4 + 30112*a^8*b^8*c^ \\
& 5 - 143360*a^9*b^6*c^6 + 365568*a^{10}*b^4*c^7 - 458752*a^{11}*b^2*c^8))*(-(9*b \\
& ^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - \\
& 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(\\
& 4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)}) \\
& /(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8 \\
& *b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)} + ((-(9*b^{13} - 9*b \\
& ^4*(-(4*a*c - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^ \\
& 3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5 \\
& *b^{12} + 4096*a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 \\
& + 3840*a^9*b^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(851968*a^{14}*b*c^8 + 192*a^ \\
& 8*b^{13}*c^2 - 4672*a^9*b^{11}*c^3 + 47360*a^{10}*b^9*c^4 - 256000*a^{11}*b^7*c^5 + \\
& 778240*a^{12}*b^5*c^6 - 1261568*a^{13}*b^3*c^7 - x*(-(9*b^{13} - 9*b^4*(-(4*a*c \\
& - b^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + \\
& 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096 \\
& *a^{11}*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b \\
& ^4*c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*(1048576*a^{16}*b*c^8 + 256*a^{10}*b^{13}*c^2 \\
& - 6144*a^{11}*b^{11}*c^3 + 61440*a^{12}*b^9*c^4 - 327680*a^{13}*b^7*c^5 + 983040*a \\
& ^{14}*b^5*c^6 - 1572864*a^{15}*b^3*c^7)) - x*(204800*a^{12}*c^9 + 144*a^6*b^{12}*c^ \\
& 3 - 3264*a^7*b^{10}*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^1 \\
& 0*b^4*c^7 - 458752*a^{11}*b^2*c^8))*(-(9*b^{13} - 9*b^4*(-(4*a*c - b^2)^9)^{(1/2)} \\
&) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5* \\
& c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 213*a*b^{11}* \\
& c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^{11}*c^6 - 24 \\
& *a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144* \\
& a^{10}*b^2*c^5))^{(1/2)} + 128000*a^{10}*c^9 + 504*a^6*b^8*c^5 - 8112*a^7*b^6*c^ \\
& 6 + 48704*a^8*b^4*c^7 - 129280*a^9*b^2*c^8))*(-(9*b^{13} - 9*b^4*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302 \\
& 40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - \\
& 213*a*b^{11}*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^5*b^{12} + 4096*a^ \\
& 11*c^6 - 24*a^6*b^{10}*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4* \\
& c^4 - 6144*a^{10}*b^2*c^5))^{(1/2)}*2i - \operatorname{atan}(\frac{(-(9*b^{13} + 9*b^4*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
\end{aligned}$$

$$\begin{aligned}
& 13c^2 - 4672a^9b^{11}c^3 + 47360a^{10}b^9c^4 - 256000a^{11}b^7c^5 + 778240a^{12}b^5c^6 - 1261568a^{13}b^3c^7 + x \cdot (-9b^{13} + 9b^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c - 51a^2b^2c \cdot (-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) + x \cdot (204800a^{12}c^9 + 144a^6b^{12}c^3 - 3264a^7b^{10}c^4 + 30112a^8b^8c^5 - 143360a^9b^6c^6 + 365568a^{10}b^4c^7 - 458752a^{11}b^2c^8) \cdot (-9b^{13} + 9b^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c - 51a^2b^2c \cdot (-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} + ((-9b^{13} + 9b^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c - 51a^2b^2c \cdot (-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (851968a^{14}b^8c^8 + 192a^8b^{13}c^2 - 4672a^9b^{11}c^3 + 47360a^{10}b^9c^4 - 256000a^{11}b^7c^5 + 778240a^{12}b^5c^6 - 1261568a^{13}b^3c^7 - x \cdot (-9b^{13} + 9b^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c - 51a^2b^2c \cdot (-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * (1048576a^{16}b^8c^8 + 256a^{10}b^{13}c^2 - 6144a^{11}b^{11}c^3 + 61440a^{12}b^9c^4 - 327680a^{13}b^7c^5 + 983040a^{14}b^5c^6 - 1572864a^{15}b^3c^7) - x \cdot (204800a^{12}c^9 + 144a^6b^{12}c^3 - 3264a^7b^{10}c^4 + 30112a^8b^8c^5 - 143360a^9b^6c^6 + 365568a^{10}b^4c^7 - 458752a^{11}b^2c^8) \cdot (-9b^{13} + 9b^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c - 51a^2b^2c \cdot (-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} + 128000a^{10}c^9 + 504a^6b^8c^5 - 8112a^7b^6c^6 + 48704a^8b^4c^7 - 129280a^9b^2c^8) \cdot (-9b^{13} + 9b^4 \cdot (-4ac - b^2)^9)^{(1/2)} + 26880a^6b^6c^6 + 2077a^2b^9c^2 - 10656a^3b^7c^3 + 30240a^4b^5c^4 - 44800a^5b^3c^5 + 25a^2c^2 \cdot (-4ac - b^2)^9)^{(1/2)} - 213ab^{11}c - 51a^2b^2c \cdot (-4ac - b^2)^9)^{(1/2)} / (32(a^5b^{12} + 4096a^{11}c^6 - 24a^6b^{10}c + 240a^7b^8c^2 - 1280a^8b^6c^3 + 3840a^9b^4c^4 - 6144a^{10}b^2c^5))^{(1/2)} * 2i - (1/a + (bx^2(11ac - 3b^2))/(2a^2(4ac - b^2))) + (cx^4(10ac - 3b^2))/(2a^2(4ac - b^2)))/(ax + bx^3 + cx^5)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**5+b*x**3+a*x)**2,x)

[Out] Timed out

$$3.101 \quad \int \frac{1}{x(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=162

$$\frac{b \log(a + bx^2 + cx^4)}{2a^3} - \frac{2b \log(x)}{a^3} - \frac{b^2 - 3ac}{a^2 x^2 (b^2 - 4ac)} - \frac{(6a^2 c^2 - 6ab^2 c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3 (b^2 - 4ac)^{3/2}} + \frac{-2ac + b^2 + b}{2ax^2 (b^2 - 4ac) (a + b)}$$

Rubi [A] time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{(6a^2 c^2 - 6ab^2 c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3 (b^2 - 4ac)^{3/2}} - \frac{b^2 - 3ac}{a^2 x^2 (b^2 - 4ac)} + \frac{b \log(a + bx^2 + cx^4)}{2a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac + b^2 + bcx^2}{2ax^2 (b^2 - 4ac) (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x + b*x^3 + c*x^5)^2), x]

[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634


```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^3(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-2(b^2 - 3ac) - 2bcx}{x^2(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{2(-b^2 + 3ac)}{ax^2} - \frac{2b(-b^2 + 4ac)}{a^2x} + \frac{2(-b^4 + 5ab^2c - 3a^2c^2)}{a^2(a + bx + cx^2)} \right) dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{2b \log(x)}{a^3} - \frac{\text{Subst} \left(\int \frac{-b^4 + 5ab^2c - 3a^2c^2}{a + bx + cx^2} dx, x, x^2 \right)}{2a^3} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{2b \log(x)}{a^3} + \frac{b \text{Subst} \left(\int \frac{b + 2c}{a + bx + cx^2} dx, x, x^2 \right)}{2a^3} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{2b \log(x)}{a^3} + \frac{b \log(a + bx^2 + cx^4)}{2a^3} \\
&= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{a^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 248, normalized size = 1.53

$$\frac{\frac{(6a^2c^2 - 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^3\sqrt{b^2 - 4ac} + b^4) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-6a^2c^2 + 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^3\sqrt{b^2 - 4ac} - b^4) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{a(-3abc - 2ac^2x^2 + b^3 + b^2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{a}{x^2} - 4b \log(x)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2), x]

[Out]
$$\begin{aligned}
& \left(-\frac{a}{x^2} - \frac{a(b^3 - 3ab^2c + b^2cx^2 - 2a^2c^2x^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - 4b \log(x) + \frac{(b^4 - 6ab^2c + 6a^2c^2 + b^3\sqrt{b^2 - 4ac} - 4ab^2c\sqrt{b^2 - 4ac}) \log(b - \sqrt{b^2 - 4ac} + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-b^4 + 6ab^2c - 6a^2c^2 + b^3\sqrt{b^2 - 4ac} - 4ab^2c\sqrt{b^2 - 4ac}) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{a(-3abc - 2ac^2x^2 + b^3 + b^2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{a}{x^2} - 4b \log(x) \right)
\end{aligned}$$

$4ac] - 4ab\sqrt{b^2 - 4ac} \cdot \text{Log}[b + \sqrt{b^2 - 4ac} + 2cx^2] / (b^2 - 4ac)^{3/2} / (2a^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a*x + b*x^3 + c*x^5)^2), x]

[Out] IntegrateAlgebraic[1/(x*(a*x + b*x^3 + c*x^5)^2), x]

fricas [B] time = 1.64, size = 1007, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*\text{sqrt}(b^2 - 4*a*c)*\text{log}((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\text{log}(c*x^4 + b*x^2 + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\text{log}(x) / ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*\text{sqrt}(-b^2 + 4*a*c)*\text{arctan}(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\text{log}(c*x^4 + b*x^2 + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\text{log}(x) / ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)] \end{aligned}$$

giac [A] time = 2.41, size = 182, normalized size = 1.12

$$\frac{(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(a^3b^2 - 4a^4c)\sqrt{-b^2 + 4ac}} - \frac{2b^2cx^4 - 6ac^2x^4 + 2b^3x^2 - 7abcx^2 + ab^2 - 4a^2c}{2(cx^6 + bx^4 + ax^2)(a^2b^2 - 4a^3c)} + \frac{b \log(cx^4 + bx^2 + a)}{2a^3} - \frac{b \log(x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right) / ((a^3b^2 - 4a^4c) \sqrt{-b^2 + 4ac}) - \frac{1}{2} (2b^2cx^4 - 6a^2cx^4 + 2b^3x^2 - 7ab^2cx^2 + ab^2 - 4a^2c) / ((cx^6 + bx^4 + ax^2)(a^2b^2 - 4a^3c)) + \frac{1}{2} b \log(cx^4 + bx^2 + a) / a^3 - b \log(x^2) / a^3$

maple [B] time = 0.02, size = 352, normalized size = 2.17

$$\frac{c^2x^2}{(cx^4+bx^2+a)(4ac-b^2)a} + \frac{b^2cx^2}{2(cx^4+bx^2+a)(4ac-b^2)a^2} - \frac{6c^2 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a} + \frac{6b^2c \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^2} - \frac{b^4 \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}a^3} - \frac{3bc}{2(cx^4+bx^2+a)(4ac-b^2)a} + \frac{b^3}{2(cx^4+bx^2+a)(4ac-b^2)a^2} + \frac{2bc \ln(cx^4+bx^2+a)}{(4ac-b^2)a^2} - \frac{b^3 \ln(cx^4+bx^2+a)}{2(4ac-b^2)a^3} - \frac{2b \ln(x)}{a^3} - \frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^5+b*x^3+a*x)^2,x)

[Out] $-1/2/a^2/x^2 - 2/a^3*b*\ln(x) - 1/a/(cx^4+bx^2+a)*c^2/(4ac-b^2)*x^2 + 1/2/a^2/(cx^4+bx^2+a)*c/(4ac-b^2)*x^2*b^2 - 3/2/a/(cx^4+bx^2+a)*b/(4ac-b^2)*c + 1/2/a^2/(cx^4+bx^2+a)*b^3/(4ac-b^2) + 2/a^2/(4ac-b^2)*c*\ln(cx^4+bx^2+a)*b - 1/2/a^3/(4ac-b^2)*\ln(cx^4+bx^2+a)*b^3 - 6/a/(4ac-b^2)^{(3/2)}*\arctan((2cx^2+b)/(4ac-b^2)^{(1/2)})*c^2 + 6/a^2/(4ac-b^2)^{(3/2)}*\arctan((2cx^2+b)/(4ac-b^2)^{(1/2)})*b^2*c - 1/a^3/(4ac-b^2)^{(3/2)}*\arctan((2cx^2+b)/(4ac-b^2)^{(1/2)})*b^4$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2(b^2c - 3ac^2)x^4 + ab^2 - 4a^2c + (2b^3 - 7abc)x^2}{2((a^2b^2c - 4a^3c^2)x^6 + (a^2b^3 - 4a^3bc)x^4 + (a^3b^2 - 4a^4c)x^2)} - \frac{-2 \int \frac{(b^3c - 4abc^2)x^3 + (b^4 - 5ab^2c + 3a^2c^2)x}{cx^4 + bx^2 + a} dx}{a^3b^2 - 4a^4c} - \frac{2b \log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $-1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*x^2) / ((a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2) - 2*integrate(-(b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*x)/(c*x^4 + b*x^2 + a), x) / (a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3$

mupad [B] time = 6.77, size = 5491, normalized size = 33.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a*x + b*x^3 + c*x^5)^2),x)

[Out] $(\log(a + bx^2 + cx^4)(b^7 - 64a^3b^3c^3 + 48a^2b^3c^2 - 12ab^5c)) / (2(a^3b^6 - 64a^6c^3 - 12a^4b^4c + 48a^5b^2c^2)) - (1/(2a) - (x$

$$\begin{aligned}
& ^2*(2*b^3 - 7*a*b*c)/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4 \\
& *a*c - b^2))/(a*x^2 + b*x^4 + c*x^6) - (2*b*\log(x))/a^3 + (\operatorname{atan}(((2*a^9*b^ \\
& 6*(4*a*c - b^2)^{(9/2)} - 128*a^{12}*c^3*(4*a*c - b^2)^{(9/2)} - 24*a^{10}*b^4*c*(4 \\
& *a*c - b^2)^{(9/2)} + 96*a^{11}*b^2*c^2*(4*a*c - b^2)^{(9/2)})*(3*b^6 - 3*a^3*c^3 \\
& + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^ \\
& 6)))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 \\
& + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - \\
& (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6 \\
& *b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9 \\
& *b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 1 \\
& 6*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2* \\
& c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64 \\
& *a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^ \\
& 3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c \\
& ^2)) + ((((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c \\
& ^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 \\
& + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^ \\
& 6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48 \\
& *a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^{(3/2)}) \\
& - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^ \\
& 2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2) \\
& ^{(3/2)}*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4* \\
& b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^ \\
& 2)^{(3/2)}) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^ \\
& 2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6 \\
& *(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 \\
& - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 \\
& - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8 \\
& *c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*(((4*(54*a^ \\
& 3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 - \\
& 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65* \\
& a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a \\
& ^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4* \\
& c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c \\
& ^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 \\
& + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5)))/((a \\
& ^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 \\
& - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 1 \\
& 2*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^ \\
& 7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - \\
& 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - ((((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^ \\
& 5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a \\
& ^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a \\
& *b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 \\
& - 672*a^9*b^3*c^5)))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
\end{aligned}$$

$$\begin{aligned}
& * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (b^4 + 6 a^2 c^2 - 6 a b^2 c) / (2 a^3 (4 a c - b^2)^{3/2}) - ((b^4 + 6 a^2 c^2 - 6 a b^2 c) * \\
& (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c) * (640 a^{10} b c^6 + 3 a^6 b^9 c^2 - 46 a^7 b^7 c^3 + 264 a^8 b^5 c^4 - 672 a^9 b^3 c^5)) / (a^3 (4 a c - b^2)^{3/2}) * (a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2) * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (b^4 + 6 a^2 c^2 - 6 a b^2 c) / (2 a^3 (4 a c - b^2)^{3/2}) + ((b^4 + 6 a^2 c^2 - 6 a b^2 c)^2 * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c) * (640 a^{10} b c^6 + 3 a^6 b^9 c^2 - 46 a^7 b^7 c^3 + 264 a^8 b^5 c^4 - 672 a^9 b^3 c^5)) / (2 a^6 (4 a c - b^2)^3 * (a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2) * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (3 b^6 - 3 a^3 c^3 + 36 a^2 b^2 c^2 - 21 a b^4 c) / (8 a^3 c^2 (4 a c - b^2)^3 * (9 a^4 c^4 - 6 b^8 - 288 a^2 b^4 c^2 + 382 a^3 b^2 c^3 + 72 a b^6 c)) - (b * ((((((4 * (480 a^8 c^7 - a^4 b^8 c^3 + 6 a^5 b^6 c^4 + 30 a^6 b^4 c^5 - 272 a^7 b^2 c^6)) / (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) - (2 * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c) * (640 a^{10} b c^6 + 3 a^6 b^9 c^2 - 46 a^7 b^7 c^3 + 264 a^8 b^5 c^4 - 672 a^9 b^3 c^5)) / ((a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2) * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (b^4 + 6 a^2 c^2 - 6 a b^2 c)) / (2 a^3 (4 a c - b^2)^{3/2}) - ((b^4 + 6 a^2 c^2 - 6 a b^2 c) * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c) * (640 a^{10} b c^6 + 3 a^6 b^9 c^2 - 46 a^7 b^7 c^3 + 264 a^8 b^5 c^4 - 672 a^9 b^3 c^5)) / (a^3 (4 a c - b^2)^{3/2}) * (a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2) * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c)) / (2 * (a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2)) - (((4 * (276 a^5 b c^7 - 6 a^2 b^7 c^4 + 65 a^3 b^5 c^5 - 233 a^4 b^3 c^6)) / (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) - (((4 * (480 a^8 c^7 - a^4 b^8 c^3 + 6 a^5 b^6 c^4 + 30 a^6 b^4 c^5 - 272 a^7 b^2 c^6)) / (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2) - (2 * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c) * (640 a^{10} b c^6 + 3 a^6 b^9 c^2 - 46 a^7 b^7 c^3 + 264 a^8 b^5 c^4 - 672 a^9 b^3 c^5)) / ((a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2) * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c)) / (2 * (a^3 b^6 - 64 a^6 c^3 - 12 a^4 b^4 c + 48 a^5 b^2 c^2)) * (b^4 + 6 a^2 c^2 - 6 a b^2 c)) / (2 a^3 (4 a c - b^2)^{3/2}) + ((b^4 + 6 a^2 c^2 - 6 a b^2 c)^3 * (640 a^{10} b c^6 + 3 a^6 b^9 c^2 - 46 a^7 b^7 c^3 + 264 a^8 b^5 c^4 - 672 a^9 b^3 c^5)) / (2 a^9 (4 a c - b^2)^{9/2} * (a^6 b^6 - 64 a^9 c^3 - 12 a^7 b^4 c + 48 a^8 b^2 c^2)) * (3 b^6 - 49 a^3 c^3 + 72 a^2 b^2 c^2 - 27 a b^4 c) / (8 a^3 c^2 (4 a c - b^2)^{7/2} * (9 a^4 c^4 - 6 b^8 - 288 a^2 b^4 c^2 + 382 a^3 b^2 c^3 + 72 a b^6 c)) * (2 a^9 b^6 (4 a c - b^2)^{9/2} - 128 a^{12} c^3 * (4 a c - b^2)^{9/2} - 24 a^{10} b^4 c * (4 a c - b^2)^{9/2} + 96 a^{11} b^2 c^2 * (4 a c - b^2)^{9/2})) / (36 a^4 c^6 + b^8 c^2 - 12 a b^6 c^3 + 48 a^2 b^4 c^4 - 72 a^3 b^2 c^5) + (b * ((((((4 * (24 a^7 b c^5 - 2 a^4 b^7 c^2 + 18 a^5 b^5 c^3 - 46 a^6 b^3 c^4)) / (a^6 b^4 + 16 a^8 c^2 - 8 a^7 b^2 c) - (2 * (a^7 b^6 c^2 - 8 a^8 b^4 c^3 + 16 a^9 b^2 c^4) * (b^7 - 64 a^3 b c^3 + 48 a^2 b^3 c^2 - 12 a b^5 c)) / ((a^6 b^4 + 16 a^8 c^2 - 8 a^7 b^2 c) * (a^3 b^6 - 64 a^6 c^3
\end{aligned}$$

$$\begin{aligned}
& - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4 \\
& *a*c - b^2)^{(3/2)}) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + \\
& 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)) \\
& /(a^3*(4*a*c - b^2)^{(3/2)}*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 6 \\
& 4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b \\
& ^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2* \\
& c^2)) - (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5)) \\
& /(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 \\
& + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - \\
& (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48* \\
& a^2*b^3*c^2 - 12*a*b^5*c)))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - \\
& 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2 \\
& *b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^ \\
& 2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^{(3/2)}) + ((a^7 \\
& *b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^3) \\
& /(2*a^9*(4*a*c - b^2)^{(9/2)}*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(2*a^9*b \\
& ^6*(4*a*c - b^2)^{(9/2)} - 128*a^12*c^3*(4*a*c - b^2)^{(9/2)} - 24*a^10*b^4*c*(\\
& 4*a*c - b^2)^{(9/2)} + 96*a^11*b^2*c^2*(4*a*c - b^2)^{(9/2)))*(3*b^6 - 49*a^3*c \\
& ^3 + 72*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^{(7/2)}*(9*a^4*c^ \\
& 4 - 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b \\
& ^8*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)))*(b^4 + 6*a^2*c^2 \\
& - 6*a*b^2*c))/(a^3*(4*a*c - b^2)^{(3/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)

[Out] Timed out

$$3.102 \quad \int \frac{1}{x^2(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=361

$$\frac{b(5b^2 - 19ac)}{2a^3x(b^2 - 4ac)} - \frac{5b^2 - 14ac}{6a^2x^3(b^2 - 4ac)} + \frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac)\sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^3(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}}$$

Rubi [A] time = 3.08, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1585, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac)\sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^3(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c - b(5b^2 - 19ac)\sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{2\sqrt{2}a^3(b^2 - 4ac)^{3/2} \sqrt{\sqrt{b^2 - 4ac} + b}} - \frac{5b^2 - 14ac}{6a^2x^3(b^2 - 4ac)} + \frac{b(5b^2 - 19ac)}{2a^3x(b^2 - 4ac)} + \frac{-2ac + b^2 + bcx^2}{2ax^3(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $-(5b^2 - 14ac)/(6a^2(b^2 - 4ac)x^3) + (b(5b^2 - 19ac))/(2a^3(b^2 - 4ac)x) + (b^2 - 2ac + bcx^2)/(2a^3(b^2 - 4ac)x^3(a + bx^2 + cx^4)) + (\text{Sqrt}[c]*(5b^4 - 29ab^2c + 28a^2c^2 + b(5b^2 - 19ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]])]/(2*\text{Sqrt}[2]*a^3(b^2 - 4ac)^{3/2}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]) - (\text{Sqrt}[c]*(5b^4 - 29ab^2c + 28a^2c^2 - b(5b^2 - 19ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]])]/(2*\text{Sqrt}[2]*a^3(b^2 - 4ac)^{3/2}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1121

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[((d*x)^(m+1)*(b^2 - 2ac + bcx^2)*(a + bx^2 + cx^4)^(p+1))/(2ad*(p+1)*(b^2 - 4ac)), x] + Dist[1/(2a*(p+1)*(b^2 - 4ac)), Int[(d*x)^m*(a + bx^2 + cx^4)^(p+1)*Simp[b^2*(m+2p+3) - 2ac*(m+4p+5) + bc*(m+4p+7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4ac, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_
))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax+bx^3+cx^5)^2} dx &= \int \frac{1}{x^4(a+bx^2+cx^4)^2} dx \\
&= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)} - \frac{\int \frac{-5b^2+14ac-5bcx^2}{x^4(a+bx^2+cx^4)} dx}{2a(b^2-4ac)} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)x^3} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)} + \frac{\int \frac{-3b(5b^2-19ac)-3c(5b^2-14ac)x^2}{x^2(a+bx^2+cx^4)} dx}{6a^2(b^2-4ac)} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)x^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)} - \frac{\int \frac{-3(5b^4}{x^2(a+bx^2+cx^4)} dx}{6a^2(b^2-4ac)} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)x^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)} - \frac{c(5b^4}{6a^2(b^2-4ac)x^3} \\
&= -\frac{5b^2-14ac}{6a^2(b^2-4ac)x^3} + \frac{b(5b^2-19ac)}{2a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^3(a+bx^2+cx^4)} + \frac{\sqrt{c}(5b^4}{6a^2(b^2-4ac)x^3}
\end{aligned}$$

Mathematica [A] time = 0.71, size = 344, normalized size = 0.95

$$\frac{6x(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}(-28a^2c^2+29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}-5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{4a}{x^3} + \frac{24b}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2), x]

[Out] ((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*sqrt[2]*sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) + (3*sqrt[2]*sqrt[c]*(-5*b^4 + 29*a*b^2*c - 28*a^2*c^2 + 5*b^3*sqrt[b^2 - 4*a*c] - 19*a*b*c*sqrt[b^2 - 4*a*c]))*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]])]/(12*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]

[Out] IntegrateAlgebraic[1/(x^2*(a*x + b*x^3 + c*x^5)^2), x]

fricas [B] time = 2.12, size = 3435, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^

$$5*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*\sqrt{(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)$$

giac [B] time = 3.92, size = 3651, normalized size = 10.11

result too large to display

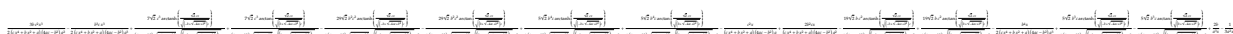
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(b^3*c*x^3 - 3*a*b*c^2*x^3 + b^4*x - 4*a*b^2*c*x + 2*a^2*c^2*x)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + \frac{1}{16}*(10*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^6*b^9 + 69*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^7*b^7*c + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^6*b^8*c - 340*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^8*b^5*c^2 - 98*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^7*b^6*c^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^6*b^7*c^2 + 688*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^9*b^3*c^3 + 288*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^8*b^4*c^3 + 49*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^7*b^5*c^3 - 448*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^10*b*c^4 - 224*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^9*b^2*c^4 - 144*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^8*b^3*c^4 + 112*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^5 + 39*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^3*c + 10*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^4*c - 76*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^2*b*c^2 - 38*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b^2*c^2 - 5*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*b^3*c^2 + 19*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2 + 2*(5*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*b^8 - 64*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^4*b^6*c - 10*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*b^7*c - 10*a^3*b^8*c + 286*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^5*b^4*c^2 + 88*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^4*b^5*c^2 + 5*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c})*$

$$\sqrt{(b^2 - 4ac)c} a^5 b^2 c^4 - 992 a^6 b^2 c^4 - 56 \sqrt{2} \sqrt{(b^2 - 4ac)c} a^6 c^5 + 448 a^7 c^5 - 10 (b^2 - 4ac) a^3 b^6 c + 8 (b^2 - 4ac) a^4 b^4 c^2 - 220 (b^2 - 4ac) a^5 b^2 c^3 + 112 (b^2 - 4ac) a^6 c^4 \cdot \text{abs}(a^3 b^2 - 4 a^4 c) \cdot \arctan\left(\frac{2 \sqrt{1/2} x / \sqrt{(a^3 b^3 - 4 a^4 b c - \sqrt{(a^3 b^3 - 4 a^4 b c)^2 - 4 (a^4 b^2 - 4 a^5 c) (a^3 b^2 c - 4 a^4 c^2)})}}{(a^3 b^2 c - 4 a^4 c^2)}\right) / ((a^7 b^6 - 12 a^8 b^4 c - 2 a^7 b^5 c + 48 a^9 b^2 c^2 + 16 a^8 b^3 c^2 + a^7 b^4 c^2 - 64 a^{10} c^3 - 32 a^9 b c^3 - 8 a^8 b^2 c^3 + 16 a^9 c^4) \cdot \text{abs}(a^3 b^2 - 4 a^4 c) \cdot \text{abs}(c)) + 1/3 (6 b x^2 - a) / (a^3 x^3)$$

maple [B] time = 0.04, size = 913, normalized size = 2.53



Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (1/x^2 / (c x^5 + b x^3 + a x))^2 dx$

[Out]
$$\begin{aligned} & -1/3/a^2/x^3 + 2/a^3*b/x + 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^3 - 1/2/a^3/ \\ & (c*x^4+b*x^2+a)*b^3*c/(4*a*c-b^2)*x^3 - 1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c \\ & ^2 + 2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*c - 1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c \\ & -b^2)*x*b^4 - 19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1 \\ & /2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b + 5/4/a^3*c/(4*a \\ & *c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^3 + 7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(\\ & 1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(\\ & 1/2)})*c)^{(1/2)}*c*x) - 29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((- \\ & b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(\\ & 1/2)}*c*x)*b^2 + 5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a \\ & *c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c \\ & *x)*b^4 + 19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{ar} \\ & \operatorname{ctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b - 5/4/a^3*c/(4*a*c-b^2)* \\ & 2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1 \\ & /2)})*c)^{(1/2)}*c*x)*b^3 + 7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+ \\ & -4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2) \\ & }*c*x) - 29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(\\ & 1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^2 + 5/ \\ & 4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(\\ & 1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x)*b^4 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3(5b^3c - 19abc^2)x^6 + (15b^4 - 62ab^2c + 14a^2c^2)x^4 - 2a^2b^2 + 8a^3c + 10(ab^3 - 4a^2bc)x^2}{6((a^3b^2c - 4a^4c^2)x^7 + (a^3b^3 - 4a^4bc)x^5 + (a^4b^2 - 4a^5c)x^3)} - \int \frac{5b^4 - 24ab^2c + 14a^2c^2 + (5b^3c - 19abc^2)x^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& 4*b^{10}*c^4 + 468480*a^{15}*b^8*c^5 - 1536000*a^{16}*b^6*c^6 + 2867200*a^{17}*b^4* \\
& c^7 - 2719744*a^{18}*b^2*c^8 + x*(-(25*b^{15} - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7 \\
& *c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2) \\
& ^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 165*a \\
& *b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10} \\
& *c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b \\
& ^2*c^5)))^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^ \\
& 3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 157286 \\
& 4*a^{20}*b^3*c^7)) - x*(401408*a^{16}*c^{10} - 400*a^9*b^{14}*c^3 + 9440*a^{10}*b^{12}* \\
& c^4 - 92816*a^{11}*b^{10}*c^5 + 488096*a^{12}*b^8*c^6 - 1458688*a^{13}*b^6*c^7 + 24 \\
& 01280*a^{14}*b^4*c^8 - 1871872*a^{15}*b^2*c^9))*(-(25*b^{15} - 25*b^6*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 1 \\
& 16928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{ \\
& (1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 \\
& - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 \\
& - 6144*a^{12}*b^2*c^5)))^{(1/2)}*i)/((((-(25*b^{15} - 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} \\
& - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4 \\
& *b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} + 1 \\
& 65*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8 \\
& *b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{ \\
& 12}*b^2*c^5)))^{(1/2)}*(320*a^{12}*b^{14}*c^2 - 917504*a^{19}*c^9 - 7936*a^{13}*b^{12}* \\
& c^3 + 82816*a^{14}*b^{10}*c^4 - 468480*a^{15}*b^8*c^5 + 1536000*a^{16}*b^6*c^6 - 286 \\
& 7200*a^{17}*b^4*c^7 + 2719744*a^{18}*b^2*c^8 + x*(-(25*b^{15} - 25*b^6*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + \\
& 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9) \\
& ^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + 4096*a^{13}*c^ \\
& 6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 \\
& - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}*c^2 - 614 \\
& 4*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^ \\
& 5*c^6 - 1572864*a^{20}*b^3*c^7)) - x*(401408*a^{16}*c^{10} - 400*a^9*b^{14}*c^3 + 9 \\
& 440*a^{10}*b^{12}*c^4 - 92816*a^{11}*b^{10}*c^5 + 488096*a^{12}*b^8*c^6 - 1458688*a^{1 \\
& 3}*b^6*c^7 + 2401280*a^{14}*b^4*c^8 - 1871872*a^{15}*b^2*c^9))*(-(25*b^{15} - 25*b \\
& ^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a \\
& ^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + \\
& 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} + 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)})/(32*(a^7*b^{12} + \\
& 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840 \\
& *a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)} - (((-(25*b^{15} - 25*b^6*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + \\
& 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3* \\
& (- (4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)
\end{aligned}$$

$$\begin{aligned}
& 6*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 \\
& - 1572864*a^{20}*b^3*c^7) - x*(401408*a^{16}*c^{10} - 400*a^9*b^{14}*c^3 + 9440*a \\
& ^{10}*b^{12}*c^4 - 92816*a^{11}*b^{10}*c^5 + 488096*a^{12}*b^8*c^6 - 1458688*a^{13}*b^6 \\
& *c^7 + 2401280*a^{14}*b^4*c^8 - 1871872*a^{15}*b^2*c^9))*(-(25*b^{15} + 25*b^6*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^ \\
& 9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a \\
& ^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096 \\
& *a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11} \\
& *b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*1i + (((-(25*b^{15} + 25*b^6*(-(4*a*c - \\
& b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 1 \\
& 16928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^ \\
& (1/2) - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 4096*a^{13}*c^6 \\
& - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 \\
& - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(917504*a^{19}*c^9 - 320*a^{12}*b^{14}*c^2 + 7936*a^ \\
& ^{13}*b^{12}*c^3 - 82816*a^{14}*b^{10}*c^4 + 468480*a^{15}*b^8*c^5 - 1536000*a^{16}*b^6* \\
& c^6 + 2867200*a^{17}*b^4*c^7 - 2719744*a^{18}*b^2*c^8 + x*(-(25*b^{15} + 25*b^6*(- \\
& (4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b \\
& ^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49* \\
& a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c \\
& - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + 409 \\
& 6*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^1 \\
& 1*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(1048576*a^{21}*b*c^8 + 256*a^{15}*b^{13}* \\
& c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b^7*c^5 + 98304 \\
& 0*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) - x*(401408*a^{16}*c^{10} - 400*a^9*b^1 \\
& 4*c^3 + 9440*a^{10}*b^{12}*c^4 - 92816*a^{11}*b^{10}*c^5 + 488096*a^{12}*b^8*c^6 - 14 \\
& 58688*a^{13}*b^6*c^7 + 2401280*a^{14}*b^4*c^8 - 1871872*a^{15}*b^2*c^9))*(-(25*b^ \\
& ^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 \\
& - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6* \\
& b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2* \\
& c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a \\
& ^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c \\
& ^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*1i)/(((-(25*b^{15} + 25*b \\
& ^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a \\
& ^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - \\
& 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4* \\
& a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)))/(32*(a^7*b^{12} + \\
& 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840 \\
& *a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5)))^{(1/2)}*(320*a^{12}*b^{14}*c^2 - 917504*a^{19} \\
& *c^9 - 7936*a^{13}*b^{12}*c^3 + 82816*a^{14}*b^{10}*c^4 - 468480*a^{15}*b^8*c^5 + 153 \\
& 6000*a^{16}*b^6*c^6 - 2867200*a^{17}*b^4*c^7 + 2719744*a^{18}*b^2*c^8 + x*(-(25*b \\
& ^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 \\
& - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6 \\
& *b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2
\end{aligned}$$

$$\begin{aligned}
& *c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)} / (32*(\\
& a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6* \\
& c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)} * (1048576*a^{21}*b*c^8 + \\
& 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 327680*a^{18}*b \\
& ^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) - x*(401408*a^{16}*c^{10} \\
& - 400*a^9*b^{14}*c^3 + 9440*a^{10}*b^{12}*c^4 - 92816*a^{11}*b^{10}*c^5 + 488096*a^{1 \\
& 2}*b^8*c^6 - 1458688*a^{13}*b^6*c^7 + 2401280*a^{14}*b^4*c^8 - 1871872*a^{15}*b^2* \\
& c^9)) * (-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366 \\
& *a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 \\
& + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c \\
& + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9) \\
& ^{(1/2)}) / (32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1 \\
& 280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)} - ((- (25* \\
& b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^ \\
& 2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^ \\
& 6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^ \\
& 2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)}) / (32* \\
& (a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6 \\
& *c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)} * (917504*a^{19}*c^9 - 32 \\
& 0*a^{12}*b^{14}*c^2 + 7936*a^{13}*b^{12}*c^3 - 82816*a^{14}*b^{10}*c^4 + 468480*a^{15}*b^ \\
& 8*c^5 - 1536000*a^{16}*b^6*c^6 + 2867200*a^{17}*b^4*c^7 - 2719744*a^{18}*b^2*c^8 \\
& + x*(-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b*c^7 + 6366*a \\
& ^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + \\
& 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 615*a*b^{13}*c + \\
& 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a*c - b^2)^9)^{(\\
& 1/2)}) / (32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9*b^8*c^2 - 128 \\
& 0*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)} * (1048576*a^ \\
& 21*b*c^8 + 256*a^{15}*b^{13}*c^2 - 6144*a^{16}*b^{11}*c^3 + 61440*a^{17}*b^9*c^4 - 32 \\
& 7680*a^{18}*b^7*c^5 + 983040*a^{19}*b^5*c^6 - 1572864*a^{20}*b^3*c^7) - x*(40140 \\
& 8*a^{16}*c^{10} - 400*a^9*b^{14}*c^3 + 9440*a^{10}*b^{12}*c^4 - 92816*a^{11}*b^{10}*c^5 + \\
& 488096*a^{12}*b^8*c^6 - 1458688*a^{13}*b^6*c^7 + 2401280*a^{14}*b^4*c^8 - 187187 \\
& 2*a^{15}*b^2*c^9)) * (-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1/2)} - 80640*a^7*b \\
& *c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744* \\
& a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b^2)^9)^{(1/2)} - 61 \\
& 5*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 165*a*b^4*c*(-(4*a* \\
& c - b^2)^9)^{(1/2)}) / (32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8*b^{10}*c + 240*a^9* \\
& b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{12}*b^2*c^5))^{(1/2)} \\
&) + 476672*a^{13}*b*c^{10} + 1800*a^9*b^9*c^6 - 29080*a^{10}*b^7*c^7 + 176032*a^{1 \\
& 1}*b^5*c^8 - 473216*a^{12}*b^3*c^9)) * (-(25*b^{15} + 25*b^6*(-(4*a*c - b^2)^9)^{(1 \\
& /2)} - 80640*a^7*b*c^7 + 6366*a^2*b^{11}*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4* \\
& b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 - 49*a^3*c^3*(-(4*a*c - b \\
& ^2)^9)^{(1/2)} - 615*a*b^{13}*c + 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^{(1/2)} - 16 \\
& 5*a*b^4*c*(-(4*a*c - b^2)^9)^{(1/2)}) / (32*(a^7*b^{12} + 4096*a^{13}*c^6 - 24*a^8* \\
& b^{10}*c + 240*a^9*b^8*c^2 - 1280*a^{10}*b^6*c^3 + 3840*a^{11}*b^4*c^4 - 6144*a^{1 \\
& 2}*b^2*c^5))^{(1/2)} * 2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)

[Out] Timed out

$$3.103 \quad \int \frac{1}{x^3(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=219

$$-\frac{(3b^2 - 2ac) \log(a + bx^2 + cx^4)}{4a^4} + \frac{\log(x)(3b^2 - 2ac)}{a^4} + \frac{b(3b^2 - 11ac)}{2a^3x^2(b^2 - 4ac)} - \frac{3b^2 - 8ac}{4a^2x^4(b^2 - 4ac)} + \frac{b(30a^2c^2 - 20ab^2c + 3b^4)}{2a^4(b^2 - 4ac)^{3/2}}$$

Rubi [A] time = 0.31, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right) + b(3b^2 - 11ac)}{2a^4(b^2 - 4ac)^{3/2}} - \frac{3b^2 - 8ac}{4a^2x^4(b^2 - 4ac)} - \frac{(3b^2 - 2ac) \log(a + bx^2 + cx^4)}{4a^4} + \frac{\log(x)(3b^2 - 2ac)}{a^4} + \frac{-2ac + b^2 + bcx^2}{2a^4(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $-(3b^2 - 8ac)/(4a^2(b^2 - 4ac)x^4) + (b(3b^2 - 11ac))/(2a^3(b^2 - 4ac)x^2) + (b^2 - 2ac + bcx^2)/(2a(b^2 - 4ac)x^4(a + bx^2 + cx^4)) + (b(3b^4 - 20ab^2c + 30a^2c^2) \operatorname{ArcTanh}[(b + 2cx^2)/\sqrt{b^2 - 4ac}])/(2a^4(b^2 - 4ac)^{3/2}) + ((3b^2 - 2ac) \log[x])/a^4 - ((3b^2 - 2ac) \log[a + bx^2 + cx^4])/(4a^4)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + bx + cx^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2cd - be, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^5 (a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3 (a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-3b^2 + 8ac - 3bcx}{x^3 (a + bx + cx^2)} dx, x, x^2 \right)}{2a (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{-3b^2 + 8ac}{ax^3} + \frac{3b^3 - 11abc}{a^2 x^2} + \frac{(b^2 - 4ac)(-3b^2 + 2ac)}{a^3 x} \right) dx, x, x^2 \right)}{2a (b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{(3b^2 - 2ac)}{2a^3 (b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{(3b^2 - 2ac)}{2a^3 (b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{(3b^2 - 2ac)}{2a^3 (b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{b(3b^2 - 2ac)}{2a^3 (b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 328, normalized size = 1.50

$$\frac{2a(2a^2c^2 - 4ab^2c - 3abc^2 + b^4 + b^3cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(8a^2c^2\sqrt{b^2 - 4ac} + 30a^2bc^2 - 20ab^3c - 14a^2c\sqrt{b^2 - 4ac} + 3b^4\sqrt{b^2 - 4ac} + 3b^5)\log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-8a^2c^2\sqrt{b^2 - 4ac} + 30a^2bc^2 - 20ab^3c + 14a^2c\sqrt{b^2 - 4ac} - 3b^4\sqrt{b^2 - 4ac} + 3b^5)\log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{a^2}{x^4} + 4\log(x)(3b^2 - 2ac) + \frac{4ab}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $(-a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(3*b^2 - 2*a*c)*\text{Log}[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 + 3*b^4*\text{Sqrt}[b^2 - 4*a*c] - 14*a*b^2*c*\text{Sqrt}[b^2 - 4*a*c] + 8*a^2*c^2*\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^{(3/2)} + ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2$

$2 - 3*b^4*\text{Sqrt}[b^2 - 4*a*c] + 14*a*b^2*c*\text{Sqrt}[b^2 - 4*a*c] - 8*a^2*c^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2]/(b^2 - 4*a*c)^{(3/2)}/(4*a^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(a*x + b*x^3 + c*x^5)^2), x]

[Out] IntegrateAlgebraic[1/(x^3*(a*x + b*x^3 + c*x^5)^2), x]

fricas [B] time = 2.28, size = 1242, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $[-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*\text{sqrt}(b^2 - 4*a*c)*\text{log}((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*\text{log}(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*\text{log}(x)]/(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4, -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*\text{sqrt}(-b^2 + 4*a*c)*\text{arctan}(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*\text{log}(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*1$

$\text{og}(x))/((a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)x^8 + (a^4b^5 - 8a^5b^3c + 16a^6b^2c^2)x^6 + (a^5b^4 - 8a^6b^2c + 16a^7c^2)x^4]$

giac [A] time = 2.04, size = 274, normalized size = 1.25

$$\frac{(3b^5 - 20ab^3c + 30a^2b^2c^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) + \frac{3b^4cx^4 - 14ab^2c^2x^4 + 8a^2c^3x^4 + 3b^5x^2 - 12ab^3cx^2 + 2a^2b^2c^2x^2 + 5ab^4 - 22a^2b^2c + 12a^3c^2}{4(a^4b^2 - 4a^5c)(cx^4 + bx^2 + a)} - \frac{(3b^2 - 2ac) \log(cx^4 + bx^2 + a)}{4a^4} + \frac{(3b^2 - 2ac) \log(x^2)}{2a^4} - \frac{9b^2x^4 - 6acx^4 - 4abx^2 + a^2}{4a^4x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $-1/2*(3b^5 - 20a*b^3*c + 30a^2*b^2*c^2)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^4*b^2 - 4*a^5*c)*\sqrt{-b^2 + 4*a*c}) + 1/4*(3b^4*c*x^4 - 14*a*b^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b^2*c^2*x^2 + 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x^2 + a)) - 1/4*(3b^2 - 2*a*c)*\log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3b^2 - 2*a*c)*\log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)$

maple [B] time = 0.02, size = 443, normalized size = 2.02

$$\frac{3b^5c^2}{2(c^4 + b^2 + a)(4ac - b^2)^2} - \frac{b^4cx^2}{2(c^4 + b^2 + a)(4ac - b^2)^2} + \frac{15b^4c \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac - b^2)^2 a^2} + \frac{10b^3c \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(4ac - b^2)^2 a^2} + \frac{3b^2 \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(4ac - b^2)^2 a^2} + \frac{c^2}{(c^4 + b^2 + a)(4ac - b^2)a} + \frac{2b^2c}{(c^4 + b^2 + a)(4ac - b^2)a^2} + \frac{2c^2 \ln(cx^4 + bx^2 + a)}{(4ac - b^2)^2 a^2} - \frac{b^4}{2(c^4 + b^2 + a)(4ac - b^2)a^2} + \frac{7b^2c \ln(cx^4 + bx^2 + a)}{2(4ac - b^2)^2 a^2} + \frac{3b^2 \ln(cx^4 + bx^2 + a)}{4(4ac - b^2)^2 a^2} + \frac{2c \ln(x)}{a^3} + \frac{3b^2 \ln(x)}{a^3} - \frac{b}{a^2} - \frac{1}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c*x^5+b*x^3+a*x)^2,x)

[Out] $-1/4/a^2/x^4 - 2/a^3*c*\ln(x) + 3/a^4*b^2*\ln(x) + 1/a^3*b/x^2 + 3/2/a^2/(c*x^4 + b*x^2 + a)*b*c^2/(4*a*c - b^2)*x^2 - 1/2/a^3/(c*x^4 + b*x^2 + a)*b^3*c/(4*a*c - b^2)*x^2 - 1/a/(c*x^4 + b*x^2 + a)/(4*a*c - b^2)*c^2 + 2/a^2/(c*x^4 + b*x^2 + a)/(4*a*c - b^2)*b^2*c - 1/2/a^3/(c*x^4 + b*x^2 + a)/(4*a*c - b^2)*b^4 + 2/a^2/(4*a*c - b^2)*c^2*\ln(c*x^4 + b*x^2 + a) - 7/2/a^3/(4*a*c - b^2)*c*\ln(c*x^4 + b*x^2 + a)*b^2 + 3/4/a^4/(4*a*c - b^2)*\ln(c*x^4 + b*x^2 + a)*b^4 + 15/a^2/(4*a*c - b^2)^(3/2)*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^(1/2))*b*c^2 - 10/a^3/(4*a*c - b^2)^(3/2)*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^(1/2))*b^3*c + 3/2/a^4/(4*a*c - b^2)^(3/2)*\arctan((2*c*x^2 + b)/(4*a*c - b^2)^(1/2))*b^5$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2(3b^3c - 11abc^2)x^6 + (6b^4 - 25ab^2c + 8a^2c^2)x^4 - a^2b^2 + 4a^3c + 3(ab^3 - 4a^2bc)x^2}{4((a^3b^2c - 4a^4c^2)x^8 + (a^3b^3 - 4a^4bc)x^6 + (a^4b^2 - 4a^5c)x^4)} - \frac{1}{4} \frac{(3b^4 - 14ab^2c + 8a^2c^2) \log(cx^4 + bx^2 + a) + \frac{(3b^5 - 20ab^3c + 30a^2b^2c^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}}{a^4b^2 - 4a^5c} + \frac{(3b^2 - 2ac) \log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 - a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^8 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - \text{integrate}(((3*b^4$

$$\begin{aligned}
& ^3 - 12*a^{10}*b^4*c + 48*a^{11}*b^2*c^2) - ((2560*a^{13}*b*c^6 + 12*a^9*b^9*c^2 \\
& - 184*a^{10}*b^7*c^3 + 1056*a^{11}*b^5*c^4 - 2688*a^{12}*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(a^9*b^6 - 64*a^{12}*c^3 - 12*a^{10}*b^4*c + 48*a^{11}*b^2*c^2)) * (4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) \\
& + (b*((b*((2240*a^{10}*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^{12}*c^3 - 12*a^{10}*b^4*c + 48*a^{11}*b^2*c^2) - ((2560*a^{13}*b*c^6 + 12*a^9*b^9*c^2 - 184*a^{10}*b^7*c^3 + 1056*a^{11}*b^5*c^4 - 2688*a^{12}*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(a^9*b^6 - 64*a^{12}*c^3 - 12*a^{10}*b^4*c + 48*a^{11}*b^2*c^2)) * (4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)) / (4*a^4*(4*a*c - b^2)^{(3/2)}) - (b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)) * (2560*a^{13}*b*c^6 + 12*a^9*b^9*c^2 - 184*a^{10}*b^7*c^3 + 1056*a^{11}*b^5*c^4 - 2688*a^{12}*b^3*c^5) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (8*a^4*(4*a*c - b^2)^{(3/2)} * (a^9*b^6 - 64*a^{12}*c^3 - 12*a^{10}*b^4*c + 48*a^{11}*b^2*c^2)) * (4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)) / (4*a^4*(4*a*c - b^2)^{(3/2)}) - (b^2*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))^2 * (2560*a^{13}*b*c^6 + 12*a^9*b^9*c^2 - 184*a^{10}*b^7*c^3 + 1056*a^{11}*b^5*c^4 - 2688*a^{12}*b^3*c^5) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (32*a^8*(4*a*c - b^2)^3 * (a^9*b^6 - 64*a^{12}*c^3 - 12*a^{10}*b^4*c + 48*a^{11}*b^2*c^2) * (4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (3*b^6 - 25*a^3*c^3 + 50*a^2*b^2*c^2 - 23*a*b^4*c)) / (8*a^3*c^2*(4*a*c - b^2)^3 * (54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 5775*a^4*b^2*c^4 - 720*a*b^8*c)) * (16*a^{12}*b^6*(4*a*c - b^2)^{(9/2)} - 1024*a^{15}*c^3*(4*a*c - b^2)^{(9/2)} - 192*a^{13}*b^4*c*(4*a*c - b^2)^{(9/2)} + 768*a^{14}*b^2*c^2*(4*a*c - b^2)^{(9/2))) / (9*b^{10}*c^2 - 120*a*b^8*c^3 + 580*a^2*b^6*c^4 - 1200*a^3*b^4*c^5 + 900*a^4*b^2*c^6) + (((b*((36*a^3*b^8*c^3 - 309*a^4*b^6*c^4 + 778*a^5*b^4*c^5 - 473*a^6*b^2*c^6)/(a^9*b^4 + 16*a^{11}*c^2 - 8*a^{10}*b^2*c) - (((12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5) / (a^9*b^4 + 16*a^{11}*c^2 - 8*a^{10}*b^2*c) + ((4*a^{10}*b^6*c^2 - 32*a^{11}*b^4*c^3 + 64*a^{12}*b^2*c^4) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(a^9*b^4 + 16*a^{11}*c^2 - 8*a^{10}*b^2*c) * (4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)) / (4*a^4*(4*a*c - b^2)^{(3/2)}) - (((b*((12*a^6*b^8*c^2 - 116*a^7*b^6*c^3 + 348*a^8*b^4*c^4 - 304*a^9*b^2*c^5) / (a^9*b^4 + 16*a^{11}*c^2 - 8*a^{10}*b^2*c) + ((4*a^{10}*b^6*c^2 - 32*a^{11}*b^4*c^3 + 64*a^{12}*b^2*c^4) * (6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c)) / (2*(a^9*b^4 + 16*a^{11}*c^2 - 8*a^{10}*b^2*c) * (4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) * (3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)) / (4*a^4*(4*a*c - b^2)^{(3/2)}) + (b*(3*b^4 + 30
\end{aligned}$$

$$\begin{aligned}
& a^2c^2 - 20ab^2c) * (4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4) * \\
& (6b^8 + 256a^4c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (8a^4(4ac - b^2)^{3/2} * (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) * (4a^4b^6 - \\
& 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) * (6b^8 + 256a^4c^4 + 336 \\
& a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (2(4a^4b^6 - 256a^7c^3 - \\
& 48a^5b^4c + 192a^6b^2c^2)) + (b^3(3b^4 + 30a^2c^2 - 20ab^2c))^3 \\
& (4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4) / (64a^{12}(4ac - b^2)^{9/2} * (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c)) * (16a^{12}b^6(4ac - b^2)^{9/2} - 1024a^{15}c^3(4ac - b^2)^{9/2} - 192a^{13}b^4c(4ac - b^2)^{9/2} + 768a^{14}b^2c^2(4ac - b^2)^{9/2}) * (9b^8 + 80a^4c^4 + 270a^2b^4c^2 - 285a^3b^2c^3 - 87ab^6c) / (8a^3c^2(4ac - b^2)^{7/2} * (9b^{10}c^2 - 120ab^8c^3 + 580a^2b^6c^4 - 1200a^3b^4c^5 + 900a^4b^2c^6) * (54b^{10} - 1600a^5c^5 + 3480a^2b^6c^2 - 7200a^3b^4c^3 + 5775a^4b^2c^4 - 720ab^8c)) - (3b(16a^{12}b^6(4ac - b^2)^{9/2} - 1024a^{15}c^3(4ac - b^2)^{9/2} - 192a^{13}b^4c(4ac - b^2)^{9/2} + 768a^{14}b^2c^2(4ac - b^2)^{9/2}) * (((36a^3b^8c^3 - 309a^4b^6c^4 + 778a^5b^4c^5 - 473a^6b^2c^6) / (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) - ((12a^6b^8c^2 - 116a^7b^6c^3 + 348a^8b^4c^4 - 304a^9b^2c^5) / (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) + ((4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4) * (6b^8 + 256a^4c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (2(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2))) * (6b^8 + 256a^4c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (2(4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) - (27b^8c^4 - 216ab^6c^5 + 495a^2b^4c^6 - 242a^3b^2c^7) / (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) + (b((b((12a^6b^8c^2 - 116a^7b^6c^3 + 348a^8b^4c^4 - 304a^9b^2c^5) / (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) + ((4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4) * (6b^8 + 256a^4c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (2(a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2))) * (3b^4 + 30a^2c^2 - 20ab^2c) / (4a^4(4ac - b^2)^{3/2})) + (b(3b^4 + 30a^2c^2 - 20ab^2c) * (4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4) * (6b^8 + 256a^4c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (8a^4(4ac - b^2)^{3/2} * (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2))) * (3b^4 + 30a^2c^2 - 20ab^2c) / (4a^4(4ac - b^2)^{3/2})) + (b^2(3b^4 + 30a^2c^2 - 20ab^2c))^2 * (4a^{10}b^6c^2 - 32a^{11}b^4c^3 + 64a^{12}b^2c^4) * (6b^8 + 256a^4c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76ab^6c) / (32a^8(4ac - b^2)^3 * (a^9b^4 + 16a^{11}c^2 - 8a^{10}b^2c) * (4a^4b^6 - 256a^7c^3 - 48a^5b^4c + 192a^6b^2c^2))) * (3b^6 - 25a^3c^3 + 50a^2b^2c^2 - 23ab^4c) / (8a^3c^2(4ac - b^2)^3 * (9b^{10}c^2 - 120ab^8c^3 + 580a^2b^6c^4 - 1200a^3b^4c^5 + 900a^4b^2c^6) * (54b^{10} - 1600a^5c^5 + 3480a^2b^6c^2 - 7200a^3b^4c^3 + 5775a^4b^2c^4 - 720ab^8c)) * (3b^4 + 30a^2c^2 - 20ab^2c) / (2a^4(4ac - b^2)^{3/2})
\end{aligned}$$

$$\begin{aligned} & - (\log(x) \cdot (2ac - 3b^2)) / a^4 - (\log(a + bx^2 + cx^4) \cdot (6b^8 + 256a^4 \\ & \cdot c^4 + 336a^2b^4c^2 - 576a^3b^2c^3 - 76a^2b^6c)) / (2(4a^4b^6 - 256 \\ & \cdot a^7c^3 - 48a^5b^4c + 192a^6b^2c^2)) - (1/(4a) - (3bx^2)/(4a^2) \\ & + (x^4(6b^4 + 8a^2c^2 - 25ab^2c)) / (4a^3(4ac - b^2))) - (bcx^6(\\ & 11ac - 3b^2)) / (2a^3(4ac - b^2))) / (ax^4 + bx^6 + cx^8) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)

[Out] Timed out

3.104 $\int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx$

Optimal. Leaf size=129

$$\frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\sqrt{x} (b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{16c^{3/2} \sqrt{ax + bx^3 + cx^5}}$$

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1918, 1914, 1107, 621, 206}

$$\frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\sqrt{x} (b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{16c^{3/2} \sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_) + (a_.)*(x_)^(q_) + (c_.)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[

$a*x^q + b*x^n + c*x^{(2*n - q)}$, Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1918

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, n - q]

Rubi steps

$$\begin{aligned} \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{(b^2 - 4ac) \int \frac{x^{3/2}}{\sqrt{ax + bx^3 + cx^5}} dx}{8c} \\ &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\left((b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \right) \int \frac{x}{\sqrt{a + bx^2 + cx^4}} dx}{8c\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\left((b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \right) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x \right)}{16c\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\left((b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \right) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x \right)}{8c\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{(b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{16c^{3/2} \sqrt{ax + bx^3 + cx^5}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.98

$$\frac{\sqrt{x(a + bx^2 + cx^4)} \left(\frac{(b + 2cx^2) \sqrt{a + bx^2 + cx^4}}{4c} - \frac{(b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{8c^{3/2}} \right)}{2\sqrt{x} \sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (Sqrt[x*(a + b*x^2 + c*x^4)]*((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2)))/(2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4])

IntegrateAlgebraic [A] time = 0.46, size = 126, normalized size = 0.98

$$\frac{\log(\sqrt{x})(4ac - b^2)}{16c^{3/2}} + \frac{(b^2 - 4ac) \log\left(-2c^{3/2}\sqrt{ax + bx^3 + cx^5} + bc\sqrt{x} + 2c^2x^{5/2}\right)}{16c^{3/2}} + \frac{(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) + ((-b^2 + 4*a*c)*Log[Sqrt[x]])/(16*c^(3/2)) + ((b^2 - 4*a*c)*Log[b*c*Sqrt[x] + 2*c^2*x^(5/2) - 2*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]])/(16*c^(3/2))

fricas [A] time = 1.04, size = 232, normalized size = 1.80

$$\left[\frac{(b^2 - 4ac)\sqrt{c}x \log\left(\frac{8c^2x^5 + 8bcx^3 + 4\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{x} + (b^2 + 4ac)x}{x}\right) - 4\sqrt{cx^5 + bx^3 + ax}(2c^2x^2 + bc)\sqrt{x}}{32c^2x}, \frac{(b^2 - 4ac)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{-c}\sqrt{x}}{2(c^2x^5 + bcx^3 + acx)}\right) + 2\sqrt{cx^5 + bx^3 + ax}(2c^2x^2 + bc)\sqrt{x}}{16c^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2), x, algorithm="fricas")

[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4*a*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x)]

giac [A] time = 0.97, size = 127, normalized size = 0.98

$$\frac{1}{8}\sqrt{cx^4 + bx^2 + a}\left(2x^2 + \frac{b}{c}\right) + \frac{(b^2 - 4ac) \log\left(\left|-2\left(\sqrt{cx^2 - \sqrt{cx^4 + bx^2 + a}}\sqrt{c} - b\right)\right|\right)}{16c^{\frac{3}{2}}} - \frac{b^2 \log\left(\left|-b + 2\sqrt{a}\sqrt{c}\right|\right) - 4ac \log\left(\left|-b + 2\sqrt{a}\sqrt{c}\right|\right) + 2\sqrt{a}b\sqrt{c}}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2), x, algorithm="giac")

[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - 1/16*(b^2*lo

$g(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) - 4*a*c*\log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) + 2*\text{sqrt}(a)*b*\text{sqrt}(c))/c^{(3/2)}$

maple [A] time = 0.01, size = 157, normalized size = 1.22

$$\frac{\sqrt{(cx^4 + bx^2 + a)}x \left(4\sqrt{cx^4 + bx^2 + a} c^{\frac{3}{2}}x^2 + 4ac \ln\left(\frac{2cx^2 + b + 2\sqrt{cx^4 + bx^2 + a} \sqrt{c}}{2\sqrt{c}}\right) - b^2 \ln\left(\frac{2cx^2 + b + 2\sqrt{cx^4 + bx^2 + a} \sqrt{c}}{2\sqrt{c}}\right) + 2\sqrt{cx^4 + bx^2 + a} b\sqrt{c} \right)}{16\sqrt{cx^4 + bx^2 + a} c^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x)`

[Out] $1/16*((c*x^4+b*x^2+a)*x)^{(1/2)}/c^{(3/2)}*(4*x^2*c^{(3/2)}*(c*x^4+b*x^2+a)^{(1/2)} + 4*\ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}+b)/c^{(1/2)})*a*c - \ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}+b)/c^{(1/2)})*b^2+2*b*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)})/x^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^5 + bx^3 + ax} \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \sqrt{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)`

[Out] `int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \sqrt{x(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)`

[Out] `Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)`

$$3.105 \quad \int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}} dx$$

Optimal. Leaf size=194

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1921, 1953, 1251, 843, 621, 206, 724}

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2), x]

[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax + bx^3 + cx^5}}{x^{3/2}} dx &= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{1}{2} \int \frac{2a + bx^2}{\sqrt{x} \sqrt{ax + bx^3 + cx^5}} dx \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{2a+bx^2}{x\sqrt{a+bx^2+cx^4}} dx}{2\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{4\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{\left(a\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{2\sqrt{ax + bx^3 + cx^5}} + \frac{\left(b\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{2\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} - \frac{\left(a\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax + bx^3 + cx^5}} + \frac{\left(b\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} - \frac{\sqrt{a} \sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax + bx^3 + cx^5}} + \frac{b\sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax + bx^3 + cx^5}} + \frac{b\sqrt{x} \sqrt{a + bx^2 + cx^4}}{4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 155, normalized size = 0.80

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \left(2\sqrt{c} \sqrt{a + bx^2 + cx^4} - 2\sqrt{a} \sqrt{c} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a} \sqrt{a+bx^2+cx^4}}\right) + b \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right) \right)}{4\sqrt{c} \sqrt{x} (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2), x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] - 2*Sqrt[a]*Sqrt[c]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])) + b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])))/(4*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 0.60, size = 148, normalized size = 0.76

$$\frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} - \frac{b \log\left(-2\sqrt{c} \sqrt{ax + bx^3 + cx^5} + b\sqrt{x} + 2cx^{5/2}\right)}{4\sqrt{c}} + \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{c} x^{5/2} - \sqrt{ax + bx^3 + cx^5}}\right) + \frac{b \log(\sqrt{x})}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) + Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[x])/(
Sqrt[c]*x^(5/2) - Sqrt[a*x + b*x^3 + c*x^5])] + (b*Log[Sqrt[x]])/(4*Sqrt[c]
) - (b*Log[b*Sqrt[x] + 2*c*x^(5/2) - 2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5]])/(
(4*Sqrt[c]))
```

```
fricas [A] time = 1.40, size = 666, normalized size = 3.43
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)
*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(
(b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^
2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*
x), -1/4*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*s
qrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - sqrt(a)*c*x*log(-((b^2 + 4*a
*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*s
qrt(a)*sqrt(x))/x^5) - 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/8*(4
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*s
qrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x
^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a
*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(2*sqrt(-a)*c
*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c
*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^
5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.01, size = 136, normalized size = 0.70

$$\frac{\sqrt{(cx^4 + bx^2 + a)x} \left(2\sqrt{a} \sqrt{c} \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a} \sqrt{a}}{x^2}\right) - b \ln\left(\frac{2cx^2 + b + 2\sqrt{cx^4 + bx^2 + a} \sqrt{c}}{2\sqrt{c}}\right) - 2\sqrt{cx^4 + bx^2 + a} \sqrt{c} \right)}{4\sqrt{cx^4 + bx^2 + a} \sqrt{c} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2), x)`

[Out]
$$-1/4*((c*x^4+b*x^2+a)*x)^(1/2)/x^(1/2)*(2*a^(1/2)*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*c^(1/2)-2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)-b*\ln(1/2*(2*c*x^2+b+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/c^(1/2)))/(c*x^4+b*x^2+a)^(1/2)/c^(1/2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2), x)`

[Out] `int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x(a + bx^2 + cx^4)}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2), x)`

[Out] `Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)`

$$3.106 \quad \int x^{3/2} (ax + bx^3 + cx^5)^{3/2} dx$$

Optimal. Leaf size=244

$$\frac{(128a^2c^2 - 100ab^2c + 15b^4) \sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{3b\sqrt{x} (b^2 - 4ac)^2 \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax + bx^3 + cx^5}} - \frac{x^{3/2}}{80c}$$

Rubi [A] time = 0.36, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1919, 1945, 1949, 12, 1914, 1107, 621, 206}

$$\frac{(128a^2c^2 - 100ab^2c + 15b^4) \sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2} (4cx^2 (5b^2 - 16ac) + b(5b^2 - 4ac)) \sqrt{ax + bx^3 + cx^5}}{640c^2} - \frac{3b\sqrt{x} (b^2 - 4ac)^2 \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax + bx^3 + cx^5}} + \frac{\sqrt{x} (3b + 8cx^2) (ax + bx^3 + cx^5)^{3/2}}{80c}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107


```
Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
  Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1919

```
Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
+ (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
```


Mathematica [A] time = 0.20, size = 192, normalized size = 0.79

$$\frac{(x(a+bx^2+cx^4))^{3/2} \left(-\frac{3b(b^2-4ac) \left((b^2-4ac) \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}} \right) - 2\sqrt{c}(b+2cx^2)\sqrt{a+bx^2+cx^4} \right)}{256c^{7/2}} - \frac{b(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{16c^2} + \frac{(a+bx^2+cx^4)^{5/2}}{5c} \right)}{2x^{3/2}(a+bx^2+cx^4)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]

[Out] ((x*(a + b*x^2 + c*x^4))^(3/2)*(-1/16*(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/c^2 + (a + b*x^2 + c*x^4)^(5/2)/(5*c) - (3*b*(b^2 - 4*a*c)*(-2*sqrt[c]*(b + 2*c*x^2)*sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*sqrt[c]*sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2)))/(2*x^(3/2)*(a + b*x^2 + c*x^4)^(3/2))

IntegrateAlgebraic [A] time = 1.92, size = 214, normalized size = 0.88

$$\frac{3(16a^2bc^2 - 8ab^3c + b^5) \log\left(\frac{-2\sqrt{c}\sqrt{ax+bx^3+cx^5} + b\sqrt{x} + 2cx^{5/2}}{512c^{7/2}}\right) - 3 \log(\sqrt{x})(16a^2bc^2 - 8ab^3c + b^5)}{512c^{7/2}} + \frac{\sqrt{ax+bx^3+cx^5}(128a^2c^2 - 100ab^2c + 56abc^2x^2 + 256ac^3x^4 + 15b^4 - 10b^2cx^2 + 8b^2c^2x^4 + 176bc^2x^6 + 128c^4x^8)}{1280c^3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]

[Out] (sqrt[a*x + b*x^3 + c*x^5]*(15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 10*b^3*c*x^2 + 56*a*b*c^2*x^2 + 8*b^2*c^2*x^4 + 256*a*c^3*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8))/(1280*c^3*sqrt[x]) - (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[sqrt[x]])/(512*c^(7/2)) + (3*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*Log[b*sqrt[x] + 2*c*x^(5/2) - 2*sqrt[c]*sqrt[a*x + b*x^3 + c*x^5]])/(512*c^(7/2))

fricas [A] time = 1.28, size = 396, normalized size = 1.62

$$\frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{c}\log\left(\frac{5\sqrt{c}\sqrt{ax+bx^3+cx^5} + 2\sqrt{c}\sqrt{x} + 2cx^{5/2}}{512c^{7/2}}\right) + 4(128c^5x^8 + 176b^2c^4x^6 + 15b^4c^3x^4 - 100a^2b^2c^2x^2 + 128a^2c^3x^4 + 8(b^2c^3 + 32a^2c^4)x^4 - 2(5b^3c^2 - 28ab^2c^3)\sqrt{c}\sqrt{x} + b^2 + ax\sqrt{c})}{512c^{7/2}} + \frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{ax+bx^3+cx^5}}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}\right) + 2(128c^5x^8 + 176b^2c^4x^6 + 15b^4c^3x^4 - 100a^2b^2c^2x^2 + 128a^2c^3x^4 + 8(b^2c^3 + 32a^2c^4)x^4 - 2(5b^3c^2 - 28ab^2c^3)\sqrt{c}\sqrt{x} + b^2 + ax\sqrt{c})}{256c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")

[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4

- 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x)]

giac [B] time = 2.11, size = 662, normalized size = 2.71

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] $\frac{1}{96}*(2*\sqrt{c*x^4 + b*x^2 + a}*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*\log(\text{abs}(-2*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2 + a})*\sqrt{c} - b))/c^{5/2} + (3*b^3*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) - 12*a*b*c*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) + 6*\sqrt{a}*b^2*\sqrt{c} - 16*a^{3/2}*c^{3/2})/c^{5/2})*a + 1/768*(2*\sqrt{c*x^4 + b*x^2 + a}*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*\log(\text{abs}(-2*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2 + a})*\sqrt{c} - b))/c^{7/2} - (15*b^4*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) - 72*a*b^2*c*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) + 48*a^2*c^2*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) + 30*\sqrt{a}*b^3*\sqrt{c} - 104*a^{3/2}*b*c^{3/2})/c^{7/2})*b + 1/7680*(2*\sqrt{c*x^4 + b*x^2 + a}*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4 - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*\log(\text{abs}(-2*(\sqrt{c}*x^2 - \sqrt{c*x^4 + b*x^2 + a})*\sqrt{c} - b))/c^{9/2} + (105*b^5*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) - 600*a*b^3*c*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) + 720*a^2*b*c^2*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) + 210*\sqrt{a}*b^4*\sqrt{c} - 920*a^{3/2}*b^2*c^{3/2} + 512*a^{5/2}*c^{5/2})/c^{9/2})*c$

maple [A] time = 0.01, size = 369, normalized size = 1.51

$$\frac{\sqrt{c^4 + b^2 + a} \left(-256\sqrt{c^4 + b^2 + a} c^2 b^2 - 352\sqrt{c^4 + b^2 + a} b c^2 - 512\sqrt{c^4 + b^2 + a} a c^2 - 16\sqrt{c^4 + b^2 + a} b^2 c^2 - 112\sqrt{c^4 + b^2 + a} a b c^2 + 20\sqrt{c^4 + b^2 + a} b^2 c^2 + 240b^2 c^2 \ln\left(\frac{2\sqrt{c^4 + b^2 + a} c^2}{2c}\right) - 120b^2 c^2 \ln\left(\frac{2\sqrt{c^4 + b^2 + a} c^2}{2c}\right) + 15b^2 c^2 \ln\left(\frac{2\sqrt{c^4 + b^2 + a} c^2}{2c}\right) - 256\sqrt{c^4 + b^2 + a} a c^2 + 200\sqrt{c^4 + b^2 + a} b c^2 - 30\sqrt{c^4 + b^2 + a} b^2 c^2 \right)}{2560\sqrt{c^4 + b^2 + a} c^2 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x)

[Out] $-1/2560*((c*x^4+b*x^2+a)*x)^{1/2}/c^{7/2}*(-256*x^8*c^{9/2}*(c*x^4+b*x^2+a)^{1/2}-352*x^6*b*c^{7/2}*(c*x^4+b*x^2+a)^{1/2}-512*x^4*a*c^{7/2}*(c*x^4+b*x^2+a)^{1/2}-16*x^4*b^2*c^{5/2}*(c*x^4+b*x^2+a)^{1/2}-112*x^2*a*b*c^{5/2}*(c*x^4+b*x^2+a)^{1/2}+20*x^2*b^3*c^{3/2}*(c*x^4+b*x^2+a)^{1/2}+240*\ln(1/2*(2*c*x^2+b+2*(c*x^4+b*x^2+a)^{1/2}*c^{1/2}))/c^{1/2})*a^2*b*c^2-120*\ln(1/2*(2*c*x^2+b+2*(c*x^4+b*x^2+a)^{1/2}*c^{1/2}))/c^{1/2})*a*b^3*c+15*\ln(1/2*(2*c*x^2+b+2*(c*x^4+b*x^2+a)^{1/2}*c^{1/2}))/c^{1/2})*b^5-256*a^2*c^{5/2}*(c*x^4+b*x^2+a)^{1/2}+200*a*b^2*c^{3/2}*(c*x^4+b*x^2+a)^{1/2}-30*b^4*c^{1/2}*(c*x^4+b*x^2+a)^{1/2})/x^{1/2}/(c*x^4+b*x^2+a)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^5 + bx^3 + ax)^{\frac{3}{2}} x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} (cx^5 + bx^3 + ax)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)

[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)

[Out] Timed out

$$3.107 \quad \int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}} dx$$

Optimal. Leaf size=177

$$\frac{3\sqrt{x} (b^2 - 4ac)^2 \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax + bx^3 + cx^5}} - \frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1918, 1914, 1107, 621, 206}

$$-\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{3\sqrt{x} (b^2 - 4ac)^2 \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax + bx^3 + cx^5}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x], x]

[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c^2*Sqrt[x]) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) + (3*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1914

```

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

```

Rule 1918

```

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]

```

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3 + cx^5)^{3/2}}{\sqrt{x}} dx &= \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} - \frac{(3(b^2 - 4ac)) \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2 - 4ac) \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2 - 4ac) \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2 - 4ac) \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2 - 4ac) \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx}{16c}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 152, normalized size = 0.86

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \left(2\sqrt{c} (b + 2cx^2) \sqrt{a + bx^2 + cx^4} (4c(5a + 2cx^4) - 3b^2 + 8bcx^2) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right) \right)}{256c^{5/2} \sqrt{x(a + bx^2 + cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x], x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(5/2)*Sqrt[x*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 1.52, size = 181, normalized size = 1.02

$$\frac{3(16a^2c^2 - 8ab^2c + b^4) \log\left(-2\sqrt{c}\sqrt{ax + bx^3 + cx^5} + b\sqrt{x} + 2cx^{5/2}\right)}{256c^{5/2}} + \frac{3 \log(\sqrt{x})(16a^2c^2 - 8ab^2c + b^4)}{256c^{5/2}} + \frac{\sqrt{ax + bx^3 + cx^5} (20abc + 40ac^2x^2 - 3b^3 + 2b^2cx^2 + 24bc^2x^4 + 16c^3x^6)}{128c^2\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x], x]

[Out] (Sqrt[a*x + b*x^3 + c*x^5]*(-3*b^3 + 20*a*b*c + 2*b^2*c*x^2 + 40*a*c^2*x^2 + 24*b*c^2*x^4 + 16*c^3*x^6))/(128*c^2*Sqrt[x]) + (3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*Log[Sqrt[x]])/(256*c^(5/2)) - (3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*Log[b*Sqrt[x] + 2*c*x^(5/2) - 2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5]])/(256*c^(5/2))

fricas [A] time = 1.02, size = 332, normalized size = 1.88

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{c} \log\left(\frac{-3b^2 + 8bcx^2 + 4c(5a + 2cx^4) - 3b^2 + 8bcx^2}{512c^2}\right) + 4(16c^4x^6 + 24b^2c^3x^4 - 3b^3c + 20a^2b^2c^2 + 2(b^2c^2 + 20ac^2)x^2)\sqrt{cx^5 + bx^3 + ax} \sqrt{x} - 3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^5 + bx^3 + ax}\sqrt{c}}{2(b^2c^2 + 20ac^2)}\right) - 2(16c^4x^6 + 24b^2c^3x^4 - 3b^3c + 20a^2b^2c^2 + 2(b^2c^2 + 20ac^2)x^2)\sqrt{cx^5 + bx^3 + ax} \sqrt{x}}{256c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2), x, algorithm="fricas")

[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 4*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x), -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x)]

giac [B] time = 1.58, size = 518, normalized size = 2.93

$\frac{1}{16} \sqrt{c^2 x^4 + b^2 x^2 + a} (2x^2 + b/c) + (b^2 - 4ac) \log(\text{abs}(-2\sqrt{c}x^2 - \sqrt{c^2 x^4 + b^2 x^2 + a})\sqrt{c} - b)/c^{3/2} - (b^2 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) - 4ac \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 2\sqrt{a}b\sqrt{c})/c^{3/2})a + 1/96(2\sqrt{c^2 x^4 + b^2 x^2 + a})(2(4x^2 + b/c)x^2 - (3b^2 - 8ac)/c^2) - 3(b^3 - 4ab^2c) \log(\text{abs}(-2(\sqrt{c}x^2 - \sqrt{c^2 x^4 + b^2 x^2 + a})\sqrt{c} - b))/c^{5/2} + (3b^3 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) - 12ab^2c \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 6\sqrt{a}b^2\sqrt{c} - 16a^{3/2}c^{3/2})/c^{5/2})b + 1/768(2\sqrt{c^2 x^4 + b^2 x^2 + a})(2(4(6x^2 + b/c)x^2 - (5b^2c - 12ac^2)/c^3)x^2 + (15b^3 - 52ab^2c)/c^3) + 3(5b^4 - 24ab^2c + 16a^2c^2) \log(\text{abs}(-2(\sqrt{c}x^2 - \sqrt{c^2 x^4 + b^2 x^2 + a})\sqrt{c} - b))/c^{7/2} - (15b^4 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) - 72ab^2c \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 48a^2c^2 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 30\sqrt{a}b^3\sqrt{c} - 104a^{3/2}b^2c^{3/2})/c^{7/2})c$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")

[Out] $\frac{1}{16} \sqrt{c^2 x^4 + b^2 x^2 + a} (2x^2 + b/c) + (b^2 - 4ac) \log(\text{abs}(-2\sqrt{c}x^2 - \sqrt{c^2 x^4 + b^2 x^2 + a})\sqrt{c} - b)/c^{3/2} - (b^2 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) - 4ac \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 2\sqrt{a}b\sqrt{c})/c^{3/2})a + 1/96(2\sqrt{c^2 x^4 + b^2 x^2 + a})(2(4x^2 + b/c)x^2 - (3b^2 - 8ac)/c^2) - 3(b^3 - 4ab^2c) \log(\text{abs}(-2(\sqrt{c}x^2 - \sqrt{c^2 x^4 + b^2 x^2 + a})\sqrt{c} - b))/c^{5/2} + (3b^3 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) - 12ab^2c \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 6\sqrt{a}b^2\sqrt{c} - 16a^{3/2}c^{3/2})/c^{5/2})b + 1/768(2\sqrt{c^2 x^4 + b^2 x^2 + a})(2(4(6x^2 + b/c)x^2 - (5b^2c - 12ac^2)/c^3)x^2 + (15b^3 - 52ab^2c)/c^3) + 3(5b^4 - 24ab^2c + 16a^2c^2) \log(\text{abs}(-2(\sqrt{c}x^2 - \sqrt{c^2 x^4 + b^2 x^2 + a})\sqrt{c} - b))/c^{7/2} - (15b^4 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) - 72ab^2c \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 48a^2c^2 \log(\text{abs}(-b + 2\sqrt{a}\sqrt{c})) + 30\sqrt{a}b^3\sqrt{c} - 104a^{3/2}b^2c^{3/2})/c^{7/2})c$

maple [A] time = 0.02, size = 295, normalized size = 1.67

$\frac{\sqrt{c^2 x^4 + b^2 x^2 + a} x \left(32\sqrt{c^2 x^4 + b^2 x^2 + a} c^2 x^6 + 48\sqrt{c^2 x^4 + b^2 x^2 + a} b c^2 x^4 + 80\sqrt{c^2 x^4 + b^2 x^2 + a} a c^2 x^2 + 4\sqrt{c^2 x^4 + b^2 x^2 + a} b^2 c^2 x^2 + 48a^2 c^2 \ln\left(\frac{2c^2 x^2 + 2\sqrt{c^2 x^4 + b^2 x^2 + a} \sqrt{c}}{2\sqrt{c}}\right) - 24a b^2 c \ln\left(\frac{2c^2 x^2 + 2\sqrt{c^2 x^4 + b^2 x^2 + a} \sqrt{c}}{2\sqrt{c}}\right) + 3b^4 \ln\left(\frac{2c^2 x^2 + 2\sqrt{c^2 x^4 + b^2 x^2 + a} \sqrt{c}}{2\sqrt{c}}\right) + 40\sqrt{c^2 x^4 + b^2 x^2 + a} a b c^2 - 6\sqrt{c^2 x^4 + b^2 x^2 + a} b^2 \sqrt{c} \right)}{256\sqrt{c^2 x^4 + b^2 x^2 + a} c^2 \sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)

[Out] $\frac{1}{256} \left((c^2 x^4 + b^2 x^2 + a) x \right)^{1/2} / c^{5/2} * (32x^6 c^{7/2} (c^2 x^4 + b^2 x^2 + a)^{1/2} + 48x^4 b^2 c^{5/2} (c^2 x^4 + b^2 x^2 + a)^{1/2} + 80x^2 a c^{5/2} (c^2 x^4 + b^2 x^2 + a)^{1/2} + 4x^2 b^2 c^{3/2} (c^2 x^4 + b^2 x^2 + a)^{1/2} + 48 \ln(1/2 * (2c^2 x^2 + b + 2(c^2 x^4 + b^2 x^2 + a)^{1/2}) * c^{1/2}) / c^{1/2}) * a^2 c^2 - 24 \ln(1/2 * (2c^2 x^2 + b + 2(c^2 x^4 + b^2 x^2 + a)^{1/2}) * c^{1/2}) / c^{1/2}) * a b^2 c + 3 \ln(1/2 * (2c^2 x^2 + b + 2(c^2 x^4 + b^2 x^2 + a)^{1/2}) * c^{1/2}) / c^{1/2}) * b^4 + 40 a b^2 c^{3/2} (c^2 x^4 + b^2 x^2 + a)^{1/2} - 6 b^3 c^{3/2} (c^2 x^4 + b^2 x^2 + a)^{1/2} / x^{1/2} / (c^2 x^4 + b^2 x^2 + a)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")

[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(cx^5 + bx^3 + ax)^{3/2}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2),x)

[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x(a + bx^2 + cx^4))^{3/2}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)

[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/sqrt(x), x)

$$3.108 \quad \int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=82

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{c} \sqrt{ax + bx^3 + cx^5}}$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1914, 1107, 621, 206}

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{c} \sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[

$a*x^q + b*x^n + c*x^{(2*n - q)}$, Int[$x^{(m - q/2)}/\text{Sqrt}[a + b*x^{(n - q)} + c*x^{(2*(n - q))}$], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rubi steps

$$\begin{aligned} \int \frac{x^{3/2}}{\sqrt{ax + bx^3 + cx^5}} dx &= \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{x}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{2\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c} \sqrt{ax + bx^3 + cx^5}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.00

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c} \sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c} \sqrt{x(a + bx^2 + cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[$x^{(3/2)}/\text{Sqrt}[a*x + b*x^3 + c*x^5]$, x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 0.32, size = 67, normalized size = 0.82

$$\frac{\log(\sqrt{x})}{2\sqrt{c}} - \frac{\log\left(-2\sqrt{c} \sqrt{ax + bx^3 + cx^5} + b\sqrt{x} + 2cx^{5/2}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] Log[Sqrt[x]]/(2*Sqrt[c]) - Log[b*Sqrt[x] + 2*c*x^(5/2) - 2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5]]/(2*Sqrt[c])

fricas [A] time = 1.05, size = 135, normalized size = 1.65

$$\left[\frac{\log\left(-\frac{8c^2x^5+8bcx^3+4\sqrt{cx^5+bx^3+ax}(2cx^2+b)\sqrt{c}\sqrt{x+(b^2+4ac)x}}{x}\right)}{4\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(2cx^2+b)\sqrt{-c}\sqrt{x}}{2(c^2x^5+bcx^3+acx)}\right)}{2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2), x, algorithm="fricas")

[Out] [1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x))*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x))/c]

giac [A] time = 0.63, size = 60, normalized size = 0.73

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{2\sqrt{c}} + \frac{\log\left(\left|-b + 2\sqrt{a}\sqrt{c}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2), x, algorithm="giac")

[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(cx^4 + bx^2 + a))*sqrt(c) - b))/sqrt(c) + 1/2*log(abs(-b + 2*sqrt(a)*sqrt(c)))/sqrt(c)

maple [A] time = 0.01, size = 72, normalized size = 0.88

$$\frac{\sqrt{(cx^4 + bx^2 + a)}x \ln\left(\frac{2cx^2 + b + 2\sqrt{cx^4 + bx^2 + a}\sqrt{c}}{2\sqrt{c}}\right)}{2\sqrt{cx^4 + bx^2 + a}\sqrt{c}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2), x)

[Out] 1/2/x^(1/2)*((c*x^4+b*x^2+a)*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^2 + b + 2*(c*x^4 + b*x^2 + a)^(1/2)*c^(1/2))/c^(1/2))/c^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)

[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)

$$3.109 \quad \int \frac{1}{\sqrt{x} \sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]

[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rubi steps

$$\int \frac{1}{\sqrt{x} \sqrt{ax + bx^3 + cx^5}} dx = -\text{Subst} \left(\int \frac{1}{4a - x^2} dx, x, \frac{\sqrt{x} (2a + bx^2)}{\sqrt{ax + bx^3 + cx^5}} \right)$$

$$= -\frac{\tanh^{-1} \left(\frac{\sqrt{x} (2a + bx^2)}{2\sqrt{a} \sqrt{ax + bx^3 + cx^5}} \right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.63

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{2\sqrt{a} \sqrt{x} (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]

[Out] -1/2*(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 0.36, size = 52, normalized size = 1.02

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{c} x^{5/2} - \sqrt{ax + bx^3 + cx^5}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]

[Out] ArcTanh[(Sqrt[a]*Sqrt[x])/(Sqrt[c]*x^(5/2) - Sqrt[a*x + b*x^3 + c*x^5])]/Sqrt[a]

fricas [A] time = 1.17, size = 137, normalized size = 2.69

$$\left[\frac{\log \left(-\frac{(b^2 + 4ac)x^5 + 8abx^3 + 8a^2x - 4\sqrt{cx^5 + bx^3 + ax}(bx^2 + 2a)\sqrt{a}\sqrt{x}}{x^5} \right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^5 + bx^3 + ax}(bx^2 + 2a)\sqrt{-a}\sqrt{x}}{2(acx^5 + abx^3 + a^2x)} \right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]

giac [A] time = 0.54, size = 56, normalized size = 1.10

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

maple [A] time = 0.02, size = 72, normalized size = 1.41

$$\frac{\sqrt{(cx^4 + bx^2 + a)}x \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a}\sqrt{a}}{x^2}\right)}{2\sqrt{cx^4 + bx^2 + a}\sqrt{a}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)

[Out] -1/2/x^(1/2)*((c*x^4+b*x^2+a)*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} \sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

$$3.110 \quad \int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}} dx$$

Optimal. Leaf size=103

$$\frac{\sqrt{x}(-2ac + b^2 + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1922, 1913, 206}

$$\frac{\sqrt{x}(-2ac + b^2 + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*a^(3/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1922

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di

```

st[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n
- q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -(n - q)*(2*p + 3
)]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(ax + bx^3 + cx^5)^{3/2}} dx &= \frac{\sqrt{x} (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) \sqrt{ax + bx^3 + cx^5}} + \frac{\int \frac{1}{\sqrt{x} \sqrt{ax + bx^3 + cx^5}} dx}{a} \\
&= \frac{\sqrt{x} (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) \sqrt{ax + bx^3 + cx^5}} - \frac{\text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{\sqrt{x}(2a + bx^2)}{\sqrt{ax + bx^3 + cx^5}}\right)}{a} \\
&= \frac{\sqrt{x} (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) \sqrt{ax + bx^3 + cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a + bx^2)}{2\sqrt{a} \sqrt{ax + bx^3 + cx^5}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 126, normalized size = 1.22

$$\frac{\sqrt{x} \left((b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}}\right) - 2\sqrt{a} (-2ac + b^2 + bcx^2) \right)}{2a^{3/2} (4ac - b^2) \sqrt{x} (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] (Sqrt[x]*(-2*Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]))/(2*a^(3/2)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 1.84, size = 123, normalized size = 1.19

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{c} x^{5/2} - \sqrt{ax + bx^3 + cx^5}}\right)}{a^{3/2}} + \frac{\sqrt{ax + bx^3 + cx^5} (2ac - b^2 - bcx^2)}{a\sqrt{x} (4ac - b^2) (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] $((-b^2 + 2*a*c - b*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) / (a*(-b^2 + 4*a*c)*\text{Sqrt}[x]*(a + b*x^2 + c*x^4)) + \text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[x]) / (\text{Sqrt}[c]*x^{5/2} - \text{Sqrt}[a*x + b*x^3 + c*x^5])] / a^{3/2}$

fricas [B] time = 1.56, size = 424, normalized size = 4.12

$$\frac{\left((b^2c - 4a^2c^2)x^5 + (b^3 - 4abc)x^3 + (ab^2 - 4a^2c)x \right) \sqrt{a} \log \left(\frac{(b^2+4ac)^2 + 8ab^3 + 8a^2c - 4\sqrt{c^3+bx^2+ax}(b^2+2a)\sqrt{a}\sqrt{x}}{x} \right) + 4\sqrt{c^3+bx^2+ax}(abcx^2+ab^2-2a^2c)\sqrt{x}}{4((a^2b^2c-4a^2c^2)x^5 + (a^2b^3-4a^2bc)x^3 + (a^2b^2-4a^2c)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2), x, algorithm="fricas")

[Out] $[1/4 * (((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x) * \text{sqrt}(a) * \log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*\text{sqrt}(c*x^5 + b*x^3 + a*x) * (b*x^2 + 2*a) * \text{sqrt}(a) * \text{sqrt}(x)) / x^5) + 4*\text{sqrt}(c*x^5 + b*x^3 + a*x) * (a*b*c*x^2 + a*b^2 - 2*a^2*c) * \text{sqrt}(x)) / ((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2 * (((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x) * \text{sqrt}(-a) * \arctan(1/2*\text{sqrt}(c*x^5 + b*x^3 + a*x) * (b*x^2 + 2*a) * \text{sqrt}(-a) * \text{sqrt}(x) / (a*c*x^5 + a*b*x^3 + a^2*x)) + 2*\text{sqrt}(c*x^5 + b*x^3 + a*x) * (a*b*c*x^2 + a*b^2 - 2*a^2*c) * \text{sqrt}(x)) / ((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)]$

giac [B] time = 0.66, size = 193, normalized size = 1.87

$$\frac{\frac{abcx^2}{a^2b^2-4a^3c} + \frac{ab^2-2a^2c}{a^2b^2-4a^3c}}{\sqrt{cx^4+bx^2+a}} - \frac{ab^2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) - 4a^2c \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a} \sqrt{a} b^2 - 2\sqrt{-a} a^{\frac{3}{2}} c}{\sqrt{-a} a^2 b^2 - 4\sqrt{-a} a^3 c} + \frac{\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2), x, algorithm="giac")

[Out] $(a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/\text{sqrt}(c*x^4 + b*x^2 + a) - (a*b^2*\arctan(\text{sqrt}(a)/\text{sqrt}(-a)) - 4*a^2*c*\arctan(\text{sqrt}(a)/\text{sqrt}(-a)) + \text{sqrt}(-a)*\text{sqrt}(a)*b^2 - 2*\text{sqrt}(-a)*a^{3/2}*c)/(\text{sqrt}(-a)*a^2*b^2 - 4*\text{sqrt}(-a)*a^3*c) + \arctan(-(\text{sqrt}(c)*x^2 - \text{sqrt}(c*x^4 + b*x^2 + a))/\text{sqrt}(-a))/(\text{sqrt}(-a)*a)$

maple [B] time = 0.02, size = 179, normalized size = 1.74

$$\frac{\sqrt{(cx^4 + bx^2 + a)} x \left(2\sqrt{a} bcx^2 + 4\sqrt{cx^4 + bx^2 + a} ac \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right) - \sqrt{cx^4 + bx^2 + a} b^2 \ln\left(\frac{bx^2+2a+2\sqrt{cx^4+bx^2+a}\sqrt{a}}{x^2}\right) - 4a^{\frac{3}{2}}c + 2\sqrt{a} b^2 \right)}{2(cx^4 + bx^2 + a)(4ac - b^2)a^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x)`

[Out]
$$-1/2*((c*x^4+b*x^2+a)*x)^(1/2)/a^(3/2)*(2*x^2*b*c*a^(1/2)+4*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a)^(1/2)-\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*b^2*(c*x^4+b*x^2+a)^(1/2)-4*a^(3/2)*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(cx^5 + bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{(cx^5 + bx^3 + ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)`

[Out] `int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(x(a + bx^2 + cx^4))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)`

[Out] `Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)`

$$3.111 \quad \int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}} dx$$

Optimal. Leaf size=154

$$\frac{3b \tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}} - \frac{(3b^2 - 8ac)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}(b^2 - 4ac)} + \frac{-2ac + b^2 + bcx^2}{ax^{3/2}(b^2 - 4ac)\sqrt{ax+bx^3+cx^5}}$$

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1951, 12, 1913, 206}

$$-\frac{(3b^2 - 8ac)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}(b^2 - 4ac)} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}} + \frac{-2ac + b^2 + bcx^2}{ax^{3/2}(b^2 - 4ac)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x]

[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]) - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])])/(4*a^(5/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1924

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
&& LtQ[m + p*q + 1, n - q]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2} (ax + bx^3 + cx^5)^{3/2}} dx &= \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) x^{3/2} \sqrt{ax + bx^3 + cx^5}} - \frac{\int \frac{-3b^2 + 8ac - 2bcx^2}{x^{5/2} \sqrt{ax + bx^3 + cx^5}} dx}{a (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) x^{3/2} \sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac) \sqrt{ax + bx^3 + cx^5}}{2a^2 (b^2 - 4ac) x^{5/2}} + \frac{\int -\frac{3b(b^2 - 4ac)}{\sqrt{x} \sqrt{ax + bx^3 + cx^5}} dx}{2a^2 (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) x^{3/2} \sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac) \sqrt{ax + bx^3 + cx^5}}{2a^2 (b^2 - 4ac) x^{5/2}} - \frac{(3b) \int \frac{1}{\sqrt{x} \sqrt{ax + bx^3 + cx^5}} dx}{2a^2 (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) x^{3/2} \sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac) \sqrt{ax + bx^3 + cx^5}}{2a^2 (b^2 - 4ac) x^{5/2}} + \frac{(3b) \operatorname{Subst} \int \frac{1}{\sqrt{x} \sqrt{ax + bx^3 + cx^5}} dx}{2a^2 (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) x^{3/2} \sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac) \sqrt{ax + bx^3 + cx^5}}{2a^2 (b^2 - 4ac) x^{5/2}} + \frac{3b \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{c} x^{5/2} - \sqrt{ax + bx^3 + cx^5}} \right)}{2a^2 (b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 160, normalized size = 1.04

$$\frac{2\sqrt{a} (-4a^2c + a(b^2 - 10bcx^2 - 8c^2x^4) + 3b^2x^2(b + cx^2)) - 3bx^2(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a} \sqrt{a + bx^2 + cx^4}} \right)}{4a^{5/2} x^{3/2} (4ac - b^2) \sqrt{x(a + bx^2 + cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]

[Out] (2*sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x^4)) - 3*b*(b^2 - 4*a*c)*x^2*sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*sqrt[a]*sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*sqrt[x*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 2.25, size = 159, normalized size = 1.03

$$\frac{(-4a^2c + ab^2 - 10abcx^2 - 8ac^2x^4 + 3b^3x^2 + 3b^2cx^4) \sqrt{ax + bx^3 + cx^5}}{2a^2x^{5/2} (4ac - b^2) (a + bx^2 + cx^4)} - \frac{3b \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{c} x^{5/2} - \sqrt{ax + bx^3 + cx^5}} \right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]

[Out] ((a*b^2 - 4*a^2*c + 3*b^3*x^2 - 10*a*b*c*x^2 + 3*b^2*c*x^4 - 8*a*c^2*x^4)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(-b^2 + 4*a*c)*x^(5/2)*(a + b*x^2 + c*x^4) - (3*b*ArcTanh[(Sqrt[a]*Sqrt[x])/(Sqrt[c]*x^(5/2) - Sqrt[a*x + b*x^3 + c*x^5])]))/(2*a^(5/2))

fricas [A] time = 1.34, size = 508, normalized size = 3.30

$$\frac{3((b^2c - 4ab^2)^2 + (b^2 - 4ab^2)^2 + (ab^2 - 4a^2bc)^2)\sqrt{a} \log\left(\frac{(b^2c - 4ab^2)^2 + (b^2 - 4ab^2)^2 + (ab^2 - 4a^2bc)^2}{4((a^2b^2c - 4a^2c^2)^2 + (a^2b^2 - 4a^2bc)^2 + (a^2b^2 - 4a^2c^2)^2)}\right) - 4\sqrt{a^2 + bc} + \arctan\left(\frac{(3ab^2c - 8a^2b^2)^2 + a^2c^2 - 4a^2c + (3ab^2 - 10a^2bc)^2}{4((a^2b^2c - 4a^2c^2)^2 + (a^2b^2 - 4a^2bc)^2 + (a^2b^2 - 4a^2c^2)^2)}\right)\sqrt{a}}{4((a^2b^2c - 4a^2c^2)^2 + (a^2b^2 - 4a^2bc)^2 + (a^2b^2 - 4a^2c^2)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")

[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(a)*log(-(b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) - 4*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.02, size = 220, normalized size = 1.43

$$\frac{\sqrt{(cx^4 + bx^2 + a)x} \left(-16a^{\frac{3}{2}}c^2x^4 + 6\sqrt{a}b^2cx^4 + 12\sqrt{c}x^4 + bx^2 + a \right) abcx^2 \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a}\sqrt{a}}{x^2}\right) - 3\sqrt{cx^4 + bx^2 + a}b^3x^2 \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a}\sqrt{a}}{x^2}\right) - 20a^{\frac{3}{2}}bcx^2 + 6\sqrt{a}b^3x^2 - 8a^{\frac{5}{2}}c + 2a^{\frac{3}{2}}b^2}{4(cx^4 + bx^2 + a)(4ac - b^2)a^{\frac{5}{2}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)

[Out] 1/4*((c*x^4+b*x^2+a)*x)^(1/2)/a^(5/2)*(-16*x^4*a^(3/2)*c^2+6*x^4*b^2*c*a^(1/2)+12*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)*x^2*a*b*c*(c*x^4

$$+b*x^2+a)^{(1/2)}-3*\ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^{(1/2)}*a^{(1/2)})/x^2)*x^2*b^3*(c*x^4+b*x^2+a)^{(1/2)}-20*a^{(3/2)}*x^2*b*c+6*x^2*b^3*a^{(1/2)}-8*a^{(5/2)}*c+2*a^{(3/2)}*b^2)/x^{(5/2)}/(c*x^4+b*x^2+a)/(4*a*c-b^2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^5 + bx^3 + ax)^{\frac{3}{2}} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2} (cx^5 + bx^3 + ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)

[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (x(a + bx^2 + cx^4))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)

[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)

$$3.112 \quad \int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}} dx$$

Optimal. Leaf size=51

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1915}

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Antiderivative was successfully verified.

[In] Int[x^((3*(-1+n))/2)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x]

[Out] (-2*x^((-1+n)/2)*(b+2*c*x))/((b^2-4*a*c)*Sqrt[a*x^(-1+n)+b*x^n+c*x^(1+n)])

Rule 1915

Int[(x_)^(m_)/((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(3/2), x_Symbol] :> Simp[(-2*x^((n-1)/2)*(b+2*c*x))/((b^2-4*a*c)*Sqrt[a*x^(n-1)+b*x^n+c*x^(n+1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3*(n-1))/2] && EqQ[q, n-1] && EqQ[r, n+1] && NeQ[b^2-4*a*c, 0]

Rubi steps

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}} dx = -\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.90

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{x^{n-1}(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2), x]

[Out] $(-2*x^{(-1 + n)/2}*(b + 2*c*x))/((b^2 - 4*a*c)*\text{Sqrt}[x^{(-1 + n)}*(a + x*(b + c*x))])$

IntegrateAlgebraic [A] time = 0.13, size = 73, normalized size = 1.43

$$\frac{2x^{\frac{3(n-1)}{2}}(b+2cx)(a+x(b+cx))^{3/2}}{(b^2-4ac)\sqrt{a+bx+cx^2}(x^{n-1}(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2), x]

[Out] $(-2*x^{((3*(-1 + n))/2)}*(b + 2*c*x)*(a + x*(b + c*x))^{3/2})/((b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]*x^{(-1 + n)}*(a + x*(b + c*x))^{3/2})$

fricas [A] time = 1.08, size = 83, normalized size = 1.63

$$-\frac{2(2cx^2 + bx)\sqrt{\frac{(cx^2+bx+a)x^{n+1}}{x^2}}}{(ab^2 - 4a^2c + (b^2c - 4ac^2)x^2 + (b^3 - 4abc)x)x^{\frac{1}{2}n + \frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2), x, algorithm="fricas")

[Out] $-2*(2*c*x^2 + b*x)*\text{sqrt}((c*x^2 + b*x + a)*x^{(n + 1)}/x^2)/((a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^{(1/2*n + 1/2)})$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n - \frac{3}{2}}}{(cx^{n+1} + ax^{n-1} + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2), x, algorithm="giac")

[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3n}{2} - \frac{3}{2}}}{(ax^{n-1} + bx^n + cx^{n+1})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2*n-3/2)/(a*x^(n-1)+b*x^n+c*x^(n+1))^(3/2), x)

[Out] int(x^(3/2*n-3/2)/(a*x^(n-1)+b*x^n+c*x^(n+1))^(3/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n - \frac{3}{2}}}{(cx^{n+1} + ax^{n-1} + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2), x, algorithm="maxima")

[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{\frac{3n}{2} - \frac{3}{2}}}{(bx^n + ax^{n-1} + cx^{n+1})^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2), x)

[Out] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2), x)

[Out] Timed out

$$3.113 \quad \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx = -\text{Subst}\left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right) \\ = -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.62

$$\frac{x\sqrt{x^4 - 3x^2 + 3} \tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]

[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

IntegrateAlgebraic [A] time = 0.16, size = 40, normalized size = 0.89

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{x^3 - \sqrt{x^6 - 3x^4 + 3x^2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]

[Out] ArcTanh[(Sqrt[3]*x)/(x^3 - Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

fricas [A] time = 1.02, size = 55, normalized size = 1.22

$$\frac{1}{6} \sqrt{3} \log\left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

giac [A] time = 0.44, size = 60, normalized size = 1.33

$$\frac{\sqrt{3} \log(x^2 + \sqrt{3} - \sqrt{x^4 - 3x^2 + 3}) - \sqrt{3} \log(-x^2 + \sqrt{3} + \sqrt{x^4 - 3x^2 + 3})}{6 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")

[Out] $1/6*(\sqrt{3}*\log(x^2 + \sqrt{3}) - \sqrt{x^4 - 3*x^2 + 3}) - \sqrt{3}*\log(-x^2 + \sqrt{3} + \sqrt{x^4 - 3*x^2 + 3}))/\text{sgn}(x)$

maple [A] time = 0.01, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4 - 3x^2 + 3} \sqrt{3} x \operatorname{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6 - 3x^4 + 3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^6-3*x^4+3*x^2)^{(1/2)}, x)$

[Out] $1/6/(x^6-3*x^4+3*x^2)^{(1/2)}*x*(x^4-3*x^2+3)^{(1/2)}*3^{(1/2)}*\operatorname{arctanh}(1/2*(x^2-2)*3^{(1/2)})/(x^4-3*x^2+3)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(x^6-3*x^4+3*x^2)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(1/\sqrt{x^6 - 3*x^4 + 3*x^2}, x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(3*x^2 - 3*x^4 + x^6)^{(1/2)}, x)$

[Out] $\text{int}(1/(3*x^2 - 3*x^4 + x^6)^{(1/2)}, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(x**6-3*x**4+3*x**2)**(1/2), x)$

[Out] $\text{Integral}(1/\sqrt{x**6 - 3*x**4 + 3*x**2}, x)$

$$3.114 \quad \int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1996

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx \\
&= -\text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}} \right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3} \tanh^{-1} \left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}} \right)}{2\sqrt{3}\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)], x]

[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

IntegrateAlgebraic [A] time = 0.04, size = 40, normalized size = 0.89

$$\frac{\tanh^{-1} \left(\frac{\sqrt{3}x}{x^3 - \sqrt{x^6 - 3x^4 + 3x^2}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[x^2*(3 - 3*x^2 + x^4)], x]

[Out] ArcTanh[(Sqrt[3]*x)/(x^3 - Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

fricas [A] time = 1.23, size = 55, normalized size = 1.22

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

giac [A] time = 0.72, size = 60, normalized size = 1.33

$$\frac{\sqrt{3} \log\left(x^2 + \sqrt{3} - \sqrt{x^4 - 3x^2 + 3}\right) - \sqrt{3} \log\left(-x^2 + \sqrt{3} + \sqrt{x^4 - 3x^2 + 3}\right)}{6 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")

[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)

maple [A] time = 0.01, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4 - 3x^2 + 3} \sqrt{3} x \operatorname{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{(x^4 - 3x^2 + 3)}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)

[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2 (x^4 - 3x^2 + 3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)`

[Out] `int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2(x^4 - 3x^2 + 3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2*(x**4-3*x**2+3))**(1/2), x)`

[Out] `Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)`

$$3.115 \quad \int \frac{1}{\sqrt{1-(1-x^2)^3}} dx$$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1996, 1904, 206}

$$\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - (1 - x^2)^3], x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1996

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-(1-x^2)^3}} dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx \\
&= -\text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}} \right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3} \tanh^{-1} \left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}} \right)}{2\sqrt{3}\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - (1 - x^2)^3], x]

[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

IntegrateAlgebraic [A] time = 0.04, size = 40, normalized size = 0.89

$$\frac{\tanh^{-1} \left(\frac{\sqrt{3}x}{x^3 - \sqrt{x^6 - 3x^4 + 3x^2}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 - (1 - x^2)^3], x]

[Out] ArcTanh[(Sqrt[3]*x)/(x^3 - Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

fricas [A] time = 0.88, size = 55, normalized size = 1.22

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

giac [A] time = 0.49, size = 60, normalized size = 1.33

$$\frac{\sqrt{3} \log(x^2 + \sqrt{3} - \sqrt{x^4 - 3x^2 + 3}) - \sqrt{3} \log(-x^2 + \sqrt{3} + \sqrt{x^4 - 3x^2 + 3})}{6 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")

[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)

maple [A] time = 0.00, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4 - 3x^2 + 3} \sqrt{3} x \operatorname{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6 - 3x^4 + 3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(-x^2+1)^3)^(1/2),x)

[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2-1)^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(x^2-1)^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^3 + 1)^(1/2), x)
```

```
[Out] int(1/((x^2 - 1)^3 + 1)^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{\sqrt{1 - (1 - x^2)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x**2+1)**3)**(1/2), x)
```

```
[Out] Integral(1/sqrt(1 - (1 - x**2)**3), x)
```

$$3.116 \quad \int \sqrt{3x^2 - 3x^4 + x^6} dx$$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3*x^2 - 3*x^4 + x^6], x]

[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
  :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3x^2 - 3x^4 + x^6} dx &= \frac{\sqrt{3x^2 - 3x^4 + x^6} \int x\sqrt{3 - 3x^2 + x^4} dx}{x\sqrt{3 - 3x^2 + x^4}} \\
&= \frac{\sqrt{3x^2 - 3x^4 + x^6} \operatorname{Subst}\left(\int \sqrt{3 - 3x + x^2} dx, x, x^2\right)}{2x\sqrt{3 - 3x^2 + x^4}} \\
&= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(3\sqrt{3x^2 - 3x^4 + x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{3 - 3x + x^2}} dx, x, x^2\right)}{16x\sqrt{3 - 3x^2 + x^4}} \\
&= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(\sqrt{3}\sqrt{3x^2 - 3x^4 + x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{3}}} dx, x, -3 + 2x\right)}{16x\sqrt{3 - 3x^2 + x^4}} \\
&= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} - \frac{3\sqrt{3x^2 - 3x^4 + x^6} \sinh^{-1}\left(\frac{3 - 2x^2}{\sqrt{3}}\right)}{16x\sqrt{3 - 3x^2 + x^4}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.81

$$\frac{x \left(4x^6 - 18x^4 + 30x^2 + 3\sqrt{x^4 - 3x^2 + 3} \sinh^{-1}\left(\frac{2x^2 - 3}{\sqrt{3}}\right) - 18 \right)}{16\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3*x^2 - 3*x^4 + x^6], x]

[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])

IntegrateAlgebraic [A] time = 0.13, size = 73, normalized size = 0.85

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (2x^2 - 3)}{8x} - \frac{3}{16} \log\left(-2x^3 + 2\sqrt{x^6 - 3x^4 + 3x^2} + 3x\right) + \frac{3 \log(x)}{16}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3*x^2 - 3*x^4 + x^6],x]

[Out] $((-3 + 2*x^2)*\text{Sqrt}[3*x^2 - 3*x^4 + x^6])/(8*x) + (3*\text{Log}[x])/16 - (3*\text{Log}[3*x - 2*x^3 + 2*\text{Sqrt}[3*x^2 - 3*x^4 + x^6]])/16$

fricas [A] time = 1.09, size = 70, normalized size = 0.81

$$\frac{12x \log\left(-\frac{2x^3 - 3x - 2\sqrt{x^6 - 3x^4 + 3x^2}}{x}\right) - 8\sqrt{x^6 - 3x^4 + 3x^2}(2x^2 - 3) - 9x}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")

[Out] $-1/64*(12*x*\log(-(2*x^3 - 3*x - 2*\text{sqrt}(x^6 - 3*x^4 + 3*x^2))/x) - 8*\text{sqrt}(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x$

giac [A] time = 0.39, size = 69, normalized size = 0.80

$$\frac{1}{16} \left(2\sqrt{x^4 - 3x^2 + 3}(2x^2 - 3) - 3 \log(-2x^2 + 2\sqrt{x^4 - 3x^2 + 3} + 3) \right) \text{sgn}(x) + \frac{3}{16} (2\sqrt{3} + \log(2\sqrt{3} + 3)) \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")

[Out] $1/16*(2*\text{sqrt}(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*\log(-2*x^2 + 2*\text{sqrt}(x^4 - 3*x^2 + 3) + 3))*\text{sgn}(x) + 3/16*(2*\text{sqrt}(3) + \log(2*\text{sqrt}(3) + 3))*\text{sgn}(x)$

maple [A] time = 0.01, size = 81, normalized size = 0.94

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} \left(4\sqrt{x^4 - 3x^2 + 3} x^2 + 3 \operatorname{arcsinh}\left(\frac{\sqrt{3}(2x^2-3)}{3}\right) - 6\sqrt{x^4 - 3x^2 + 3} \right)}{16\sqrt{x^4 - 3x^2 + 3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6-3*x^4+3*x^2)^(1/2),x)

[Out] $1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*\operatorname{arcsinh}(1/3*3^(1/2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6 - 3x^4 + 3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^6 - 3x^4 + 3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - 3*x^4 + x^6)^(1/2),x)

[Out] int((3*x^2 - 3*x^4 + x^6)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6 - 3x^4 + 3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**6-3*x**4+3*x**2)**(1/2),x)

[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)

$$3.117 \quad \int \sqrt{x^2 (3 - 3x^2 + x^4)} dx$$

Optimal. Leaf size=86

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
  :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x^2(3-3x^2+x^4)} dx &= \int \sqrt{3x^2-3x^4+x^6} dx \\
&= \frac{\sqrt{3x^2-3x^4+x^6} \int x\sqrt{3-3x^2+x^4} dx}{x\sqrt{3-3x^2+x^4}} \\
&= \frac{\sqrt{3x^2-3x^4+x^6} \text{Subst}\left(\int \sqrt{3-3x+x^2} dx, x, x^2\right)}{2x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{(3\sqrt{3x^2-3x^4+x^6}) \text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}} dx, x, x^2\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{(\sqrt{3}\sqrt{3x^2-3x^4+x^6}) \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, -3+\frac{x^2}{3}\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 0.81

$$\frac{x \left(4x^6 - 18x^4 + 30x^2 + 3\sqrt{x^4 - 3x^2 + 3} \sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right) - 18 \right)}{16\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)], x]
```

[Out] $(x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*\sqrt{3 - 3*x^2 + x^4})*\text{ArcSinh}[(-3 + 2*x^2)/\sqrt{3}]))/(16*\sqrt{x^2*(3 - 3*x^2 + x^4)})$

IntegrateAlgebraic [A] time = 0.03, size = 73, normalized size = 0.85

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (2x^2 - 3)}{8x} - \frac{3}{16} \log\left(-2x^3 + 2\sqrt{x^6 - 3x^4 + 3x^2} + 3x\right) + \frac{3 \log(x)}{16}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x^2*(3 - 3*x^2 + x^4)], x]

[Out] $((-3 + 2*x^2)*\sqrt{3*x^2 - 3*x^4 + x^6})/(8*x) + (3*\text{Log}[x])/16 - (3*\text{Log}[3*x - 2*x^3 + 2*\sqrt{3*x^2 - 3*x^4 + x^6}])/16$

fricas [A] time = 1.01, size = 70, normalized size = 0.81

$$\frac{12x \log\left(-\frac{2x^3 - 3x - 2\sqrt{x^6 - 3x^4 + 3x^2}}{x}\right) - 8\sqrt{x^6 - 3x^4 + 3x^2}(2x^2 - 3) - 9x}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2*(x^4-3*x^2+3))^(1/2), x, algorithm="fricas")

[Out] $-1/64*(12*x*\log(-(2*x^3 - 3*x - 2*\sqrt{x^6 - 3*x^4 + 3*x^2}))/x) - 8*\sqrt{x^6 - 3*x^4 + 3*x^2}*(2*x^2 - 3) - 9*x)/x$

giac [A] time = 0.51, size = 69, normalized size = 0.80

$$\frac{1}{16} \left(2\sqrt{x^4 - 3x^2 + 3}(2x^2 - 3) - 3 \log\left(-2x^2 + 2\sqrt{x^4 - 3x^2 + 3} + 3\right) \right) \text{sgn}(x) + \frac{3}{16} \left(2\sqrt{3} + \log(2\sqrt{3} + 3) \right) \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2*(x^4-3*x^2+3))^(1/2), x, algorithm="giac")

[Out] $1/16*(2*\sqrt{x^4 - 3*x^2 + 3}*(2*x^2 - 3) - 3*\log(-2*x^2 + 2*\sqrt{x^4 - 3*x^2 + 3} + 3))*\text{sgn}(x) + 3/16*(2*\sqrt{3} + \log(2*\sqrt{3} + 3))*\text{sgn}(x)$

maple [A] time = 0.01, size = 81, normalized size = 0.94

$$\frac{\sqrt{(x^4 - 3x^2 + 3)} x^2 \left(4\sqrt{x^4 - 3x^2 + 3} x^2 + 3 \operatorname{arcsinh}\left(\frac{\sqrt{3}(2x^2 - 3)}{3}\right) - 6\sqrt{x^4 - 3x^2 + 3} \right)}{16\sqrt{x^4 - 3x^2 + 3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^4-3*x^2+3)*x^2)^(1/2),x)`

[Out] $\frac{1}{16} * ((x^4 - 3x^2 + 3) * x^2)^{1/2} * (4 * (x^4 - 3x^2 + 3)^{1/2} * x^2 + 3 * \operatorname{arcsinh}(1/3 * 3^{1/2} * (2 * x^2 - 3)) - 6 * (x^4 - 3x^2 + 3)^{1/2}) / x / (x^4 - 3x^2 + 3)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^4 - 3x^2 + 3)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^2 (x^4 - 3x^2 + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(x^4 - 3*x^2 + 3))^(1/2),x)`

[Out] `int((x^2*(x^4 - 3*x^2 + 3))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2*(x**4-3*x**2+3))**(1/2),x)`

[Out] Timed out

$$3.118 \quad \int \sqrt{1 - (1 - x^2)^3} dx$$

Optimal. Leaf size=86

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - (1 - x^2)^3], x]

[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
  :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1 - (1 - x^2)^3} dx &= \int \sqrt{3x^2 - 3x^4 + x^6} dx \\
&= \frac{\sqrt{3x^2 - 3x^4 + x^6} \int x\sqrt{3 - 3x^2 + x^4} dx}{x\sqrt{3 - 3x^2 + x^4}} \\
&= \frac{\sqrt{3x^2 - 3x^4 + x^6} \operatorname{Subst}\left(\int \sqrt{3 - 3x + x^2} dx, x, x^2\right)}{2x\sqrt{3 - 3x^2 + x^4}} \\
&= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(3\sqrt{3x^2 - 3x^4 + x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{3 - 3x + x^2}} dx, x, x^2\right)}{16x\sqrt{3 - 3x^2 + x^4}} \\
&= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(\sqrt{3}\sqrt{3x^2 - 3x^4 + x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{3}}} dx, x, -3 + 2x^2\right)}{16x\sqrt{3 - 3x^2 + x^4}} \\
&= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} - \frac{3\sqrt{3x^2 - 3x^4 + x^6} \sinh^{-1}\left(\frac{3 - 2x^2}{\sqrt{3}}\right)}{16x\sqrt{3 - 3x^2 + x^4}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 70, normalized size = 0.81

$$\frac{x \left(4x^6 - 18x^4 + 30x^2 + 3\sqrt{x^4 - 3x^2 + 3} \sinh^{-1}\left(\frac{2x^2 - 3}{\sqrt{3}}\right) - 18 \right)}{16\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - (1 - x^2)^3], x]

[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])

IntegrateAlgebraic [A] time = 0.03, size = 73, normalized size = 0.85

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} (2x^2 - 3)}{8x} - \frac{3}{16} \log\left(-2x^3 + 2\sqrt{x^6 - 3x^4 + 3x^2} + 3x\right) + \frac{3 \log(x)}{16}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - (1 - x^2)^3], x]

[Out] ((-3 + 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) + (3*Log[x])/16 - (3*Log[3*x - 2*x^3 + 2*Sqrt[3*x^2 - 3*x^4 + x^6]])/16

fricas [A] time = 0.97, size = 70, normalized size = 0.81

$$\frac{12x \log\left(-\frac{2x^3 - 3x - 2\sqrt{x^6 - 3x^4 + 3x^2}}{x}\right) - 8\sqrt{x^6 - 3x^4 + 3x^2}(2x^2 - 3) - 9x}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(-x^2+1)^3)^(1/2), x, algorithm="fricas")

[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x

giac [A] time = 0.38, size = 69, normalized size = 0.80

$$\frac{1}{16} \left(2\sqrt{x^4 - 3x^2 + 3}(2x^2 - 3) - 3 \log\left(-2x^2 + 2\sqrt{x^4 - 3x^2 + 3} + 3\right) \right) \operatorname{sgn}(x) + \frac{3}{16} \left(2\sqrt{3} + \log(2\sqrt{3} + 3) \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(-x^2+1)^3)^(1/2), x, algorithm="giac")

[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)

maple [A] time = 0.00, size = 81, normalized size = 0.94

$$\frac{\sqrt{x^6 - 3x^4 + 3x^2} \left(4\sqrt{x^4 - 3x^2 + 3} x^2 + 3 \operatorname{arcsinh}\left(\frac{\sqrt{3}(2x^2 - 3)}{3}\right) - 6\sqrt{x^4 - 3x^2 + 3} \right)}{16\sqrt{x^4 - 3x^2 + 3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-(-x^2+1)^3)^(1/2),x)`

[Out] $\frac{1}{16}(x^6-3x^4+3x^2)^{1/2}(4(x^4-3x^2+3)^{1/2}x^2+3\operatorname{arcsinh}(1/3\cdot 3^{1/2})\cdot(2x^2-3))-6(x^4-3x^2+3)^{1/2}}{x(x^4-3x^2+3)^{1/2}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2 - 1)^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt((x^2 - 1)^3 + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(x^2 - 1)^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^2 - 1)^3 + 1)^(1/2),x)`

[Out] `int(((x^2 - 1)^3 + 1)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - (1 - x^2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(-x**2+1)**3)**(1/2),x)`

[Out] `Integral(sqrt(1 - (1 - x**2)**3), x)`

$$3.119 \quad \int \frac{1}{x\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a+bx+cx^2}} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.97

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]/Sqrt[a])

IntegrateAlgebraic [A] time = 0.00, size = 42, normalized size = 1.11

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}} - \frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] (2*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/Sqrt[a]

fricas [A] time = 0.88, size = 111, normalized size = 2.92

$$\left[\frac{\log\left(-\frac{8abx+(b^2+4ac)x^2-4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a))*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]

giac [A] time = 0.39, size = 35, normalized size = 0.92

$$\frac{2 \arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$-\frac{\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\sqrt{a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(1/2),x)

[Out] -1/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 0.08, size = 34, normalized size = 0.89

$$-\frac{\ln\left(\frac{b}{2} + \frac{a}{x} + \frac{\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x + c*x^2)^(1/2)),x)

[Out] -log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x)/a^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)

$$3.120 \quad \int \frac{1}{\sqrt{x^2(a+bx+cx^2)}} dx$$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]

[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1996

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}} dx &= \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}} dx \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.56

$$-\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)], x]

[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))

IntegrateAlgebraic [A] time = 0.08, size = 49, normalized size = 1.09

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{cx^2-\sqrt{ax^2+bx^3+cx^4}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[x^2*(a + b*x + c*x^2)], x]

[Out] (2*ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^2 - Sqrt[a*x^2 + b*x^3 + c*x^4])])/Sqrt[a]

fricas [A] time = 0.96, size = 130, normalized size = 2.89

$$\left[\frac{\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{-a}}{2(acx^3+abx^2+a^2x)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]

giac [A] time = 0.47, size = 59, normalized size = 1.31

$$-\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a} \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")

[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))

maple [A] time = 0.01, size = 64, normalized size = 1.42

$$\frac{\sqrt{cx^2 + bx + a} x \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a} \sqrt{a}}{x}\right)}{\sqrt{(cx^2 + bx + a)x^2} \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c*x^2+b*x+a))^(1/2),x)

[Out] -1/(x^2*(c*x^2+b*x+a))^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx^2 + bx + a)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2 (c x^2 + b x + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)`

[Out] `int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 (a + b x + c x^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2*(c*x**2+b*x+a))**(1/2), x)`

[Out] `Integral(1/sqrt(x**2*(a + b*x + c*x**2)), x)`

$$3.121 \quad \int \frac{1}{\sqrt{x} \sqrt{x(ax+bx^2+cx^2)}} dx$$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1997, 1913, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]

[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x} \sqrt{x(a+bx+cx^2)}} dx &= \int \frac{1}{\sqrt{x} \sqrt{ax+bx^2+cx^3}} dx \\
&= - \left(2 \operatorname{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{\sqrt{x}(2a+bx)}{\sqrt{ax+bx^2+cx^3}} \right) \right) \\
&= - \frac{\tanh^{-1} \left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a} \sqrt{ax+bx^2+cx^3}} \right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.53

$$-\frac{\sqrt{x} \sqrt{a+bx+cx^2} \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a} \sqrt{a+bx+cx^2}} \right)}{\sqrt{a} \sqrt{x(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a+b*x+c*x^2)]),x]

[Out] -((Sqrt[x]*Sqrt[a+b*x+c*x^2]*ArcTanh[(2*a+b*x)/(2*Sqrt[a]*Sqrt[a+b*x+c*x^2])])/(Sqrt[a]*Sqrt[x*(a+x*(b+c*x))]))

IntegrateAlgebraic [A] time = 0.31, size = 53, normalized size = 1.13

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{c} x^{3/2} - \sqrt{ax+bx^2+cx^3}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[x*(a+b*x+c*x^2)]),x]

[Out] (2*ArcTanh[(Sqrt[a]*Sqrt[x])/(Sqrt[c]*x^(3/2) - Sqrt[a*x+b*x^2+c*x^3])])/Sqrt[a]

fricas [A] time = 1.34, size = 131, normalized size = 2.79

$$\left[\frac{\log \left(\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^3+bx^2+ax}(bx+2a)\sqrt{a}\sqrt{x}}{x^3} \right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2a)\sqrt{-a}\sqrt{x}}{2(acx^3+abx^2+a^2x)} \right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")

[Out] [1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]

giac [A] time = 0.49, size = 53, normalized size = 1.13

$$\frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")

[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

maple [A] time = 0.01, size = 64, normalized size = 1.36

$$-\frac{\sqrt{cx^2 + bx + a} \sqrt{x} \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a} \sqrt{a}}{x}\right)}{\sqrt{(cx^2 + bx + a)x} \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x)

[Out] -x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx^2 + bx + a)x} \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x} \sqrt{x(c x^2 + b x + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)),x)`

[Out] `int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)`

[Out] Timed out

$$3.122 \quad \int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}} dx$$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)], x]

[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}} dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^4+cx^5}} dx \\
&= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x^{3/2}(2a+bx)}{\sqrt{ax^3+bx^4+cx^5}}\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.51

$$-\frac{x^{3/2}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)], x]

[Out] -((x^(3/2)*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))]))

IntegrateAlgebraic [A] time = 0.32, size = 55, normalized size = 1.12

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{c}x^{5/2}-\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)], x]

[Out] (2*ArcTanh[(Sqrt[a]*x^(3/2))/(Sqrt[c]*x^(5/2) - Sqrt[a*x^3 + b*x^4 + c*x^5])])/Sqrt[a]

fricas [A] time = 1.15, size = 139, normalized size = 2.84

$$\left[\frac{\log\left(\frac{8abx^3+(b^2+4ac)x^4+8a^2x^2-4\sqrt{cx^5+bx^4+ax^3}(bx+2a)\sqrt{a}\sqrt{x}}{x^4}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}(bx+2a)\sqrt{-a}\sqrt{x}}{2(acx^4+abx^3+a^2x^2)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")

[Out] [1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3 + a^2*x^2))/a]

giac [A] time = 0.49, size = 53, normalized size = 1.08

$$\frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")

[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

maple [A] time = 0.01, size = 66, normalized size = 1.35

$$-\frac{\sqrt{cx^2 + bx + a} x^{\frac{3}{2}} \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a} \sqrt{a}}{x}\right)}{\sqrt{(cx^2 + bx + a)} x^3 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x)

[Out] -1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{(cx^2 + bx + a)} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (cx^2 + bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)`

[Out] `int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2), x)`

[Out] Timed out

$$3.123 \quad \int \frac{1}{x\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1114, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m-1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$-\frac{\tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]

IntegrateAlgebraic [A] time = 0.00, size = 45, normalized size = 1.02

$$\frac{\tanh^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*x^2 + c*x^4]/Sqrt[a]]/Sqrt[a]

fricas [A] time = 1.37, size = 124, normalized size = 2.82

$$\left[\frac{\log \left(-\frac{(b^2+4ac)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a}+8a^2}{x^4} \right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)} \right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]

giac [A] time = 0.45, size = 38, normalized size = 0.86

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)

maple [A] time = 0.01, size = 39, normalized size = 0.89

$$\frac{\ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a}\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)

[Out] -1/2/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?

mupad [B] time = 2.23, size = 44, normalized size = 1.00

$$\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}} - \frac{\ln\left(2a + 2\sqrt{a}\sqrt{cx^4 + bx^2 + a} + bx^2\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)`

[Out] $-\log(1/x^2)/(2*a^{(1/2)}) - \log(2*a + 2*a^{(1/2)}*(a + b*x^2 + c*x^4)^{(1/2)} + b*x^2)/(2*a^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)`

$$3.124 \quad \int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}} dx$$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]

[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/(2*Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1996

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}} dx &= \int \frac{1}{\sqrt{ax^2+bx^4+cx^6}} dx \\
&= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx^2)}{\sqrt{ax^2+bx^4+cx^6}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}} \right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.65

$$-\frac{x\sqrt{a+bx^2+cx^4} \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}\sqrt{x^2(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)], x]

[Out] -1/2*(x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 0.09, size = 48, normalized size = 0.98

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a}x}{\sqrt{c}x^3 - \sqrt{ax^2+bx^4+cx^6}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[x^2*(a + b*x^2 + c*x^4)], x]

[Out] ArcTanh[(Sqrt[a]*x)/(Sqrt[c]*x^3 - Sqrt[a*x^2 + b*x^4 + c*x^6])]/Sqrt[a]

fricas [A] time = 1.32, size = 135, normalized size = 2.76

$$\left[\frac{\log \left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^6+bx^4+ax^2}(bx^2+2a)\sqrt{a}}{x^5} \right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^6+bx^4+ax^2}(bx^2+2a)\sqrt{-a}}{2(acx^5+abx^3+a^2x)} \right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]

giac [A] time = 0.44, size = 62, normalized size = 1.27

$$-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")

[Out] -arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(x))

maple [A] time = 0.01, size = 72, normalized size = 1.47

$$-\frac{\sqrt{cx^4 + bx^2 + a} x \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a} \sqrt{a}}{x^2}\right)}{2\sqrt{(cx^4 + bx^2 + a)} x^2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x)

[Out] -1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2 (c x^4 + b x^2 + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2),x)`

[Out] `int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)`

[Out] Timed out

$$3.125 \quad \int \frac{1}{\sqrt{x} \sqrt{x(a+bx^2+cx^4)}} dx$$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]

[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x} \sqrt{x(a+bx^2+cx^4)}} dx &= \int \frac{1}{\sqrt{x} \sqrt{ax+bx^3+cx^5}} dx \\
&= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}} \right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.63

$$-\frac{\sqrt{x} \sqrt{a+bx^2+cx^4} \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a+b*x^2+c*x^4)]),x]

[Out] -1/2*(Sqrt[x]*Sqrt[a+b*x^2+c*x^4]*ArcTanh[(2*a+b*x^2)/(2*Sqrt[a]*Sqrt[a+b*x^2+c*x^4])])/(Sqrt[a]*Sqrt[x*(a+b*x^2+c*x^4)])

IntegrateAlgebraic [A] time = 0.34, size = 52, normalized size = 1.02

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{c}x^{5/2}-\sqrt{ax+bx^3+cx^5}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[x*(a+b*x^2+c*x^4)]),x]

[Out] ArcTanh[(Sqrt[a]*Sqrt[x])/(Sqrt[c]*x^(5/2)-Sqrt[a*x+bx^3+cx^5])]/Sqrt[a]

fricas [A] time = 1.36, size = 137, normalized size = 2.69

$$\left[\frac{\log \left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5} \right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^5+abx^3+a^2x)} \right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]

giac [A] time = 0.40, size = 56, normalized size = 1.10

$$\frac{\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

maple [A] time = 0.01, size = 72, normalized size = 1.41

$$-\frac{\sqrt{cx^4 + bx^2 + a} \sqrt{x} \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a} \sqrt{a}}{x^2}\right)}{2\sqrt{(cx^4 + bx^2 + a)x} \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/((c*x^4+b*x^2+a)*x)^(1/2),x)

[Out] -1/2*x^(1/2)/((c*x^4+b*x^2+a)*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x} \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x} \sqrt{x (c x^4 + b x^2 + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)),x)`

[Out] `int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)`

[Out] Timed out

$$3.126 \quad \int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx$$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1997, 1913, 206}

$$\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)], x]

[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7])]/(2*Sqrt[a])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^5+cx^7}} dx \\
&= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{x^{3/2}(2a+bx^2)}{\sqrt{ax^3+bx^5+cx^7}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}} \right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 85, normalized size = 1.60

$$\frac{x^{3/2}\sqrt{a+bx^2+cx^4} \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}\sqrt{x^3(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)], x]

[Out] -1/2*(x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])

IntegrateAlgebraic [A] time = 0.35, size = 54, normalized size = 1.02

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a}x^{3/2}}{\sqrt{cx^{7/2}-\sqrt{ax^3+bx^5+cx^7}}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)], x]

[Out] ArcTanh[(Sqrt[a]*x^(3/2))/(Sqrt[c]*x^(7/2) - Sqrt[a*x^3 + b*x^5 + c*x^7])]/Sqrt[a]

fricas [A] time = 1.19, size = 145, normalized size = 2.74

$$\left[\frac{\log \left(-\frac{(b^2+4ac)x^6+8abx^4+8a^2x^2-4\sqrt{cx^7+bx^5+ax^3}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^6} \right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{cx^7+bx^5+ax^3}(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^6+abx^4+a^2x^2)} \right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^7 + b*x^5 + a*x^3))*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^6)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^7 + b*x^5 + a*x^3)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^6 + a*b*x^4 + a^2*x^2))/a]

giac [A] time = 0.55, size = 56, normalized size = 1.06

$$\frac{\arctan\left(-\frac{\sqrt{cx^2-\sqrt{cx^4+bx^2+a}}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

maple [A] time = 0.01, size = 74, normalized size = 1.40

$$\frac{\sqrt{cx^4 + bx^2 + a} x^{\frac{3}{2}} \ln\left(\frac{bx^2 + 2a + 2\sqrt{cx^4 + bx^2 + a} \sqrt{a}}{x^2}\right)}{2\sqrt{(cx^4 + bx^2 + a)} x^3 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x)

[Out] -1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((b*x^2+2*a+2*(c*x^4+b*x^2+a)^(1/2)*a^(1/2))/x^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{(cx^4 + bx^2 + a)}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (c x^4 + b x^2 + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2), x)`

[Out] `int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2), x)`

[Out] Timed out

$$3.127 \quad \int \frac{1}{x\sqrt{3-3x^2+x^4}} dx$$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]

[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{3-3x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{3-3x+x^2}} dx, x, x^2 \right) \\
&= -\text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{3(2-x^2)}{\sqrt{3-3x^2+x^4}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}} \right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{\tanh^{-1} \left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}} \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]

[Out] -1/2*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])]/Sqrt[3]

IntegrateAlgebraic [A] time = 0.00, size = 38, normalized size = 0.95

$$\frac{\tanh^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{x^4-3x^2+3}}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]

[Out] ArcTanh[x^2/Sqrt[3] - Sqrt[3 - 3*x^2 + x^4]/Sqrt[3]]/Sqrt[3]

fricas [A] time = 1.15, size = 47, normalized size = 1.18

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^2 + 2\sqrt{3}(x^2 - 2) + 2\sqrt{x^4 - 3x^2 + 3}(\sqrt{3} + 2) - 6}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{6}\sqrt{3}\log(-3x^2 + 2\sqrt{3}(x^2 - 2) + 2\sqrt{x^4 - 3x^2 + 3})(\sqrt{3} + 2) - 6/x^2)$

giac [A] time = 0.47, size = 55, normalized size = 1.38

$$\frac{1}{6}\sqrt{3}\log\left(x^2 + \sqrt{3} - \sqrt{x^4 - 3x^2 + 3}\right) - \frac{1}{6}\sqrt{3}\log\left(-x^2 + \sqrt{3} + \sqrt{x^4 - 3x^2 + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{6}\sqrt{3}\log(x^2 + \sqrt{3} - \sqrt{x^4 - 3x^2 + 3}) - \frac{1}{6}\sqrt{3}\log(-x^2 + \sqrt{3} + \sqrt{x^4 - 3x^2 + 3})$

maple [A] time = 0.00, size = 31, normalized size = 0.78

$$\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{(-3x^2+6)\sqrt{3}}{6\sqrt{x^4-3x^2+3}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(x^4-3*x^2+3)^(1/2),x)`

[Out] $-1/6*3^{(1/2)}*\operatorname{arctanh}(1/6*(-3*x^2+6)*3^{(1/2)}/(x^4-3*x^2+3)^{(1/2)})$

maxima [A] time = 0.95, size = 20, normalized size = 0.50

$$-\frac{1}{6}\sqrt{3}\operatorname{arsinh}\left(-\sqrt{3} + \frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")`

[Out] $-1/6*\sqrt{3}*\operatorname{arsinh}(-\sqrt{3} + 2*\sqrt{3}/x^2)$

mupad [B] time = 0.43, size = 33, normalized size = 0.82

$$\frac{\sqrt{3}\left(\ln\left(x^2 - \frac{2\sqrt{3}\sqrt{x^4-3x^2+3}}{3} - 2\right) + \ln\left(\frac{1}{x^2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(x^4 - 3*x^2 + 3)^(1/2)),x)`

[Out] $-(3^{1/2} * (\log(x^2 - (2*3^{1/2})*(x^4 - 3*x^2 + 3)^{1/2}))/3 - 2) + \log(1/x^2)))/6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^4 - 3x^2 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(x**4-3*x**2+3)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)`

$$3.128 \quad \int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 1996

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx \\
&= -\text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}} \right) \\
&= -\frac{\tanh^{-1} \left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}} \right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3} \tanh^{-1} \left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}} \right)}{2\sqrt{3}\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)], x]

[Out] -1/2*(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

IntegrateAlgebraic [A] time = 0.00, size = 40, normalized size = 0.89

$$\frac{\tanh^{-1} \left(\frac{\sqrt{3}x}{x^3 - \sqrt{x^6 - 3x^4 + 3x^2}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[x^2*(3 - 3*x^2 + x^4)], x]

[Out] ArcTanh[(Sqrt[3]*x)/(x^3 - Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

fricas [A] time = 1.05, size = 55, normalized size = 1.22

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

giac [A] time = 0.43, size = 60, normalized size = 1.33

$$\frac{\sqrt{3} \log(x^2 + \sqrt{3} - \sqrt{x^4 - 3x^2 + 3}) - \sqrt{3} \log(-x^2 + \sqrt{3} + \sqrt{x^4 - 3x^2 + 3})}{6 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")

[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)

maple [A] time = 0.00, size = 58, normalized size = 1.29

$$\frac{\sqrt{x^4 - 3x^2 + 3} \sqrt{3} x \operatorname{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{(x^4 - 3x^2 + 3)x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^4-3*x^2+3)*x^2)^(1/2),x)

[Out] 1/6/((x^4-3*x^2+3)*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2 (x^4 - 3x^2 + 3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)`

[Out] `int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2(x^4 - 3x^2 + 3)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2*(x**4-3*x**2+3))**(1/2), x)`

[Out] `Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)`

$$3.129 \quad \int \frac{1}{\sqrt{x} \sqrt{x(3-3x+x^2)}} dx$$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]

[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x} \sqrt{x(3-3x+x^2)}} dx &= \int \frac{1}{\sqrt{x} \sqrt{3x-3x^2+x^3}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{12-x^2} dx, x, \frac{(6-3x)\sqrt{x}}{\sqrt{3x-3x^2+x^3}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.44

$$\frac{\sqrt{x} \sqrt{x^2 - 3x + 3} \tanh^{-1}\left(\frac{\sqrt{3}(x-2)}{2\sqrt{x^2-3x+3}}\right)}{\sqrt{3} \sqrt{x(x^2 - 3x + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]), x]

[Out] (Sqrt[x]*Sqrt[3 - 3*x + x^2]*ArcTanh[(Sqrt[3]*(-2 + x))/(2*Sqrt[3 - 3*x + x^2])])/(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])

IntegrateAlgebraic [A] time = 0.31, size = 45, normalized size = 1.05

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3} \sqrt{x}}{x^{3/2} - \sqrt{x^3 - 3x^2 + 3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]), x]

[Out] (2*ArcTanh[(Sqrt[3]*Sqrt[x])/(x^(3/2) - Sqrt[3*x - 3*x^2 + x^3])])/Sqrt[3]

fricas [A] time = 1.16, size = 49, normalized size = 1.14

$$\frac{1}{6} \sqrt{3} \log\left(\frac{7x^3 + 4\sqrt{3}\sqrt{x^3 - 3x^2 + 3x}(x-2)\sqrt{x} - 24x^2 + 24x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2), x, algorithm="fricas")

[Out] $\frac{1}{6}\sqrt{3}\log((7x^3 + 4\sqrt{3})\sqrt{x^3 - 3x^2 + 3x})(x - 2)\sqrt{x} - 24x^2 + 24x)/x^3)$

giac [A] time = 0.33, size = 47, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(x + \sqrt{3} - \sqrt{x^2 - 3x + 3}\right) - \frac{1}{3}\sqrt{3}\log\left(-x + \sqrt{3} + \sqrt{x^2 - 3x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{3}\sqrt{3}\log(x + \sqrt{3} - \sqrt{x^2 - 3x + 3}) - \frac{1}{3}\sqrt{3}\log(-x + \sqrt{3} + \sqrt{x^2 - 3x + 3})$

maple [A] time = 0.01, size = 50, normalized size = 1.16

$$\frac{\sqrt{x^2 - 3x + 3} \sqrt{3} \sqrt{x} \operatorname{arctanh}\left(\frac{(x-2)\sqrt{3}}{2\sqrt{x^2-3x+3}}\right)}{3\sqrt{(x^2 - 3x + 3)}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x)`

[Out] $\frac{1}{3}x^{1/2}/(x*(x^2-3*x+3))^{1/2}*(x^2-3*x+3)^{1/2}*3^{1/2}*\operatorname{arctanh}(1/2*(x-2)*3^{1/2}/(x^2-3*x+3)^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2 - 3x + 3)}x\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x} \sqrt{x} (x^2 - 3x + 3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

```
[Out] Timed out
```


$$3.130 \quad \int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}} dx$$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1913, 206}

$$\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]

[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n - q) + a*x^q])]/(Sqrt[a]*(n - q)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rubi steps

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}} dx = -\frac{2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x^{q/2}(2a+bx^{n-q})}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\right)}{n-q}$$

$$= -\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n + cx^{2n-q} + ax^q}}\right)}{\sqrt{a}(n-q)}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}} dx$$

Verification is not applicable to the result.

[In] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]

[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]

[Out] Defer[IntegrateAlgebraic][x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*q)/(b*xⁿ+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac")

[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*xⁿ + a*x^q), x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{q}{2}-1}}{\sqrt{ax^q + bx^n + cx^{2n-q}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/2*q)/(b*xⁿ+c*x^(2*n-q)+a*x^q)^(1/2),x)

[Out] int(x^(-1+1/2*q)/(b*xⁿ+c*x^(2*n-q)+a*x^q)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/2*q)/(b*xⁿ+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*xⁿ + a*x^q), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{q}{2}-1}}{\sqrt{bx^n + ax^q + cx^{2n-q}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(q/2 - 1)/(b*xⁿ + a*x^q + c*x^(2*n - q))^(1/2),x)

[Out] int(x^(q/2 - 1)/(b*xⁿ + a*x^q + c*x^(2*n - q))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)

[Out] Timed out

Chapter 4

Appendix

Local contents

4.1	Download section	718
4.2	Listing of Grading functions	718

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
```

```

If[ExpnType[result]<=ExpnType[optimal],
  If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
    If[LeafCount[result]<=2*LeafCount[optimal],
      "A",
      "B"],
    "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],

```

```

    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```



```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```



```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```